Solutions

1 Number and language

Exercise 1.1 page 5

Prime numbers are:

1. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Exercise 1.2 page 6

1. **a)** 1, 2, 3, 6
b) 1, 3, 9
c) 1, 7
d) 1, 3, 5, 15
e) 1, 2, 3, 4, 6, 8, 12, 24
f) 1, 2, 3, 4, 6, 9, 12, 18, 36
g) 1, 5, 7, 35
h) 1, 5, 25
i) 1, 2, 3, 6, 7, 14, 21, 42
j) 1, 2, 4, 5, 10, 20, 25, 50, 100

Exercise 1.3 page 6

1. **a)** 3, 5 **b)** 2, 3 **c)** 2, 3 **d)** 2
e) 2, 5 **f)** 13 **g)** 3, 11 **h)** 5, 7
i) 2, 5, 7 **j)** 2, 7

Exercise 1.4 page 7

1. **a)** $2^2 \times 3$ **b)** 2^5 **c)** $2^2 \times 3^2$
d) $2^3 \times 5$ **e)** $2^2 \times 11$ **f)** $2^3 \times 7$
g) $3^2 \times 5$ **h)** 3×13 **i)** $3 \times 7 \times 11$
j) $3^2 \times 7$

Exercise 1.5 page 7

1. **a)** 4 **b)** 5 **c)** 6 **d)** 3 **e)** 9
f) 22 **g)** 8 **h)** 13 **i)** 17 **j)** 12

2. **a)** 42 **b)** 60 **c)** 70 **d)** 90 **e)** 120
f) 105 **g)** 20 **h)** 231 **i)** 240 **j)** 200

Exercise 1.6 page 8

1. **a)** Rational **b)** Rational **c)** Irrational
d) Rational **e)** Rational **f)** Rational
g) Irrational **h)** Rational **i)** Rational

2. **a)** Irrational **b)** Irrational **c)** Rational
d) Rational **e)** Rational **f)** Rational

3. **a)** Rational **b)** Irrational **c)** Rational
d) Rational

Exercise 1.7 page 9

1. **a)** 5 **b)** 3 **c)** 7 **d)** 10 **e)** 11
f) 13 **g)** 0.1 **h)** 0.2 **i)** 0.3 **j)** 0.5

2. Students check their answers

3. **a)** $\frac{1}{3}$ **b)** $\frac{1}{4}$ **c)** $\frac{1}{5}$ **d)** $\frac{1}{7}$ **e)** $\frac{1}{10}$
f) $\frac{2}{3}$ **g)** $\frac{3}{10}$ **h)** $\frac{7}{9}$ **i)** $\frac{5}{3}$ **j)** $\frac{5}{2}$

Exercise 1.8 page 9

1. The following answers are correct to 1 d.p.:
a) 5.9 **b)** 6.7 **c)** 7.4 **d)** 7.7 **e)** 1.4

2. Students check their answers

Exercise 1.9 page 10

1. **a)** 2 **b)** 5 **c)** 3 **d)** 0.1 **e)** 0.3
f) 6 **g)** 10 **h)** 100 **i)** –2 **j)** –3
k) –10 **l)** –1

Exercise 1.10 page 10

1. 146 °C

2. **a)** −$35 **b)** −$318 **c)** −$88
d) −$160 **e)** $90

3. 165 m

4. 695 m

Student assessment 1 page 11

1. **a)** Rational **b)** Irrational **c)** Rational
d) Rational **e)** Rational **f)** Irrational

2. **a)** $\frac{5}{8}$ **b)** 3 **c)** $\frac{11}{25}$

3. **a)** 81 **b)** 225 **c)** 0.04
d) 0.49

4. **a)** 12.25 **b)** 16.81 **c)** 0.0225

5. **a)** 15 **b)** 0.1 **c)** 0.9
d) $\frac{3}{5}$ **e)** $\frac{7}{3}$ **f)** $\frac{11}{7}$

6. **a)** 64 **b)** 0.001 **c)** $\frac{8}{27}$

7. **a)** 3 **b)** 100 **c)** $\frac{4}{5}$

8. **a)** −\$84 **b)** \$91 **c)** \$45
d) \$74 **e)** −\$43 **f)** −\$15

2 Accuracy

Exercise 2.1 page 12

1. **a)** 69 000 **b)** 74 000 **c)** 89 000
d) 4000 **e)** 100 000 **f)** 1 000 000

2. **a)** 78 500 **b)** 6900 **c)** 14 100
d) 8100 **e)** 1000 **f)** 3000

3. **a)** 490 **b)** 690 **c)** 8850
d) 80 **e)** 0 **f)** 1000

Exercise 2.2 page 13

1. **a)** 5.6 **b)** 0.7 **c)** 11.9
d) 157.4 **e)** 4.0 **f)** 15.0
g) 3.0 **h)** 1.0 **i)** 12.0

2. **a)** 6.47 **b)** 9.59 **c)** 16.48
d) 0.09 **e)** 0.01 **f)** 9.30
g) 100.00 **h)** 0.00 **i)** 3.00

Exercise 2.3 page 13

1. **a)** 50 000 **b)** 48 600 **c)** 7000
d) 7500 **e)** 500 **f)** 2.57
g) 1000 **h)** 2000 **i)** 15.0

2. **a)** 0.09 **b)** 0.6 **c)** 0.94
d) 1 **e)** 0.95 **f)** 0.003
g) 0.0031 **h)** 0.0097 **i)** 0.01

Exercise 2.4 page 14

1. **a)** 419.6 **b)** 5.0 **c)** 166.3
d) 23.8 **e)** 57.8 **f)** 4427.1
g) 1.9 **h)** 4.1 **i)** 0.6

Answers to Q.2–4 may vary slightly from those given below:

2. **a)** 1200 **b)** 3000 **c)** 3000
d) 150 000 **e)** 0.8 **f)** 100

3. **a)** 200 **b)** 200 **c)** 30
d) 550 **e)** 500 **f)** 3000

4. **a)** 130 **b)** 80 **c)** 9
d) 4 **e)** 200 **f)** 250

5. **c)**, **e)** and **f)** are incorrect.

Answers to Q.6 and 7 may vary slightly from those given below:

6. **a)** 120 m^2 **b)** 40 m^2 **c)** 400 cm^2

7. **a)** 200 cm^3 **b)** 4000 cm^3 **c)** 2000 cm^3

Exercise 2.5 page 17

1. **a)** i) Lower bound = 5.5
Upper bound = 6.5
ii) $5.5 \leqslant x < 6.5$
b) i) Lower bound = 82.5
Upper bound = 83.5
ii) $82.5 \leqslant x < 83.5$
c) i) Lower bound = 151.5
Upper bound = 152.5
ii) $151.5 \leqslant x < 152.5$
d) i) Lower bound = 999.5
Upper bound = 1000.5
ii) $999.5 \leqslant x < 1000.5$
e) i) Lower bound = 99.5
Upper bound = 100.5
ii) $99.5 \leqslant x < 100.5$

2. **a)** i) Lower bound = 3.75
Upper bound = 3.85
ii) $3.75 \leqslant x < 3.85$
b) i) Lower bound = 15.55
Upper bound = 15.65
ii) $15.55 \leqslant x < 15.65$
c) i) Lower bound = 0.95
Upper bound = 1.05
ii) $0.95 \leqslant x < 1.05$
d) i) Lower bound = 9.95
Upper bound = 10.05
ii) $9.95 \leqslant x < 10.05$
e) i) Lower bound = 0.25
Upper bound = 0.35
ii) $0.25 \leqslant x < 0.35$

3. **a)** i) Lower bound = 4.15
Upper bound = 4.25
ii) $4.15 \leqslant x < 4.25$
b) i) Lower bound = 0.835
Upper bound = 0.845
ii) $0.835 \leqslant x < 0.845$
c) i) Lower bound = 415
Upper bound = 425
ii) $415 \leqslant x < 425$

d) i) Lower bound = 4950
Upper bound = 5050
ii) $4950 \leqslant x < 5050$

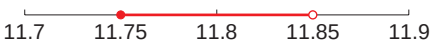
e) i) Lower bound = 0.0445
Upper bound = 0.0455
ii) $0.0445 \leqslant x < 0.0455$

f) i) Lower bound = 24 500
Upper bound = 25 500
ii) $24\,500 \leqslant x < 25\,500$

4. **a)**

b) $5.35 \leqslant M < 5.45$

5. **a)**



b) $11.75 \leqslant T < 11.85$

6. **a)** Lower bound = 615 m^3
Upper bound = 625 m^3
b) $615 \leqslant x < 625$

7. **a)** Lower bound = 625 m
Upper bound = 635 m
b) $395 \leqslant W < 405$

Exercise 2.6 page 18

1. **a)** Lower bound = 263.25
Upper bound = 297.25
b) Lower bound = 3295.25
Upper bound = 3455.25
c) Lower bound = 4925.25
Upper bound = 5075.25
d) Lower bound = 3.76 (2 d.p.)
Upper bound = 4.26 (2 d.p.)
e) Lower bound = 2.83 (2 d.p.)
Upper bound = 3.19 (2 d.p.)
f) Lower bound = 8.03 (2 d.p.)
Upper bound = 8.66 (2 d.p.)
g) Lower bound = 44.95 (2 d.p.)
Upper bound = 52.82 (2 d.p.)
h) Lower bound = 39.77 (2 d.p.)
Upper bound = 42.23 (2 d.p.)
i) Lower bound = 16.14 (2 d.p.)
Upper bound = 18.88 (2 d.p.)
j) Lower bound = 3.55 (2 d.p.)
Upper bound = 7.12
k) Lower bound = 1.47 (2 d.p.)
Upper bound = 1.63 (2 d.p.)
l) Lower bound = 18.51 (2 d.p.)
Upper bound = 28.59 (2 d.p.)

2. **a)** Lower bound = 6.7
Upper bound = 6.9
b) Lower bound = 29.69 (2 d.p.)
Upper bound = 30.80 (2 d.p.)
c) Lower bound = 147.76 (2 d.p.)
Upper bound = 150.25 (2 d.p.)
d) Lower bound = 13.3
Upper bound = 13.5
e) Lower bound = 1.75 (2 d.p.)
Upper bound = 1.81 (2 d.p.)
f) Lower bound = 0.39 (2 d.p.)
Upper bound = 0.46 (2 d.p.)
g) Lower bound = 34.10 (2 d.p.)
Upper bound = 40.03 (2 d.p.)
h) Lower bound = 0.98 (2 d.p.)
Upper bound = 1.02 (2 d.p.)
i) Lower bound = 0
Upper bound = 0.04

3. **a)** Lower bound = 20 002.5
Upper bound = 20 962.5
b) Lower bound = 1.06 (2 d.p.)
Upper bound = 1.15 (2 d.p.)
c) Lower bound = 1 116 250
Upper bound = 1 188 250
d) Lower bound = 88.43 (2 d.p.)
Upper bound = 91.60 (2 d.p.)
e) Lower bound = 131.75 (2 d.p.)
Upper bound = 139.34 (2 d.p.)
f) Lower bound = 18.10 (2 d.p.)
Upper bound = 20.39 (2 d.p.)
g) Lower bound = 2.24 (2 d.p.)
Upper bound = 2.53 (2 d.p.)
h) Lower bound = 3.45 (2 d.p.)
Upper bound = 3.97 (2 d.p.)
i) Lower bound = 60.34 (2 d.p.)
Upper bound = 68.52 (2 d.p.)

Exercise 2.7 page 19

1. Lower bound = 3.5 kg
Upper bound = 4.5 kg

2. Lower bound = 21.8 cm
Upper bound = 22.2 cm

3. Lower bound = 13.7 m
Upper bound = 13.74 m

4. Lower bound = 74.13 cm^2 (2 d.p.)
Upper bound = 75.88 cm^2 (2 d.p.)

5. Lower bound = 68 425 m^2
Upper bound = 75 625 m^2

6. Lower bound = 13.1 (1 d.p.)
Upper bound = 13.5 (1 d.p.)

7. Lower bound = 9.0 cm (1 d.p.)
Upper bound = 11.1 cm (1 d.p.)

8. **a)** Lower bound = 53.1 cm (1 d.p.)
Upper bound = 53.7 cm (1 d.p.)
b) Lower bound = 224.3 cm^2 (1 d.p.)
Upper bound = 229.7 cm^2 (1 d.p.)

9. **a)** Lower bound = 11.2 cm (1 d.p.)
Upper bound = 11.4 cm (1 d.p.)
b) Lower bound = 70.5 cm (1 d.p.)
Upper bound = 71.3 cm (1 d.p.)

10. Lower bound = 11.5 g/cm^3 (1 d.p.)
Upper bound = 13.6 g/cm^3 (1 d.p.)

11. Least = 93.3 h (1 d.p.)
Greatest = 138.5 h (1 d.p.)

Student assessment 1 page 21

1. **a)** 2800 **b)** 7290 **c)** 49 000
d) 1000

2. **a)** 3.8 **b)** 6.8 **c)** 0.85
d) 1.58 **e)** 10.0 **f)** 0.008

3. **a)** 4 **b)** 6.8 **c)** 0.8
d) 10 **e)** 830 **f)** 0.005

Answers to Q.4–6 may vary from those given below:

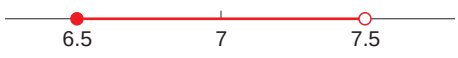
4. 18 000 yards

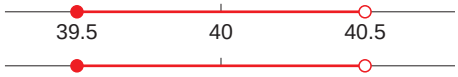
5. 40 m^2

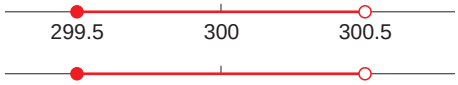
6. **a)** 25 **b)** 4 **c)** 4

7. 92.3 cm^3 (1 d.p.)

Student assessment 2 page 22

1. **a)**


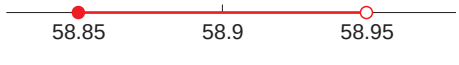
b)


c)


d)


2. **a)** $205 \leqslant x < 215$ **b)** $63.5 \leqslant x < 64.5$
c) $2.95 \leqslant x < 3.05$ **d)** $0.875 \leqslant x < 0.885$

3. Length: $349.5 \leqslant L < 350.5$
Width: $199.5 \leqslant W < 200.5$

4.


5. $0.0035 \leqslant x < 0.0045$

6. **a)** $4.825 \leqslant x < 4.835$
b) $5.045 \leqslant y < 5.055$
c) $9.95 \leqslant z < 10.05$
d) $99.995 \leqslant p < 100.005$

Student assessment 3 page 22

1. **a)** Lower bound = 965.25
Upper bound = 1035.25
b) Lower bound = 6218.75
Upper bound = 6381.75
c) Lower bound = 6.2 (1 d.p.)
Upper bound = 6.3 (1 d.p.)
d) Lower bound = 47.7 (1 d.p.)
Upper bound = 54.7 (1 d.p.)
e) Lower bound = 0.8 (1 d.p.)
Upper bound = 1.2 (1 d.p.)
f) Lower bound = 0.5 (1 d.p.)
Upper bound = 0.9 (1 d.p.)

2. Lower bound = 118.7 cm^2 (1 d.p.)
Upper bound = 121.1 cm^2 (1 d.p.)

3. Lower bound = 10.5 cm
Upper bound = 13.5 cm

4. $0.8 \leqslant x < 1$

5. Lower bound = 0.46 kg (2 d.p.)
Upper bound = 0.48 kg (2 d.p.)

Student assessment 4 page 23

1. 255 kg

2. 31 575 cm^3 (5 s.f.)

3. **a)** $124.5 \leqslant V < 125.5$ cm^3
b) $4.99 \leqslant L < 5.01$ cm (2 d.p.)

4. **a)** $25.10 \leqslant C < 25.16$ (2 d.p.)
b) $50.14 \leqslant A < 50.39$ cm^2 (2 d.p.)

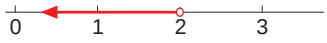
5. 22 cups

6. **a)** Lower bound = 4.5 cm
Upper bound = 5.5 cm
b) Upper and lower bounds of 100 matches ÷ 100
Lower bound = 5.43 cm
Upper bound = 5.44 cm

3 Calculations and order

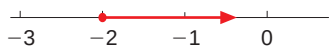
Exercise 3.1 page 25

1. **a)** < **b)** = **c)** > **d)** < **e)** = **f)** >

2. **a)**


b)


c)


d)


e)


f)


g)


h)


3. **a)** $x > 0$ **b)** $x \leqslant 3$ **c)** $0 \leqslant x \leqslant 4$
d) $-4 < x \leqslant -1$

4. **a)** $x \leqslant 20\,000$ **b)** $135 \leqslant x \leqslant 180$
c) $5x + 3 < 20$ **d)** $x \leqslant 25$
e) $350 \leqslant x \leqslant 400$ **f)** $11 \leqslant x \leqslant 28$

Exercise 3.2 page 26

1. 0.06 0.6 0.606 0.66 6.0 6.6 6.606

2. $\frac{4}{5}$ $\frac{1}{2}$ $\frac{6}{13}$ $\frac{7}{18}$ $\frac{1}{3}$ $\frac{2}{19}$

3. 60 cm 0.75 m 800 mm 180 cm 2 m

4. 4 kg 3500 g 1 kg $\frac{3}{4}$kg 700 g

5. 150 cm^3 430 ml 800 cm^3 1litre 120 cl

Exercise 3.3 page 27

1. **a)** 26 **b)** 10 **c)** 42 **d)** 16 **e)** 8 **f)** −6

2. **a)** 20 **b)** 34 **c)** 32 **d)** 31 **e)** 20 **f)** 23

3. **a)** 27 **b)** 64 **c)** 30 **d)** 3 **e)** 144 **f)** 1.6

Exercise 3.4 page 27

1. **a)** $6 \times (2 + 1) = 18$
b) $1 + 3 \times 5 = 16$
c) $(8 + 6) \div 2 = 7$
d) $(9 + 2) \times 4 = 44$
e) $9 \div 3 \times 4 + 1 = 13$
f) $(3 + 2) \times (4 - 1) = 15$

2. **a)** $12 \div 4 - 2 + 6 = 7$
b) $12 \div (4 - 2) + 6 = 12$
c) $12 \div 4 - (2 + 6) = -5$
d) $12 \div (4 - 2 + 6) = 1.5$
e) $4 + 5 \times 6 - 1 = 33$
f) $4 + 5 \times (6 - 1) = 29$
g) $(4 + 5) \times 6 - 1 = 53$
h) $(4 + 5) \times (6 - 1) = 45$

Exercise 3.5 page 28

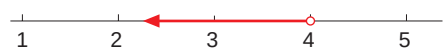
1. **a)** 2 **b)** 3 **c)** 7 **d)** 4 **e)** 23 **f)** 0

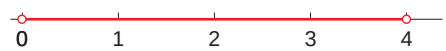
2. **a)** 1 **b)** 5 **c)** 2 **d)** 50 **e)** 7
f) −1.5

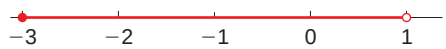
Student assessment 1 page 28

1. **a)** $x \geqslant -1$ **b)** $x < 2$
c) $-2 \leqslant x < 2$ **d)** $-1 \leqslant x \leqslant 1$

2. **a)**


b)


c)


d)


3. $\frac{3}{14}$ $\frac{2}{5}$ $\frac{1}{2}$ $\frac{4}{7}$ $\frac{9}{10}$

Student assessment 2 page 29

1. **a)** 44 **b)** 13 **c)** 25 **d)** 19
e) 49 **f)** 3

2. **a)** $(7 - 4) \times 2 = 6$
b) $12 + 3 \times (3 + 4) = 33$
c) $(5 + 5) \times (6 - 4) = 20$
d) $(5 + 5) \times 6 - 4 = 56$

3. **a)** 3.5 **b)** 10

Student assessment 3 page 29

1. **a)** 20 **b)** 15 **c)** 11 **d)** 13 **e)** 1 **f)** 4

2. **a)** $(7 - 5) \times 3 = 6$
b) $16 + 4 \times (2 + 4) = 40$
c) $(4 + 5) \times (6 - 1) = 45$
d) $(1 + 5) \times 6 - 6 = 30$

3. **a)** 5.5 **b)** 9

4 Integers, fractions, decimals and percentages

Exercise 4.1 page 30

1. **a)** 9 **b)** 16 **c)** 20 **d)** 40 **e)** 18
f) 72 **g)** 30 **h)** 48 **i)** 210 **j)** 52

Exercise 4.2 page 31

1. **a)** $\frac{14}{3}$ **b)** $\frac{18}{5}$ **c)** $\frac{47}{8}$ **d)** $\frac{17}{6}$ **e)** $\frac{17}{2}$
f) $\frac{68}{7}$ **g)** $\frac{58}{9}$ **h)** $\frac{17}{4}$ **i)** $\frac{59}{11}$ **j)** $\frac{55}{7}$
k) $\frac{43}{10}$ **l)** $\frac{146}{13}$

2. **a)** $7\frac{1}{4}$ **b)** $6\frac{3}{5}$ **c)** $6\frac{5}{6}$ **d)** $6\frac{5}{8}$ **e)** $5\frac{4}{9}$
f) $1\frac{5}{12}$ **g)** $9\frac{3}{7}$ **h)** $3\frac{3}{10}$ **i)** $9\frac{1}{2}$ **j)** $6\frac{1}{12}$

Exercise 4.3 page 31

1. **a)** 4.5 **b)** 6.3 **c)** 17.8 **d)** 3.07
e) 9.27 **f)** 11.36 **g)** 4.006 **h)** 5.027
i) 4.356 **j)** 9.204

2. **a)** 19.14 **b)** 83.812 **c)** 6.6 **d)** 11.16
e) 35.81 **f)** 5.32 **g)** 67.14 **h)** 6.06
i) 1.4 **j)** 0.175

Exercise 4.4 page 32

1. **a)** $\frac{29}{50} = \frac{58}{100} = 58\%$ **b)** $\frac{17}{25} = \frac{68}{100} = 68\%$
c) $\frac{11}{20} = \frac{55}{100} = 55\%$ **d)** $\frac{3}{10} = \frac{30}{100} = 30\%$
e) $\frac{23}{25} = \frac{92}{100} = 92\%$ **f)** $\frac{19}{50} = \frac{38}{100} = 38\%$
g) $\frac{3}{4} = \frac{75}{100} = 75\%$ **h)** $\frac{2}{5} = \frac{40}{100} = 40\%$

2.

Fraction	Decimal	Percentage
$\frac{1}{10}$	0.1	10
$\frac{1}{5}$	0.2	20
$\frac{3}{10}$	0.3	30
$\frac{4}{10} = \frac{2}{5}$	0.4	40
$\frac{1}{2}$	0.5	50
$\frac{3}{5}$	0.6	60
$\frac{7}{10}$	0.7	70
$\frac{4}{5}$	0.8	80
$\frac{9}{10}$	0.9	90
$\frac{1}{4}$	0.25	25
$\frac{3}{4}$	0.75	75

Exercise 4.5 page 34

1. **a)** 9 **b)** 3

2. **a)** $6 \times (4 + 6) \div 3 = 20$
b) $(9 - 3) \times (7 + 2) = 54$

3. **a)** 29 830 **b)** 41 492

4. **a)** 2 **b)** 7

5. **a)** 224 **b)** 28

6. **a)** 1127.4 **b)** 526.1

Exercise 4.6 page 35

1. **a)** $\frac{1}{2}$ **b)** $\frac{1}{3}$ **c)** $\frac{2}{3}$ **d)** $\frac{4}{9}$ **e)** $\frac{3}{4}$ **f)** $\frac{9}{10}$

Exercise 4.7 page 35

1. **a)** $1\frac{2}{5}$ **b)** $\frac{10}{11}$ **c)** $\frac{11}{12}$ **d)** $1\frac{2}{45}$ **e)** $1\frac{1}{65}$ **f)** $1\frac{11}{12}$

2. **a)** $1\frac{1}{8}$ **b)** $1\frac{5}{7}$ **c)** $1\frac{1}{12}$ **d)** $\frac{47}{60}$ **e)** $1\frac{29}{40}$ **f)** $\frac{51}{52}$

3. **a)** $\frac{1}{7}$ **b)** $\frac{1}{10}$ **c)** $\frac{5}{9}$ **d)** $\frac{1}{12}$ **e)** $\frac{9}{40}$ **f)** $1\frac{1}{20}$

4. **a)** $\frac{17}{60}$ **b)** $\frac{45}{88}$ **c)** $\frac{17}{20}$ **d)** $\frac{44}{195}$ **e)** $\frac{9}{20}$ **f)** $\frac{1}{18}$

5. **a)** $5\frac{3}{4}$ **b)** $5\frac{3}{10}$ **c)** $3\frac{1}{10}$ **d)** $6\frac{7}{24}$ **e)** $1\frac{1}{8}$ **f)** $\frac{25}{36}$

6. **a)** $5\frac{1}{8}$ **b)** $7\frac{9}{40}$ **c)** $-\frac{3}{8}$ **d)** $\frac{7}{20}$ **e)** $-2\frac{39}{140}$ **f)** $1\frac{1}{4}$

Exercise 4.8 page 37

1. **a)** 8 **b)** $\frac{12}{7}$ **c)** $\frac{5}{3}$ **d)** $\frac{2}{3}$ **e)** $\frac{4}{15}$ **f)** $\frac{1}{6}$

2. **a)** $\frac{1}{6}$ **b)** $\frac{3}{5}$ **c)** $\frac{4}{21}$ **d)** $\frac{2}{3}$ **e)** $\frac{1}{4}$ **f)** $\frac{7}{20}$

3. **a)** $\frac{5}{6}$ **b)** $2\frac{1}{2}$ **c)** $1\frac{1}{7}$ **d)** $4\frac{1}{6}$ **e)** $\frac{1}{5}$ **f)** $\frac{2}{3}$

4. **a)** $\frac{3}{5}$ **b)** $\frac{7}{12}$ **c)** $\frac{9}{70}$ **d)** $\frac{21}{25}$ **e)** $\frac{3}{8}$ **f)** $1\frac{25}{56}$

5. **a)** $\frac{3}{5}$ **b)** $3\frac{107}{120}$ **c)** $\frac{8}{15}$ **d)** $12\frac{1}{4}$

Exercise 4.9 page 37

1. **a)** 0.75 **b)** 0.8 **c)** 0.45 **d)** 0.34
e) $0.\dot{3}$ **f)** 0.375 **g)** 0.4375 **h)** $0.\dot{2}$
i) $0.\dot{6}\dot{3}$

2. **a)** 2.75 **b)** 3.6 **c)** 4.35 **d)** 6.22
e) $5.\dot{6}$ **f)** 6.875 **g)** 5.5625 **h)** $4.\dot{2}$
i) $5.\dot{4}28571\dot{4}$

Exercise 4.10 page 38

1. **a)** $\frac{1}{2}$ **b)** $\frac{7}{10}$ **c)** $\frac{3}{5}$ **d)** $\frac{3}{4}$ **e)** $\frac{33}{40}$
f) $\frac{1}{20}$ **g)** $\frac{1}{20}$ **h)** $\frac{201}{500}$ **i)** $\frac{1}{5000}$

2. **a)** $2\frac{2}{5}$ **b)** $6\frac{1}{2}$ **c)** $8\frac{1}{5}$ **d)** $3\frac{3}{4}$ **e)** $10\frac{11}{20}$
f) $9\frac{51}{250}$ **g)** $15\frac{91}{200}$ **h)** $30\frac{1}{1000}$ **i)** $1\frac{41}{2000}$

Exercise 4.11 page 40

1. **a)** $\frac{1}{3}$ **b)** $\frac{7}{9}$ **c)** $\frac{42}{99} = \frac{14}{33}$ **d)** $\frac{65}{99}$

2. **a)** $\frac{55}{990} = \frac{1}{18}$ **b)** $\frac{62}{990} = \frac{31}{495}$
c) $\frac{92}{90} = 1\frac{1}{45}$ **d)** $\frac{39\,638}{9900} = 4\frac{19}{4950}$

3. $\frac{1}{5}$

4. $\frac{1}{6}$

Student assessment 1 page 41

1. **a)** 12 **b)** 33 **c)** 6 **d)** 90

2. **a)** 30% **b)** 29% **c)** 50% **d)** 70%
e) 80% **f)** 219% **g)** 6% **h)** 75%
i) 31% **j)** 7% **k)** 340% **l)** 200%

3. **a)** 23 **b)** 18

4. 22 977

5. 360.2

6. **a)** $2\frac{1}{16}$ **b)** 9

7. **a)** 0.4 **b)** 1.75 **c)** $0.\dot{8}\dot{1}$ **d)** $1.6\dot{}$

8. **a)** $4\frac{1}{5}$ **b)** $\frac{3}{50}$ **c)** $1\frac{17}{20}$ **d)** $2\frac{1}{200}$

9. **a)** $\frac{37}{99}$ **b)** $\frac{8}{99}$ **c)** $1\frac{19}{90}$

10. $\frac{650}{900} = \frac{13}{18}$

Student assessment 2 page 42

1. **a)** 21 **b)** 27 **c)** 22 **d)** 39

2. **a)** 60% **b)** 49% **c)** 25% **d)** 90%
e) 150% **f)** 327% **g)** 5% **h)** 35%
i) 77% **j)** 3% **k)** 290% **l)** 400%

3. **a)** 0 **b)** 19

4. 18 032

5. 340.7

6. **a)** $1\frac{7}{10}$ **b)** 2

7. **a)** 0.875 **b)** 1.4 **c)** $0.\dot{8}$ **d)** $3.\dot{2}85714\dot{}$

8. **a)** $6\frac{1}{2}$ **b)** $\frac{1}{25}$ **c)** $3\frac{13}{20}$ **d)** $3\frac{1}{125}$

9. **a)** $\frac{7}{99}$ **b)** $\frac{1}{1000}$ **c)** $3\frac{2}{99}$

10. $\frac{11}{50}$

5 Further percentages

Exercise 5.1 page 44

1. **a)** 60% **b)** 40%

2. **a)** 87.5% **b)** 73.3%
c) 29.16% **d)** 14.29% (2 d.p.)

3. **a)** 0.39 **b)** 0.47 **c)** 0.83 **d)** 0.07
e) 0.02 **f)** 0.2

4. **a)** 31% **b)** 67% **c)** 9% **d)** 5%
e) 20% **f)** 75%

Exercise 5.2 page 44

1. **a)** 25% **b)** $66.\dot{6}\%$ **c)** 62.5%
d) 180% **e)** 490% **f)** 387.5%

2. **a)** 0.75 **b)** 0.8 **c)** 0.2
d) 0.07 **e)** 1.875 **f)** $0.1\dot{6}$

3. **a)** 20 **b)** 100 **c)** 50
d) 36 **e)** 4.5 **f)** 7.5

4. **a)** 8.5 **b)** 8.5 **c)** 52
d) 52 **e)** 17.5 **f)** 17.5

5. **a)** Black 6 **b)** Blonde 3 **c)** Brown 21

6. Lamb 66 Chicken 24 Turkey 12 Duck 18

7. Australian 143 Pakistani 44 Greek 11 Other 22

8. Newspapers 69 Pens 36 Books 18 Other 27

Exercise 5.3 page 45

1. **a)** 48% **b)** 36.8% **c)** 35%
d) 50% **e)** 45% **f)** 40%
g) $33\frac{1}{3}\%$ **h)** 57% (2 s.f.)

2. Win 50% Lose $33\frac{1}{3}\%$ Draw $16\frac{2}{3}\%$

3. A = 34.5% (1 d.p.) B = 25.6% (1 d.p.)
C = 23.0% (1 d.p.) D = 16.9% (1 d.p.)

4. Red 35.5% Blue 31.0% White 17.7%
Silver 6.6% Green 6.0% Black 3.2%

Exercise 5.4 page 47

1. **a)** 187.5 **b)** 322 **c)** 7140
d) 245 **e)** 90 **f)** 121.5

2. **a)** 90 **b)** 38 **c)** 9
d) 900 **e)** 50 **f)** 43.5

3. **a)** 20% **b)** 80% **c)** 110%
d) 5% **e)** 85% **f)** 225%

4. **a)** 50% **b)** 30% **c)** 5%
d) 100% **e)** 36% **f)** 5%

5. 7475 tonnes

6. $6825

7. **a)** $75 **b)** $93.75

8. **a)** 43 **b)** 17.2%

9. 1100

Exercise 5.5 page 48

1. **a)** 600 **b)** 350 **c)** 900
d) 250 **e)** 125 **f)** 1.5

2. **a)** 56 **b)** 65 **c)** 90
d) 20 **e)** 0.25 **f)** −38

3. 280 pages

4. 12 500 families

5. 22

6. 12 200 000 m^3

Student assessment 1 page 49

1. 640 m

2. $345.60

3. $10 125

4. **a)** 20% **b)** 41.7% (3 s.f.)
c) 22.5% **d)** 85.7% (3 s.f.)
e) 7% **f)** 30%

5. 16% profit

6. **a)** $36 **b)** 25%

Student assessment 2 page 49

1. 750 m

2. $525

3. $97 200

4. **a)** 29.2% (3 s.f.) **b)** 21.7% (3 s.f.)
c) 125% **d)** 8.33% (3 s.f.)
e) 20% **f)** 10%

5. 8.3%

6. **a)** $650 **b)** 61.8% (3 s.f.)

Student assessment 3 page 50

1. $3500 $12 000 $1 $56

2. $500 $250 $20 000 $137 500

3. 15

4. $15 000

5. 40 000 tonnes

Student assessment 4 page 51

1. $200 $25 $524 $10

2. $462 $4000 $4500 $5500

3. 15 marks

4. 35 000

5. 25 000 units

6. 46 500 units

6 Ratio and proportion

Exercise 6.1 page 52

1. 48

2. 16 h 40 min

3. 11 units

4. **a)** 7500 bricks **b)** 53 h

5. **a)** 6250 litres **b)** 128 km

6. 1110 km (3 s.f.)

7. **a)** 450 **b)** 75 **c)** 120

8. **a)** 480 km **b)** 96 km/h

Exercise 6.2 page 54

1. **a)** 450 kg **b)** 1250 kg

2. **a)** Butter 600 g Flour 2 kg Sugar 200 g
Currants 400 g
b) 120 cakes

3. **a)** 16.8 litres
b) Red 1.2 litres White 14.3 litres

4. **a)** 125
b) Red 216 Yellow 135
c) 20

5. **a)** 42 litres
b) Orange juice 495 litres
Mango juice 110 litres

Exercise 6.3 page 55

1. 60 : 90

2. 16 : 24 : 32

3. 3.25 : 1.75

4. 18 : 27

5. 10 : 50

6. 7 : 1

7. Orange 556 ml (3 s.f.) Water 444 ml (3 s.f.)

8. **a)** 11 : 9
b) 440 boys 360 girls

9. $\frac{3}{5}$

10. 32 cm

11. 4 km and 3 km

12. 40°, 80°, 120°, 120°

13. 45°, 75°, 60°

14. 24 yr old $400 000 28 yr old $466 667
32 yr old $533 333

15. Alex $2000 Maria $3500 Ahmet $2500

Exercise 6.4 page 56

1. 4

2.

Speed (km/h)	60	40	30	120	90	50	10
Time (h)	2	3	4	1	$1\frac{1}{3}$	$2\frac{2}{5}$	12

3. **a)** i) 12 h ii) 4 h iii) 48 h
b) i) 16 ii) 3 iii) 48

4. **a)** 30 rows **b)** 42 chairs

5. 6 h 40 min

6. 4

7. 18 h

Exercise 6.5 page 57

1. **a)** 160 **b)** 250 **c)** 175
d) 110 **e)** 225 **f)** 128

2. **a)** $93\frac{1}{3}$ **b)** $116\frac{2}{3}$ **c)** 80
d) $157\frac{1}{2}$ **e)** 154 **f)** 85

3. **a)** 40 **b)** 50 **c)** 35
d) 36 **e)** 15 **f)** 52

4. **a)** $22\frac{1}{2}$ **b)** $6\frac{2}{3}$ **c)** $17\frac{1}{2}$
d) $5\frac{5}{8}$ **e)** $18\frac{3}{4}$ **f)** $13\frac{1}{2}$

5. 50

6. 32

7. 210

8. 90

Exercise 6.6 page 58

1. 22 cm by 16.5 cm

2. 28 cm by 21 cm

3. 5 : 2

4. Min. = 32 : 1 Max. = 35 : 1

5. **a)** i) 28 cm^2 ii) 112 cm^2 **b)** 4 : 1

6. **a)** i) 9 cm^2 ii) 81 cm^2 **b)** 9 : 1

7. **a)** i) 30 cm^3 ii) 240 cm^3 **b)** 8 : 1

8. **a)** i) 64 cm^3 ii) 1728 cm^3 **b)** 27 : 1

9. **a)** 16 cm^2 **b)** 4 cm^2 **c)** 1 : 4

10. Student's own answer.

Student assessment 1 page 59

1. **a)** 15 km **b)** 6 km/h

2. 16 cm and 14 cm

3. 1200 g

4. 200 g

5. **a)** 2 km **b)** 48 cm

6. **a)** 26 litres of petrol and 4 litres of oil
b) 3250 ml

7. **a)** 1 : 40 **b)** 13.75 cm

8. Girl $1040 Boy $960

9. 24°, 60°, 96°

10. **a)** 15 s **b)** 8 copiers

11. 6 h

12. 2

Student assessment 2 page 60

1. **a)** 30 km **b)** 30 km/h

2. **a)** $\frac{7}{10}$ **b)** 45 cm

3. **a)** 375 g **b)** 625 g

4. **a)** 450 m **b)** 80 cm

5. **a)** 1 : 25 **b)** 1.75 m

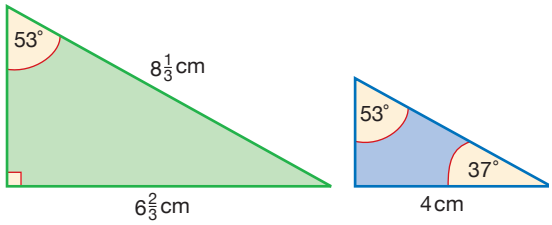
6. 300 : 750 : 1950

7. 60°, 90°, 90°, 120°

8. 150°

9. **a)** 13.5 h **b)** 12 pumps

10.



11. **a)** 4 min 48 s **b)** 1.6 litres/min

12. 21.6 cm by 9 cm

7 Indices and standard form

Exercise 7.1 page 62

1. **a)** 3^3 **b)** 2^5 **c)** 4^2
d) 6^4 **e)** 8^6 **f)** 5^1

2. **a)** $2^3 \times 3^2$ **b)** $4^5 \times 5^2$ **c)** $3^2 \times 4^3 \times 5^2$
d) 2×7^4 **e)** 6^2 **f)** $3^3 \times 4^2 \times 6^5$

3. **a)** 4×4
b) $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
c) $3 \times 3 \times 3 \times 3 \times 3$
d) $4 \times 4 \times 4 \times 6 \times 6 \times 6$
e) $7 \times 7 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
f) $3 \times 3 \times 4 \times 4 \times 4 \times 2 \times 2 \times 2 \times 2$

4. **a)** 32 **b)** 81 **c)** 64
d) 216 **e)** 1 000 000 **f)** 256
g) 72 **h)** 125 000

Exercise 7.2 page 63

1. **a)** 3^6 **b)** 8^7 **c)** 5^9
d) 4^{10} **e)** 2^4 **f)** $3^5 \times 6^6$
g) $4^8 \times 5^9 \times 6^2$ **h)** $2^4 \times 5^{10} \times 6^8$

2. **a)** 4^4 **b)** 5^3 **c)** 2
d) 6^3 **e)** 6^3 **f)** 8
g) 4^3 **h)** 3^7

3. **a)** 5^4 **b)** 4^{12} **c)** 10^{10}
d) 3^{15} **e)** 6^8 **f)** 8^6

4. **a)** 2^3 **b)** 3 **c)** 5^3
d) 4^5 **e)** $4^3 \times 2^2 (= 2^8)$ **f)** $6^4 \times 8^5$
g) $4^3 \times 5^2$ **h)** 4×6^7

Exercise 7.3 page 64

1. **a)** 8 **b)** 25 **c)** 1
d) 1 **e)** 1 **f)** $\frac{1}{4}$

2. **a)** $\frac{1}{4}$ **b)** $\frac{1}{9}$ **c)** $\frac{3}{50}$
d) $\frac{1}{200}$ **e)** 1 **f)** $\frac{1}{1000}$

3. **a)** 1 **b)** 2 **c)** 4
d) $\frac{1}{2}$ **e)** $\frac{1}{6}$ **f)** 10

4. **a)** 12 **b)** 32 **c)** 225
d) 80 **e)** 7 **f)** 64

Exercise 7.4 page 65

1. **a)** 2 **b)** 4 **c)** 3
d) 3 **e)** 4 **f)** 0

2. **a)** 4 **b)** 1 **c)** $\frac{3}{2}$
d) -1 **e)** 2 **f)** 0

3. **a)** -3 **b)** -4 **c)** -5
d) $-\frac{2}{3}$ **e)** -2 **f)** $-\frac{1}{2}$

4. **a)** -3 **b)** -7 **c)** -3
d) 2 **e)** 8 **f)** 5

Exercise 7.5 page 67

1. d and e

2. **a)** 6×10^5 **b)** 4.8×10^7 **c)** 7.84×10^{11}
d) 5.34×10^5 **e)** 7×10^6 **f)** 8.5×10^6

3. **a)** 6.8×10^6 **b)** 7.2×10^8 **c)** 8×10^5
d) 7.5×10^7 **e)** 4×10^9 **f)** 5×10^7

4. **a)** 3800 **b)** 4250000 **c)** 90030000
d) 101000

5. **a)** 6×10^5 **b)** 2.4×10^7 **c)** 1.4×10^8
d) 3×10^9 **e)** 1.2×10^{13} **f)** 1.8×10^7

6. 1.44×10^{11} m = 1.44×10^8 km

7. **a)** 8.8×10^8 **b)** 2.04×10^{11} **c)** 3.32×10^{11}
d) 4.2×10^{22} **e)** 5.1×10^{22} **f)** 2.5×10^{25}

8. **a)** 2×10^2 **b)** 3×10^5 **c)** 4×10^6
d) 2×10^4 **e)** 2.5×10^6 **f)** 4×10^4

9. **a)** 4.26×10^5 **b)** 8.48×10^9 **c)** 6.388×10^7
d) 3.157×10^9 **e)** 4.5×10^8 **f)** 6.01×10^7
g) 8.15×10^{10} **h)** 3.56×10^7

Exercise 7.6 page 68

1. **a)** 6×10^{-4} **b)** 5.3×10^{-5} **c)** 8.64×10^{-4}
d) 8.8×10^{-8} **e)** 7×10^{-7} **f)** 4.145×10^{-4}

2. **a)** 6.8×10^{-4} **b)** 7.5×10^{-7} **c)** 4.2×10^{-10}
d) 8×10^{-9} **e)** 5.7×10^{-11} **f)** 4×10^{-11}

3. **a)** 0.008 **b)** 0.000 42 **c)** 0.0903
d) 0.000 010 1

4. **a)** -4 **b)** -3 **c)** -8
d) -5 **e)** -7 **f)** 3

5. 6.8×10^5 6.2×10^3 8.414×10^2
6.741×10^{-4} 3.2×10^{-4} 5.8×10^{-7}
5.57×10^{-9}

Exercise 7.7 page 70

1. **a)** 4 **b)** 5 **c)** 10 **d)** 3 **e)** 9 **f)** 10

2. **a)** 2 **b)** 3 **c)** 2 **d)** 2 **e)** 6 **f)** 4

3. **a)** 8 **b)** 32 **c)** 27 **d)** 64 **e)** 1 **f)** 9

4. **a)** 25 **b)** 8 **c)** 32 **d)** 100 **e)** 32 **f)** 27

5. **a)** $\frac{1}{2}$ **b)** $\frac{1}{3}$ **c)** $\frac{1}{2}$ **d)** $\frac{1}{3}$ **e)** $\frac{1}{2}$ **f)** $\frac{1}{6}$

6. **a)** $\frac{1}{3}$ **b)** $\frac{1}{2}$ **c)** $\frac{1}{4}$ **d)** $\frac{1}{3}$ **e)** $\frac{1}{6}$ **f)** $\frac{1}{3}$

Exercise 7.8 page 70

1. **a)** 1 **b)** 7 **c)** 2 **d)** 1 **e)** 81 **f)** 6

2. **a)** 25 **b)** 2 **c)** 2 **d)** 27 **e)** 4 **f)** 2

3. **a)** 4 **b)** 2 **c)** 64 **d)** 9 **e)** 3 **f)** 27

Student assessment 1 page 71

1. **a)** $2^3 \times 5^2$ **b)** $2^2 \times 3^5$

2. **a)** $4 \times 4 \times 4$ **b)** $6 \times 6 \times 6 \times 6$

3. **a)** 800 **b)** 27

4. **a)** 3^7 **b)** $6^5 \times 3^9$ **c)** 2^7
d) 6 **e)** $3^2 \times 4^2$ **f)** 1

5. **a)** 4 **b)** 9 **c)** 5 **d)** 1

6. **a)** 7 **b)** -2 **c)** -1 **d)** $\frac{1}{3}$

Student assessment 2 page 71

1. **a)** $2^2 \times 3^5$ **b)** 2^{14}

2. **a)** $6 \times 6 \times 6 \times 6 \times 6$

b) $\dfrac{1}{2 \times 2 \times 2 \times 2 \times 2}$

3. **a)** 27 000 **b)** 125

4. **a)** 2^7 **b)** $7^7 \times 3^{12}$ **c)** 2^6
d) 3^3 **e)** 4^{-1} **f)** 2^2

5. **a)** 5 **b)** 16 **c)** 49 **d)** 48

6. **a)** 2.5 **b)** -0.5 **c)** 0 **d)** $\frac{2}{9}$

Student assessment 3 page 72

1. **a)** 8×10^6 **b)** 7.2×10^{-4} **c)** 7.5×10^{10}
d) 4×10^{-4} **e)** 4.75×10^9 **f)** 6.4×10^{-7}

2. **a)** 20 700 **b)** 0.001 45 **c)** 0.0523

3. 7.41×10^{-9} 3.6×10^{-5} 5.5×10^{-3}
4.21×10^7 6.2×10^7 4.9×10^8

4. **a)** 6×10^6 8.2×10^5 4.4×10^{-3}
8×10^{-1} 5.2×10^4

b) 6×10^6 8.2×10^5 5.2×10^4
8×10^{-1} 4.4×10^{-3}

5. **a)** 2 **b)** 8 **c)** -4
d) 5 **e)** -5 **f)** -5

6. **a)** 1.2×10^8 **b)** 5.6×10^8 **c)** 2×10^5
d) 2.5×10^5

7. 6 minutes

8. 4.73×10^{15} km correct to three significant figures (3 s.f.)

Student assessment 4 page 73

1. **a)** 6×10^6 **b)** 4.5×10^{-3} **c)** 3.8×10^9
d) 3.61×10^{-7} **e)** 4.6×10^8 **f)** 3×10^0

2. **a)** 8 112 000 **b)** 440 000 **c)** 0.000 305

3. 4.05×10^8 3.6×10^2 9×10^1
1.5×10^{-2} 7.2×10^{-3} 2.1×10^{-3}

4. **a)** 1.5×10^7 4.3×10^5 4.35×10^{-4}
4.8×10^0 8.5×10^{-3}
b) 4.35×10^{-4} 8.5×10^{-3} 4.8×10^0
4.3×10^5 1.5×10^7

5. **a)** 3 **b)** 9 **c)** -3
d) 6 **e)** -1 **f)** 8

6. **a)** 1.2×10^8 **b)** 1.48×10^{11} **c)** 6.73×10^7
d) 3.88×10^6

7. 43.2 minutes (3 s.f.)

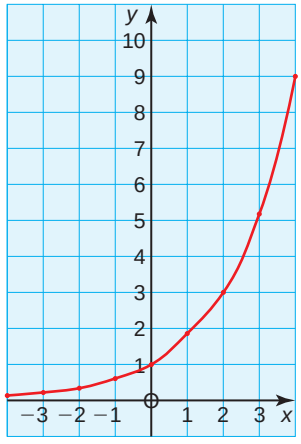
8. 2.84×10^{15} km (3 s.f.)

Student assessment 5 page 74

1. **a)** 9 **b)** 3 **c)** 3 **d)** 125
e) 49 **f)** 0.5 **g)** 5 **h)** 16

2. **a)** 1 **b)** 9 **c)** 4 **d)** 25
e) 2 **f)** 8 **g)** 1 **h)** 45

3. **a)**



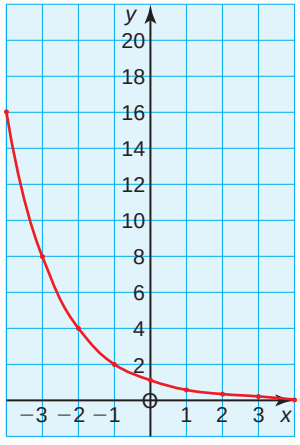
b) Approx. 2.9

Student assessment 6 page 74

1. **a)** 2 **b)** 81 **c)** $\frac{1}{3}$ **d)** 64
e) 3 **f)** 2 **g)** 6 **h)** 32

2. **a)** 15 **b)** 4 **c)** 3 **d)** 3125
e) 4 **f)** 1 **g)** 27 **h)** 10

3. **a)**



b) Approx. −2.6

8 Money and finance

Exercise 8.1 page 75

1. **a)** A$31.25 **b)** 3500 rupees
c) ZIM$9400 **d)** 3300 rand
e) L299 **f)** 4120 yen
g) 160 dinar **h)** US$195

2. **a)** €400 **b)** €2.86
c) €2.13 **d)** €45.45
e) €326.09 **f)** €11.65
g) €125 **h)** €115.38

Exercise 8.2 page 76

1. $154.82

2. $182

3. $131

4. $290.50

5. **a)** $195.05 **b)** $132.63

6. $137.50

7. €525

8. **a)** 298 rand **b)** 253.30 rand

Exercise 8.3 page 77

1. $1.80 loss

2. $2.88 profit

3. $54.65 profit per seat

4. $240 extra

5. $250 loss

Exercise 8.4 page 78

1. **a)** 11% **b)** 25%

2. **a)** 30% **b)** 20%

3. Type A = 30%
Type B = 15.4%
Type C = 33.3%
Type C makes most profit.

4. 80%

Exercise 8.5 page 80

1. **a)** $72 **b)** $420

2. **a)** $t = 5$ **b)** $t = 7$

3. **a)** $r = 7$ **b)** $r = 4$
4. **a)** $P = 200$ **b)** $P = 850$
5. $r = 4$
6. $t = 2$
7. $r = 4.5$
8. $r = 9.5$
9. \$315
10. $r = 6$

Exercise 8.6 page 82

1. \$11 033 750
2. \$52 087.50
3. \$10 368
4. 1331 students
5. 3 276 800 tonnes
6. 2 years
7. 5 years
8. 3 years

Student assessment 1 page 83

1. **a)** 2880 rupees **b)** HK\$83.33
2. **a)** €625 **b)** €192.31
3. \$122.40
4. \$26.16 (\$12.96 + \$13.20)
5. **a)** \$72 **b)** 8% **c)** 4 years
   d) 7.5% **e)** \$1250
6. \$6000
7. 30%

Student assessment 2 page 84

1. **a)** \$82.50 **b)** \$4800 **c)** \$2187.50
2. 3.5%
3. 5 years
4. **a)** \$24.36 **b)** \$2969.24 **c)** \$9953.45
5. **a)** i) \$10 625
   ii) \$9031.25
   b) 16 years

9 Time

Exercise 9.1 page 85

1. 0840
2. 1845
3. 0825
4. **a)** 2 h 18 min
   b) 1 h 24 min
5. 1st: 30 min 2nd: 32 min 44 s
   3rd: 34 min 17 s 4th: 35 min 7s
   5th: 36 min
6. 2.10 a.m. on Wednesday
7. 2145
8. 1115

Student assessment 1 page 86

1. 09 03
2. 5 h 54 min
3. 49 km
4. 11 h 5 min

Student assessment 2 page 86

1. 08 18
2. 2 h 15 min
3. 154 km
4. **a)** 3 h 30 min or 3.5 h
   b) 90 km/h

10 Set notation and Venn diagrams

Exercise 10.1 page 88

1. **a)** i) Continents of the world
   ii) Student's own answers
   b) i) Even numbers
   ii) Student's own answers
   c) i) Days of the week
   ii) Student's own answers
   d) i) Months with 31 days
   ii) Student's own answers

e) i) Triangle numbers
ii) Student's own answers

f) i) Boy's names beginning with the letter m
ii) Student's own answers

g) i) Prime numbers greater than 7
ii) Student's own answers

h) i) Vowels
ii) o, u

i) i) Planets of the solar system
ii) Student's own answers

j) i) Numbers between 3 and 12
ii) Student's own answers

k) i) Numbers between −5 and 5
ii) Student's own answers

2. **a)** 7 **c)** 7 **d)** 7
f) Unquantifiably finite, though theoretically infinite
h) 5 **i)** 9

Exercise 10.2 page 89

1. **a)** $Q = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28\}$
b) $R = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29\}$
c) $S = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$
d) $T = \{1, 4, 9, 16, 25\}$
e) $U = \{1, 3, 6, 10, 15, 21, 28\}$

2. **a)** $B = \{55, 60, 65\}$
b) $C = \{51, 54, 57, 60, 63, 66, 69\}$
c) $D = \{64\}$

3. **a)** {p, q, r}, {p, q}, {p, r}, {q, r}, {p}, {q}, {r}, { }
b) {p, q}, {p, r}, {q, r}, {p}, {q}, {r}

4. **a)** True **b)** True **c)** True **d)** False
e) False **f)** True **g)** True **h)** False

Exercise 10.3 page 91

1. **a)** True **b)** True **c)** False
d) False **e)** False **f)** True

2. **a)** $A \cap B = \{4, 6\}$
b) $A \cap B = \{4, 9\}$
c) $A \cap B = \{\text{yellow, green}\}$

3. **a)** $A \cup B = \{2, 3, 4, 6, 8, 9, 10, 13, 18\}$
b) $A \cup B = \{1, 4, 5, 6, 7, 8, 9, 16\}$
c) $A \cup B = \{\text{red, orange, blue, indigo, violet, yellow, green, purple, pink}\}$

4. **a)** $\mathscr{E} = \{a, b, p, q, r, s, t\}$
b) $A' = \{a, b\}$

5. **a)** $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8\}$
b) $A' = \{1, 4, 6, 8\}$
c) $A \cap B = \{2, 3\}$
d) $A \cup B = \{1, 2, 3, 4, 5, 7, 8\}$
e) $(A \cap B)' = \{1, 4, 5, 6, 7, 8\}$
f) $A \cap B' = \{5, 7\}$

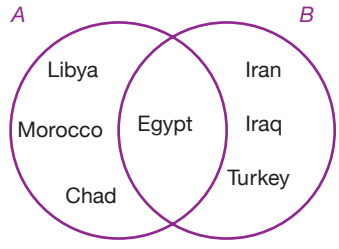
6. **a)** i) $A = \{\text{even numbers from 2 to 14}\}$
ii) $B = \{\text{multiples of 3 from 3 to 15}\}$
iii) $C = \{\text{multiples of 4 from 4 to 20}\}$
b) i) $A \cap B = \{6, 12\}$
ii) $A \cap C = \{4, 8, 12\}$
iii) $B \cap C = \{12\}$
iv) $A \cap B \cap C = \{12\}$
v) $A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12, 14, 15\}$
vi) $C \cup B = \{3, 4, 6, 8, 9, 12, 15, 16, 20\}$

7. **a)** i) $A = \{1, 2, 4, 5, 6, 7\}$
ii) $B = \{3, 4, 5, 8, 9\}$
iii) $C' = \{1, 2, 3, 4, 5, 8, 9\}$
iv) $A \cap B = \{4, 5\}$
v) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
vi) $(A \cap B)' = \{1, 2, 3, 6, 7, 8, 9\}$
b) $C \subset A$

8. **a)** i) $W = \{1, 2, 4, 5, 6, 7, 9, 10\}$
ii) $X = \{2, 3, 6, 7, 8, 9\}$
iii) $Z' = \{1, 4, 5, 6, 7, 8, 10\}$
iv) $W \cap Z = \{2, 9\}$
v) $W \cap X = \{2, 6, 7, 9\}$
vi) $Y \cap Z = \{\ \}$ or $\varnothing$
b) Z

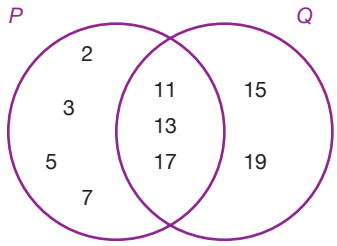
Exercise 10.4 93

1. **a)**



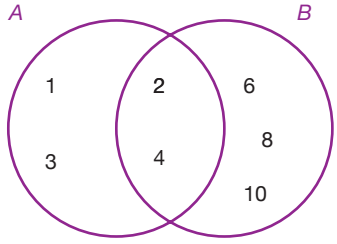
b) i) $A \cap B = \{\text{Egypt}\}$
ii) $A \cup B = \{\text{Libya, Morocco, Chad, Egypt, Iran, Iraq, Turkey}\}$

2. **a)**

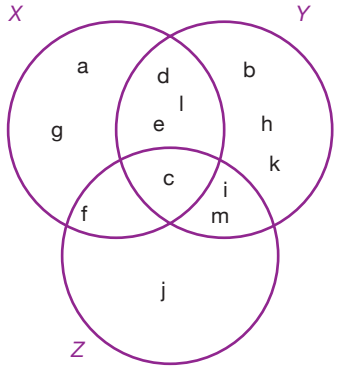


b) i) $P \cap Q = \{11, 13, 17\}$
ii) $P \cup Q = \{2, 3, 5, 7, 11, 13, 15, 17, 19\}$

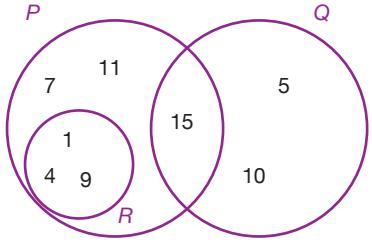
3.



4.



5.



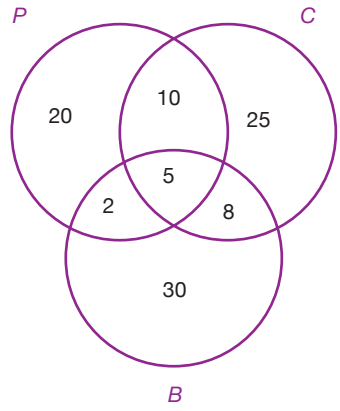
Exercise 10.5 page 94

1. **a)** 5 **b)** 14 **c)** 13

2. 45

3. **a)** 10 **b)** 50

4. **a)**



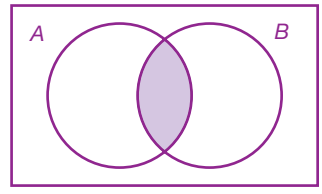
b) 100

Student assessment 1 page 94

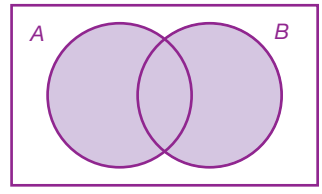
1. **a)** {even numbers from 2 to 8}
b) {even numbers}
c) {square numbers}
d) {oceans}

2. **a)** 7 **b)** 2 **c)** 6 **d)** 366

3. **a)**



b)



4. {a, b}, {a}, {b}, { }

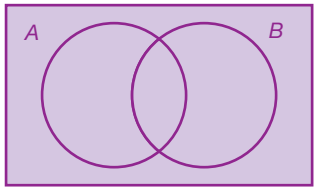
5. $A' = \{m, t, h\}$

Student assessment 2 page 95

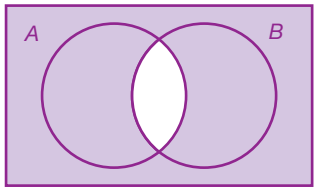
1. **a)** {odd numbers from 1 to 7}
b) {odd numbers}
c) {triangle numbers}
d) {countries in South America}

2. **a)** 12 **b)** 3 **c)** 7
d) Student's own answer

3. **a)** $\mathscr{E}$



b) $\mathscr{E}$



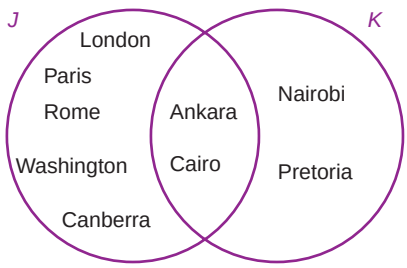
4. {o, r, k}, {w, r, k}, {w, o, k}, {w, o, r}, {w, o, r, k}

5. $P' = \{1, 3, 5, 7\}$

Student assessment 3 page 95

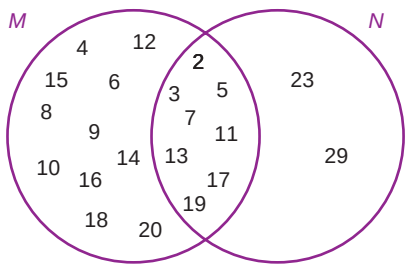
1. {2, 4}, {2, 6}, {2, 8}, {4, 6}, {4, 8}, {6, 8}, {2, 4, 6}, {2, 4, 8}, {2, 6, 8}, {4, 6, 8}

2. **a)**



b) {Ankara, Cairo} **c)** {Nairobi, Pretoria}

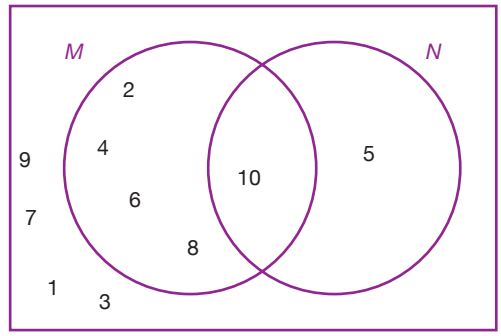
3. **a)**



b) {2, 3, 5, 7, 11, 13, 17, 19}

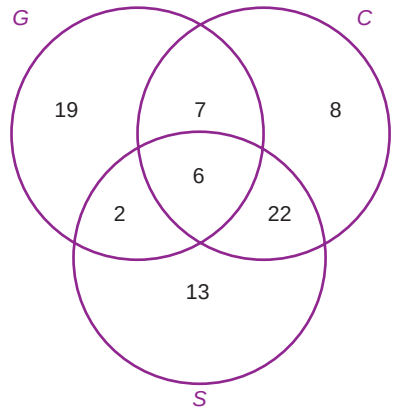
c) {4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 23, 29}

4. **a)** $\mathscr{E}$



b) X = {multiples of 10}

5. **a)**



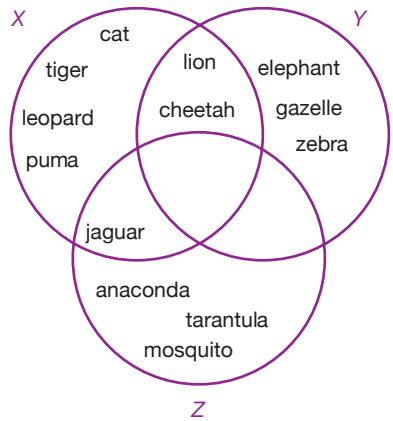
b) 6

Student assessment 4 page 96

1. **a)** 32

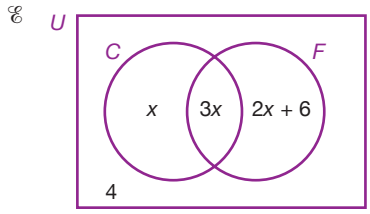
b) {a, e, i, o, u}, {a, e, i, o}, {a, e, i, u}, {a, e, o, u}, {a, i, o, u}, {e, i, o, u}

2. **a)**



b) {lion, cheetah} **c)** ∅ **d)** ∅

3. **a)** Let the number liking only cricket be x.



b) 15 **c)** 5 **d)** 16

4. **a)** 5 **b)** 35 **c)** 40 **d)** 50 **e)** 15
f) 12 **g)** 10 **h)** 78 **i)** 78

Topic 1 Mathematical investigations and ICT

Primes and squares page 100

1. $2^2 + 3^2 = 13$
$4^2 + 5^2 = 41$

2. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

3. 5, 13, 17, 29, 37, 41, 53, 61, 73, 89, 97

4. If a number generated by the rule $4n + 1$ is a prime, then it can be expressed as the sum of two square numbers. (This was proved by Fermat in the 17th century.) Alternatively, add 1 to the prime number and divide the result by 2. If the answer is even then the prime number cannot be expressed as the sum of two squares.

5. The rule works for the numbers shown, but this does not prove that it always works.

Football leagues page 100

1. $(17 + 16 + 15 + \ldots + 1) \times 2 = 306$
2. $n = t(t - 1)$

ICT activity 1 page 101

2. Student's spreadsheet

3. e.g. in cell C4 enter =B4/B$3*100

4. Student's spreadsheet

5. Student's report

ICT activity 2 page 101

Student's graphs and responses will vary except:

6. The gradient of the tangent at a point represents the speed of the runner at that point.

11 Algebraic representation and manipulation

Exercise 11.1 page 104

1. **a)** $4x - 12$ **b)** $10p - 20$
c) $-42x + 24y$ **d)** $6a - 9b - 12c$
e) $-14m + 21n$ **f)** $-16x + 6y$

2. **a)** $3x^2 - 9xy$ **b)** $a^2 + ab + ac$
c) $8m^2 - 4mn$ **d)** $-15a^2 + 20ab$
e) $4x^2 - 4xy$ **f)** $24p^2 - 8pq$

3. **a)** $-2x^2 + 3y^2$ **b)** $a - b$
c) $7p - 2q$ **d)** $3x - 4y + 2z$
e) $3x - \frac{3}{2}y$ **f)** $2x^2 - 3xy$

4. **a)** $12r^3 - 15rs + 6rt$
b) $a^3 + a^2b + a^2c$
c) $6a^3 - 9a^2b$
d) $p^2q + pq^2 - p^2q^2$
e) $m^3 - m^2n + m^3n$ **f)** $a^6 + a^5b$

Exercise 11.2 page 104

1. **a)** $-a - 8$ **b)** $4x - 20$
c) $3p - 16$ **d)** $21m - 6n$
e) 3 **f)** $-p - 3p^2$

2. **a)** $8m^2 + 28m + 2$ **b)** $x - 4$
c) $2p + 22$ **d)** $m - 12$
e) $a^2 + 6a + 2$ **f)** $7ab - 16ac + 3c$

3. **a)** $4x + 4$ **b)** $5x - \frac{3}{2}y$
c) $\frac{9}{4}x - \frac{5}{2}y$ **d)** $\frac{9}{2}x + \frac{1}{2}y$
e) $7x - 4y$ **f)** 0

Exercise 11.3 page 105

1. **a)** $x^2 + 5x + 6$ **b)** $x^2 + 7x + 12$
c) $x^2 + 7x + 10$ **d)** $x^2 + 7x + 6$
e) $x^2 + x - 6$ **f)** $x^2 + 5x - 24$

2. **a)** $x^2 + 2x - 24$ **b)** $x^2 - 3x - 28$
c) $x^2 - 2x - 35$ **d)** $x^2 - 2x - 15$
e) $x^2 - 2x - 3$ **f)** $x^2 + 2x - 63$

3. **a)** $x^2 - 5x + 6$ **b)** $x^2 - 7x + 10$
c) $x^2 - 12x + 32$ **d)** $x^2 + 6x + 9$
e) $x^2 - 6x + 9$ **f)** $x^2 - 12x + 35$

4. **a)** $x^2 - 9$ **b)** $x^2 - 49$
c) $x^2 - 64$ **d)** $x^2 - y^2$
e) $a^2 - b^2$ **f)** $p^2 - q^2$

Exercise 11.4 page 106

1. **a)** $2(2x - 3)$ **b)** $6(3 - 2p)$
c) $3(2y - 1)$ **d)** $2(2a - 3b)$
e) $3(p - q)$ **f)** $4(2m + 3n + 4r)$

2. **a)** $a(3b + 4c - 5d)$ **b)** $2p(4q + 3r - 2s)$
c) $a(a - b)$ **d)** $2x(2x - 3y)$
e) $ab(c + d + f)$ **f)** $3m(m + 3)$

3. **a)** $3pq(r - 3s)$ **b)** $5m(m - 2n)$
c) $4xy(2x - y)$ **b)** $b^2(2a^2 - 3c^2)$
e) $12(p - 3)$ **f)** $6(7x - 9)$

4. **a)** $6(3 + 2y)$ **b)** $7(2a - 3b)$
c) $11x(1 + y)$ **d)** $4(s - 4t + 5r)$
e) $5q(p - 2r + 3s)$ **f)** $4y(x + 2y)$

5. **a)** $m(m + n)$ **b)** $3p(p - 2q)$
c) $qr(p + s)$ **d)** $ab(1 + a + b)$
e) $p^3(3 - 4p)$ **f)** $b^2c(7b + c)$

6. **a)** $m(m^2 - mn + n^2)$
b) $2r^2(2r - 3 + 4s)$
c) $28xy(2x - y)$
d) $18mn(4m + 2n - mn)$

Exercise 11.5 page 106

1. **a)** 0 **b)** 30 **c)** 14
d) 20 **e)** -13 **f)** -4

2. **a)** -3 **b)** -30 **c)** 20
d) -16 **e)** -40 **f)** 42

3. **a)** -160 **b)** -23 **c)** 42
d) -17 **e)** -189 **f)** 113

4. **a)** 48 **b)** -8 **c)** 15
d) 16 **e)** -5 **f)** 9

5. **a)** 12 **b)** -5 **c)** -5
d) 7 **e)** 7 **f)** 36

Exercise 11.6 page 107

1. **a)** $n = r - m$ **b)** $m = p - n$
c) $n = 3p - 2m$ **d)** $q = 3x - 2p$
e) $a = \frac{cd}{b}$ **f)** $d = \frac{ab}{c}$

2. **a)** $x = \frac{4m}{3y}$ **b)** $r = \frac{7pq}{5}$
c) $x = \frac{c}{3}$ **d)** $x = \frac{y - 7}{3}$
e) $y = \frac{3r + 9}{5}$ **f)** $x = \frac{5y - 9}{3}$

3. **a)** $b = \frac{2a - 5}{6}$ **b)** $a = \frac{6b + 5}{2}$
c) $z = \frac{3x - 7y}{4}$ **d)** $x = \frac{4z + 7y}{3}$
e) $y = \frac{3x - 4z}{7}$ **f)** $p = \frac{8 + q}{2r}$

4. **a)** $p = 4r$ **b)** $p = \frac{4}{3r}$ **c)** $p = \frac{n}{10}$
d) $n = 10p$ **e)** $p = \frac{2t}{q + r}$ **f)** $q = \frac{2t}{p} - r$

5. **a)** $r = \frac{3m - n}{t(p + q)}$ **b)** $t = \frac{3m - n}{r(p + q)}$
c) $m = \frac{rt(p + q) + n}{3}$
d) $n = 3m - rt(p + q)$
e) $p = \frac{3m - n}{rt} - q$
f) $q = \frac{3m - n}{rt} - p$

6. **a)** $d = \frac{ab}{ce}$ **b)** $a = \frac{dec}{b}$
c) $c = \frac{ab}{de}$ **d)** $a = cd - b$
e) $b = d - \frac{a}{c}$ **f)** $c = \frac{a}{d - b}$

Exercise 11.7 page 108

1. **a)** $2y^2 + 7y + 6$ **b)** $3y^2 + 25y + 28$
c) $2y^2 + 17y + 8$ **d)** $4y^2 + 6y + 2$
e) $6y^2 + 23y + 20$ **f)** $18y^2 + 15y + 3$

2. **a)** $2p^2 + 13p - 24$ **b)** $4p^2 + 23p - 35$
c) $6p^2 + p - 12$ **d)** $12p^2 + 13p - 35$
e) $18p^2 - 2$ **f)** $28p^2 + 44p - 24$

3. **a)** $4x^2 - 4x + 1$ **b)** $9x^2 + 6x + 1$
c) $16x^2 - 16x + 4$ **d)** $25x^2 - 40x + 16$
e) $4x^2 + 24x + 36$ **f)** $4x^2 - 9$

4. **a)** $-4x^2 + 9$ **b)** $16x^2 - 9$
c) $-16x^2 + 9$ **d)** $-25y^2 + 49$
e) $8y^2 - 18$ **f)** $25y^2 - 70y + 49$

Exercise 11.8 page 109

1. **a)** $(x + y)(a + b)$ **b)** $(x - y)(a + b)$
c) $(3 + x)(m + n)$ **d)** $(m + n)(4 + x)$
e) $(m - n)(3 + x)$ **f)** $(x + z)(6 + y)$

2. **a)** $(p + q)(r - s)$ **b)** $(p + 3)(q - 4)$
c) $(q - 4)(p + 3)$ **d)** $(r + 2t)(s + t)$
e) $(s + t)(r - 2t)$ **f)** $(b + c)(a - 4c)$

3. **a)** $(y + x)(x + 4)$ **b)** $(x - 2)(x - y)$
c) $(a - 7)(b + 3)$ **d)** $(b - 1)(a - 1)$
e) $(p - 4)(q - 4)$ **f)** $(m - 5)(n - 5)$

4. **a)** $(m - 3)(n - 2)$ **b)** $(m - 3r)(n - 2r)$
c) $(p - 4q)(r - 4)$ **d)** $(a - c)(b - 1)$
e) $(x - 2y)(x - 2z)$ **f)** $(2a + b)(a + b)$

Exercise 11.9 page 109

1. **a)** $(a - b)(a + b)$ **b)** $(m - n)(m + n)$
c) $(x - 5)(x + 5)$ **d)** $(m - 7)(m + 7)$
e) $(9 - x)(9 + x)$ **f)** $(10 - y)(10 + y)$

2. **a)** $(12 - y)(12 + y)$ **b)** $(q - 13)(q + 13)$
c) $(m - 1)(m + 1)$ **d)** $(1 - t)(1 + t)$
e) $(2x - y)(2x + y)$ **f)** $(5p - 8q)(5p + 8q)$

3. **a)** $(3x - 2y)(3x + 2y)$
b) $(4p - 6q)(4p + 6q)$
c) $(8x - y)(8x + y)$
d) $(x - 10y)(x + 10y)$
e) $(qr - 2p)(qr + 2p)$
f) $(ab - cd)(ab + cd)$

4. **a)** $(mn - 3y)(mn + 3y)$
b) $(\frac{1}{2}x - \frac{1}{3}y)(\frac{1}{2}x + \frac{1}{3}y)$
c) $(2x - 9y^2)(2x + 9y^2)$
d) $(p^2 - q^2)(p^2 + q^2) = (p - q)(p + q)(p^2 + q^2)$
e) $4(m^2 - 3y^2)(m^2 + 3y^2)$
f) $(4x^2 - 9y^2)(4x^2 + 9y^2)$
$= (2x - 3y)(2x + 3y)\,(4x^2 + 9y^2)$

Exercise 11.10 page 110

1. **a)** 60 **b)** 240 **c)** 2400
d) 280 **e)** 7600 **f)** 9200

2. **a)** 2000 **b)** 9800 **c)** 200
d) 3200 **e)** 998 000 **f)** 161

3. **a)** 68 **b)** 86 **c)** 1780
d) 70 **e)** 55 **f)** 5

4. **a)** 72.4 **b)** 0.8 **c)** 65
d) 15 **e)** 1 222 000 **f)** 231

Exercise 11.11 page 111

1. **a)** $(x + 4)(x + 3)$ **b)** $(x + 6)(x + 2)$
c) $(x + 12)(x + 1)$ **d)** $(x - 3)(x - 4)$
e) $(x - 6)(x - 2)$ **f)** $(x - 12)(x - 1)$

2. **a)** $(x + 5)(x + 1)$ **b)** $(x + 4)(x + 2)$
c) $(x + 3)^2$ **d)** $(x + 5)^2$
e) $(x + 11)^2$ **f)** $(x - 6)(x - 7)$

3. **a)** $(x + 12)(x + 2)$ **b)** $(x + 8)(x + 3)$
c) $(x - 6)(x - 4)$ **d)** $(x + 12)(x + 3)$
e) $(x + 18)(x + 2)$ **f)** $(x - 6)^2$

4. **a)** $(x + 5)(x - 3)$ **b)** $(x - 5)(x + 3)$
c) $(x + 4)(x - 3)$ **d)** $(x - 4)(x + 3)$
e) $(x + 6)(x - 2)$ **f)** $(x - 12)(x - 3)$

5. **a)** $(x - 4)(x + 2)$ **b)** $(x - 5)(x + 4)$
c) $(x + 6)(x - 5)$ **d)** $(x + 6)(x - 7)$
e) $(x + 7)(x - 9)$ **f)** $(x + 9)(x - 6)$

6. **a)** $(2x + 1)(x + 1)$ **b)** $(2x + 3)(x + 2)$
c) $(2x - 3)(x + 2)$ **d)** $(2x - 3)(x - 2)$
e) $(3x + 2)(x + 2)$ **f)** $(3x - 1)(x + 4)$
g) $(2x + 3)^2$ **h)** $(3x - 1)^2$
i) $(3x + 1)(2x - 1)$

Exercise 11.12 page 112

1. **a)** $x = \dfrac{P}{2m}$ **b)** $x = \pm\sqrt{\dfrac{T}{3}}$ **c)** $x = \pm\sqrt{\dfrac{y^2}{m}}$

d) $x = \pm\sqrt{p^2 - q^2 - y^2}$

e) $x = \pm\sqrt{y^2 - n^2 - m^2}$

f) $x = \pm\sqrt{\dfrac{p^2 - q^2 + y^2}{4}}$

2. **a)** $x = \dfrac{P}{Q\mathrm{r}}$ **b)** $x = \pm\sqrt{\dfrac{P}{Qr}}$ **c)** $x = \pm\sqrt{\dfrac{Pr}{Q}}$

d) $x = \pm\sqrt{\dfrac{n}{m}}$ **e)** $x = \pm\sqrt{\dfrac{wst}{r}}$

f) $x = \pm\sqrt{\dfrac{wr}{p + q}}$

3. **a)** $x = (rp)^2$ **b)** $x = \left(\dfrac{mn}{p}\right)^2$ **c)** $x = \dfrac{k}{g^2}$

d) $x = \dfrac{r^2 g}{4\pi^2}$ **e)** $x = \dfrac{4m^2 r}{p^2}$

f) $x = \dfrac{4m^2 r}{p^2}$

Exercise 11.13 page 113

1. **a)** $t = \dfrac{v - u}{a}$ **b)** $u = \pm\sqrt{v^2 - 2as}$

c) $s = \dfrac{v^2 - u^2}{2a}$ **d)** $u = \dfrac{s - \frac{1}{2}at^2}{t}$

e) $a = \dfrac{2(s - ut)}{t^2}$ **f)** $t = \pm\sqrt{\dfrac{2(s - ut)}{a}}$

2. **a)** $r = \dfrac{A}{\pi\sqrt{s^2 + t^2}}$ **b)** $h = \pm\sqrt{\left(\dfrac{A}{\pi r}\right)^2 - r^2}$

c) $u = \dfrac{vf}{v - f}$ **d)** $v = \dfrac{fu}{u - f}$

e) $l = \left(\dfrac{t}{2\pi}\right)^2 g$ **f)** $g = l\left(\dfrac{2\pi}{t}\right)^2$

3. **a)** $x = \pm\sqrt{\dfrac{7(p+2)}{3t}}$ **b)** $a = \pm\sqrt{4 + (b-3)^2}$

Exercise 11.14 page 113

1. **a)** 0.53 m^3 (2 d.p.) **b)** $r = \sqrt{\dfrac{V}{\pi h}}$

c) 5.05 cm

2. **a)** 66 °C (2 s.f.) **b)** −11 °C (2 s.f.)

c) $F = \dfrac{9C}{5} + 32$ **d)** 320 °F

3. **a)** 15 hours **b)** $H = 1200(T - k)$

c) 5000 m

4. **a)** 524 cm^3 (3 s.f.) **b)** $r = \sqrt[3]{\dfrac{3V}{4\pi}}$

c) r = 8.42 m (3 s.f.)

5. **a)** \$251.50 **b)** $n = \dfrac{x - 1.5}{0.05}$

c) $n = 470$

Exercise 11.15 page 114

1. **a)** $\dfrac{xp}{yq}$ **b)** $\dfrac{q}{y}$ **c)** $\dfrac{p}{r}$

d) $\dfrac{d}{c}$ **e)** $\dfrac{bd}{c^2}$ **f)** p

2. **a)** m^2 **b)** r^5 **c)** x^6

d) xy^2 **e)** abc^3 **f)** $\dfrac{r^3}{pq}$

3. **a)** $\dfrac{2x}{y}$ **b)** $4q^2$ **c)** $5n$

d) $3x^3y$ **e)** $3p$ **f)** $\dfrac{2}{3mn}$

4. **a)** $\dfrac{2a}{3b}$ **b)** $\dfrac{2y}{x}$ **c)** 2

d) $3xy$ **e)** 3 **f)** $6xy$

5. **a)** $\dfrac{8y}{3}$ **b)** $\dfrac{5p}{2}$ **c)** $\dfrac{p^3}{s}$

d) $\dfrac{ef}{c^2}$

6. **a)** $\dfrac{20x}{3}$ **b)** $\dfrac{9b}{8}$

Exercise 11.16 page 115

1. **a)** $\dfrac{4}{7}$ **b)** $\dfrac{a+b}{7}$ **c)** $\dfrac{11}{13}$

d) $\dfrac{c+d}{13}$ **e)** $\dfrac{x+y+z}{3}$ **f)** $\dfrac{p^2+q^2}{5}$

2. **a)** $\dfrac{3}{11}$ **b)** $\dfrac{c-d}{11}$ **c)** $\dfrac{4}{a}$

d) $\dfrac{2a-5b}{3}$ **e)** $\dfrac{2x-3y}{7}$ **f)** $-\dfrac{1}{2x}$

3. **a)** $\dfrac{1}{2}$ **b)** $\dfrac{3}{2a}$ **c)** $\dfrac{5}{3c}$

d) $\dfrac{7}{2x}$ **e)** $\dfrac{3}{2p}$ **f)** $-\dfrac{1}{2w}$

4. **a)** $\dfrac{3p-q}{12}$ **b)** $\dfrac{x-2y}{4}$ **c)** $\dfrac{3m-n}{9}$

d) $\dfrac{x-2y}{12}$ **e)** $\dfrac{5r+m}{10}$ **f)** $\dfrac{5s-t}{15}$

5. **a)** $\dfrac{7x}{12}$ **b)** $\dfrac{9x-2y}{15}$ **c)** $\dfrac{m}{2}$

d) $\dfrac{m}{p}$ **e)** $\dfrac{x}{2y}$ **f)** $\dfrac{4r}{7s}$

Exercise 11.17 page 116

1. **a)** $\dfrac{3a+2b}{6}$ **b)** $\dfrac{5a+3b}{15}$ **c)** $\dfrac{7p+4q}{28}$

d) $\dfrac{6a+5b}{15}$ **e)** $\dfrac{9x+20y}{36}$ **f)** $\dfrac{10x+14y}{35}$

2. **a)** $\dfrac{a}{6}$ **b)** $\dfrac{2a}{15}$ **c)** $\dfrac{11p}{28}$

d) $\dfrac{11a}{15}$ **e)** $\dfrac{29x}{36}$ **f)** $\dfrac{24x}{35}$

3. **a)** $\dfrac{m}{10}$ **b)** $\dfrac{r}{10}$ **c)** $-\dfrac{x}{4}$

d) $\dfrac{29x}{28}$ **e)** $\dfrac{23x}{6}$ **f)** $\dfrac{p}{6}$

4. **a)** $\dfrac{p}{2}$ **b)** $\dfrac{2c}{3}$ **c)** $\dfrac{4x}{5}$

d) $\dfrac{m}{3}$ **e)** $\dfrac{q}{5}$ **f)** $\dfrac{w}{4}$

5. **a)** $\dfrac{3m}{2}$ **b)** $\dfrac{7m}{3}$ **c)** $-\dfrac{m}{2}$

d) $\dfrac{5m}{2}$ **e)** $\dfrac{p}{3}$ **f)** $\dfrac{36q}{7}$

6. **a)** $\frac{pr-p}{r}$ **b)** $\frac{x+xy}{y}$ **c)** $\frac{mn+m}{n}$
d) $\frac{a+ab}{b}$ **e)** $\frac{2xy-x}{y}$ **f)** $\frac{2pq-3p}{q}$

7. **a)** $\frac{5a+12}{6}$ **b)** $\frac{11b-20}{15}$
c) $\frac{3c-2}{4}$ **d)** $\frac{25d-54}{14}$

Exercise 11.18 page 118

1. **a)** $\frac{3x+4}{(x+2)(x+1)}$ **b)** $\frac{m-7}{(m+2)(m-1)}$
c) $\frac{3p-7}{(p-3)(p-2)}$ **d)** $\frac{w+11}{(w-1)(w+3)}$
e) $\frac{-12}{(y+4)(y+1)}$ **f)** $\frac{12-m}{(m-2)(m+3)}$

2. **a)** $\frac{x}{x+2}$ **b)** $\frac{y}{y+3}$ **c)** $\frac{m+2}{m-3}$
d) $\frac{p}{p-5}$ **e)** $\frac{m}{m+4}$ **f)** $\frac{m+1}{m+2}$

3. **a)** $\frac{x}{x+3}$ **b)** $\frac{x}{x+4}$ **c)** $\frac{y}{y-3}$
d) $\frac{x}{x+3}$ **e)** $\frac{x}{x+2}$ **f)** $\frac{x}{x+1}$

4. **a)** $\frac{x}{x+1}$ **b)** $\frac{x}{x+3}$ **c)** $\frac{x}{x-3}$
d) $\frac{x}{x+2}$ **e)** $\frac{x}{x-3}$ **f)** $\frac{x}{x+7}$

Student assessment 1 page 119

1. **a)** $10a-30b+15c$ **b)** $15x^2-27x$
c) $-15xy^2-5y^3$ **d)** $15x^3y+9x^2y^2-3x^5$
e) $12-p$ **f)** $14m^2-14m$
g) $4x+3$ **h)** $\frac{13}{2}m^2-3m$

2. **a)** $4(3a-b)$ **b)** $x(x-4y)$
c) $4p^2(2p-q)$ **d)** $8xy(3-2x+y)$

3. **a)** -21 **b)** 26 **c)** 43
d) 7 **e)** 12 **f)** 12

4. **a)** $q=x-3p$ **b)** $n=\frac{3m-8r}{5}$
c) $y=\frac{2mt}{3}$ **d)** $w=\frac{2y}{x}-y$
e) $p=\frac{xyt}{2rs}$ **f)** $x=w(m+n)-y$

Student assessment 2 page 119

1. **a)** $6x-9y+15z$ **b)** $8pm-28p$
c) $-8m^2n+4mn^2$ **d)** $20p^3q-8p^2q^2-8p^3$
e) $-2x-2$ **f)** $22x^2-14x$
g) 2 **h)** $\frac{5}{2}x^2-x$

2. **a)** $8(2p-q)$ **b)** $p(p-6q)$
c) $5pq(p-2q)$ **d)** $3pq(3-2p+4q)$

3. **a)** 0 **b)** -7 **c)** 29
d) 7 **e)** 7 **f)** 35

4. **a)** $n=p-4m$ **b)** $y=\frac{4x-5z}{3}$
c) $y=\frac{10px}{3}$ **d)** $y=\frac{3w}{m}-x$
e) $r=\frac{pqt}{4mn}$ **f)** $q=r(m-n)-p$

Student assessment 3 page 120

1. **a)** $(m-5n)(x-5)$
b) $(2x-9y)(2x+9y)$
c) 7600
d) $(x^2-y^2)(x^2+y^2)=(x-y)(x+y)(x^2+y^2)$

2. **a)** $x^2+8x+15$ **b)** $x^2-14x+49$
c) $x^2+10x+25$ **d)** $x^2-5x-14$
e) $6x^2+13x-8$ **f)** $25y^2-70y+49$

3. **a)** $(x-16)(x-2)$ **b)** $(x+4)(x-6)$
c) $(x-6)(x-3)$ **d)** $(x-1)^2$
e) $(2x-1)(x+3)$ **f)** $(3x-2)^2$

4. **a)** $a=\frac{v^2-u^2}{2s}$ **b)** $h=\pm\sqrt{p^2-r^2}$
c) $s=\pm\sqrt{\frac{rn}{m}}$ **d)** $g=\frac{4\pi^2 l}{t^2}$
e) $x=\frac{7-2y}{3}$ **f)** $x=\frac{5w}{3}$

5. **a)** b^2 **b)** x^4 **c)** $\frac{4n^2}{m}$
d) $\frac{p^3q}{r^2}$ **e)** $\frac{5t^2}{6}$ **f)** $\frac{2s^3}{3r}$

6. **a)** $\frac{12m+5n}{16}$ **b)** $\frac{3m}{4y}$ **c)** $\frac{5r}{12x}$
d) $\frac{8x-9y}{12}$ **e)** $\frac{6r}{7}$ **f)** $\frac{-5x-2}{3}$

7. **a)** $\frac{5m+13}{(m+2)(m+3)}$ **b)** $\frac{y+3}{y-3}$
c) $\frac{x}{x+7}$

Student assessment 4 page 121

1. **a)** $(q + r)(p - 3r)$
b) $(1 - t^2)(1 + t^2) = (1 - t)(1 + t)(1 + t^2)$
c) 750 000 **d)** 50

2. **a)** $x^2 - 2x - 8$ **b)** $x^2 - 16x + 64$
c) $x^2 + 2xy + y^2$ **d)** $x^2 - 121$
e) $6x^2 - 13x + 6$ **f)** $9x^2 - 30x + 25$

3. **a)** $(x - 11)(x + 7)$ **b)** $(x - 3)^2$
c) $(x - 12)(x + 12)$ **d)** $3(x - 2)(x + 3)$
e) $(2x - 3)(x + 4)$ **f)** $(2x - 5)^2$

4. **a)** $f = \pm\sqrt{\frac{p}{m}}$ **b)** $t = \pm\sqrt{\frac{m}{5}}$
c) $p = \left(\frac{A}{\pi r}\right)^2 - q$ **d)** $x = \frac{ty}{y - t}$
e) $p = 4q$ **f)** $t = \frac{r(s-1)}{2+r}$

5. **a)** x^4 **b)** nq **c)** y^3
d) $\frac{4}{q}$ **e)** $\frac{n^3}{2}$ **f)** $\frac{21bc^2}{4}$

6. **a)** $\frac{2m}{11}$ **b)** $-\frac{3p}{16}$ **c)** $\frac{3x}{4y}$
d) $\frac{6m + 13n}{30p}$ **e)** $\frac{14 - y}{3}$ **e)** $\frac{4y+9}{2}$

7. **a)** $\frac{7x - 23}{(x - 5)(x - 2)}$ **b)** $\frac{a - b}{a + b}$
c) $\frac{1}{x + 3}$

Student assessment 5 page 122

1. **a)** 0.204 m^3 (3 s.f.) **b)** $r = \sqrt{\frac{V}{\pi h}}$
c) r = 5.40 cm

2. **a)** 2410 cm^2 **b)** $h = \frac{A}{2\pi r} - r$
c) 10.9 cm

3. **a)** 5.39 cm (3 s.f.)
b) 3.68 m (3 s.f.)
c) $x = \sqrt{d^2 - y^2 - z^2}$
d) 0.713 m (3 s.f.)

4. **a)** 4.44 s (3 s.f.) **b)** $l = \frac{T^2 g}{4\pi^2}$
c) 2.28 m (3 s.f.)

12 Algebraic indices

Exercise 12.1 page 123

1. **a)** c^8 **b)** m^2 **c)** b^9
d) m^3n^6 **e)** $2a^4b$ **f)** $3x^3y^2$
g) $\frac{uv^3}{2}$ **h)** $\frac{x^2y^3z^2}{3}$

2. **a)** $12a^5$ **b)** $8a^5b^3$ **c)** $8p^6$
d) $16m^4n^6$ **e)** $200p^{13}$ **f)** $32m^5n^{11}$
g) $24xy^3$ **h)** $ab^{(d+e)}$

Exercise 12.2 page 124

1. **a)** c^3 **b)** g
c) q^{-2} **d)** m^{-1} or $\frac{1}{m}$

2. **a)** a^2 **b)** $\frac{r^{-6}}{p^{-6}}$ or $\frac{p^6}{r^6}$
c) t^{16} **d)** m^9

Exercise 12.3 page 125

1. **a)** $a^{\frac{3}{5}}$ **b)** $a^{\frac{1}{3}}$ **c)** a **d)** $a^{\frac{3}{7}}$

2. **a)** $(\sqrt[7]{b})^2$ **b)** $(\sqrt[3]{b})^8$ **c)** $(\sqrt[5]{b})^{-2}$ **d)** $(\sqrt[3]{b})^{-4}$

3. **a)** $a^{\frac{3}{4}}$ **b)** $a^{\frac{3}{20}}$ **c)** $a^{\frac{5}{2}}$ **d)** $a^{-\frac{1}{6}}$

4. **a)** $(\sqrt[20]{b})^{19}$ **b)** $(\sqrt[5]{b})^{-7}$ **c)** $(\sqrt[3]{b})^{14}$ **d)** $(\sqrt[6]{b})^{-11}$

5. **a)** $\frac{x^{\frac{5}{2}}}{12}$ **b)** $2y^{-\frac{1}{3}}$ **c)** $4p^{-\frac{5}{2}}$ **d)** $\frac{9}{2x^{\frac{1}{3}}}$

Student assessment 1 page 126

1. **a)** a^3b^2 **b)** d^2e^5

2. **a)** $m \times m \times m$ **b)** $r \times r \times r \times r$

3. **a)** a^7 **b)** $p^5 \times q^9$ **c)** b^3 **d)** e^6

4. **a)** r^4 **b)** $\frac{1}{b^2}$ or b^{-2} **c)** $\frac{1}{n^{-3}}$ or n^3

5. **a)** p^{-9} **b)** h^7

Student assessment 2 page 126

1. **a)** $a^{\frac{7}{8}}$ **b)** $a^{-\frac{2}{5}}$

2. **a)** $(\sqrt[9]{b^4})$ **b)** $(\sqrt[3]{b^{-2}})$

3. **a)** $a^{\frac{11}{6}}$ **b)** $a^{\frac{19}{6}}$

4. **a)** $(\sqrt{t})^3$ **b)** $(\sqrt[15]{t})^{-19}$

13 Equations and inequalities

Exercise 13.1 page 127

1. **a)** $x = -4$ **b)** $y = 5$ **c)** $y = -5$
d) $p = -4$ **e)** $y = 8$ **f)** $x = -5.5$

2. **a)** $x = 4\frac{1}{3}$ **b)** $x = 5$ **c)** $x = 6$
d) $y = -8$ **e)** $y = 4$ **f)** $m = 10$

3. **a)** $m = 1$ **b)** $p = 3$ **c)** $k = -1$
d) $x = -21$ **e)** $x = 2$ **f)** $y = 3$

4. **a)** $x = 6$ **b)** $y = 14$ **c)** $x = 4$
d) $m = 12$ **e)** $x = 35$ **f)** $p = 20$

5. **a)** $x = 15$ **b)** $x = -5$ **c)** $x = 7.5$
d) $x = 8$ **e)** $x = 2.5$ **f)** $x = 10$

6. **a)** $x = 5$ **b)** $x = 14$ **c)** $x = 22$
d) $x = 5$ **e)** $x = 8$ **f)** $x = 2$

7. **a)** $y = 10$ **b)** $x = 17$ **c)** $x = 13$
d) $y = -5$ **e)** $x = 4$ **f)** $x = 6.5$

Exercise 13.2 page 129

1. **a)** i) $3x + 60 = 180$ ii) $x = 40$
iii) 40°, 60°, 80°
b) i) $3x = 180$ ii) $x = 60$
iii) 20°, 80°, 80°
c) i) $18x = 180$ ii) $x = 10$
iii) 20°, 50°, 110°
d) i) $6x = 180$ ii) $x = 30$
iii) 30°, 60°, 90°
e) i) $7x - 30 = 180$ ii) $x = 30$
iii) 10°, 40°, 130°
f) i) $9x - 45 = 180$ ii) $x = 25$
iii) 25°, 55°, 100°

2. **a)** i) $12x = 360$ ii) $x = 30$
iii) 90°, 120°, 150°
b) i) $11x + 30 = 360$ ii) $x = 30$
iii) 90°, 135°, 135°
c) i) $12x + 60 = 360$ ii) $x = 25$
iii) 35°, 80°, 90°, 155°
d) i) $10x + 30 = 360$ ii) $x = 33$
iii) 33°, 94°, 114°, 119°

3. **a)** i) $11x - 80 = 360$ ii) $x = 40$
iii) 40°, 80°, 80°, 160°
b) i) $10x + 60 = 360$ ii) $x = 30$
iii) 45°, 90°, 95°, 130°
c) i) $16x + 8 = 360$ ii) $x = 22$
iii) 44°, 96°, 100°, 120°
d) i) $9x + 45 = 360$ ii) $x = 35$
iii) 35°, 55°, 120°, 150°
e) i) $6x + 18 = 180$ ii) $x = 27$
iii) 50°, 50°, 130°, 130°

4. **a)** 20 **b)** 25 **c)** 14 **d)** 25
e) 31 **f)** 40

5. **a)** 50 **b)** 13 **c)** 40 **d)** 40

6. **a)** 5 **b)** 2 **c)** 7 **d)** 1.1
e) 25 **f)** 15

Exercise 13.3 page 133

1. **a)** $x = 4$ $y = 2$ **b)** $x = 6$ $y = 5$
c) $x = 6$ $y = -1$ **d)** $x = 5$ $y = 2$
e) $x = 5$ $y = 2$ **f)** $x = 4$ $y = 9$

2. **a)** $x = 3$ $y = 2$ **b)** $x = 7$ $y = 4$
c) $x = 1$ $y = 1$ **d)** $x = 1$ $y = 5$
e) $x = 1$ $y = 10$ **f)** $x = 8$ $y = 2$

3. **a)** $x = 5$ $y = 4$ **b)** $x = 4$ $y = 3$
c) $x = 10$ $y = 5$ **d)** $x = 6$ $y = 4$
e) $x = 4$ $y = 4$ **f)** $x = 10$ $y = -2$

4. **a)** $x = 5$ $y = 4$ **b)** $x = 4$ $y = 2$
c) $x = 5$ $y = 3$ **d)** $x = 5$ $y = -2$
e) $x = 1$ $y = 5$ **f)** $x = -3$ $y = -3$

5. **a)** $x = -5$ $y = -2$ **b)** $x = -3$ $y = -4$
c) $x = 4$ $y = 3\frac{2}{3}$ **d)** $x = 2$ $y = 7$
e) $x = 1$ $y = 1$ **f)** $x = 2$ $y = 9$

6. **a)** $x = 2$ $y = 3$ **b)** $x = 5$ $y = 10$
c) $x = 4$ $y = 6$ **d)** $x = 4$ $y = 4$
e) $x = 5$ $y = 1$ **f)** $x = -3$ $y = -3$

7. **a)** $x = 1$ $y = -1$ **b)** $x = 11\frac{2}{3}$ $y = 8$
c) $x = 4$ $y = 0$ **d)** $x = 3$ $y = 4$
e) $x = 2$ $y = 8$ **f)** $x = 1$ $y = 1$

Exercise 13.4 page 135

1. **a)** $x = 2$ $y = 3$ **b)** $x = 1$ $y = 4$
c) $x = 5$ $y = 2$ **d)** $x = 3$ $y = 3$
e) $x = 4$ $y = 2$ **f)** $x = 6$ $y = 1$

2. **a)** $x = 1$ $y = 4$ **b)** $x = 5$ $y = 2$
c) $x = 3$ $y = 3$ **d)** $x = 6$ $y = 1$
e) $x = 2$ $y = 3$ **f)** $x = 2$ $y = 3$

3. **a)** $x = 0$ $y = 3$ **b)** $x = 5$ $y = 2$
c) $x = 1$ $y = 7$ **d)** $x = 6$ $y = 4$
e) $x = 2$ $y = 5$ **f)** $x = 3$ $y = 0$

4. **a)** $x = 1$ $y = 0.5$ **b)** $x = 2.5$ $y = 4$
c) $x = \frac{1}{5}$ $y = 4$ **d)** $x = \frac{3}{4}$ $y = \frac{1}{2}$
e) $x = 5$ $y = \frac{1}{3}$ **f)** $x = \frac{1}{2}$ $y = 1$

Exercise 13.5 page 135

1. 10 and 7

2. 16 and 9

3. $x = 1$ $y = 4$

4. $x = 5$ $y = 2$

5. 60 and 20 years old

6. 60 and 6 years old

Exercise 13.6 page 137

1. **a)** 7 **b)** 3 **c)** 6
d) 7 **e)** 10 **f)** 15

2. **a)** 7 **b)** 6 **c)** 6
d) 10 **e)** 3

3. **a)** 8 **b)** 5 **c)** 8
d) 6 **e)** 6

4. **a)** 6, 18, 26
b) 160, 214, 246, 246
c) 50°, 80°, 100°, 140°, 170°
d) 80°, 80°, 80°, 160°, 160°, 160°
e) 150°, 150°, 150°, 150°, 120°, 120°, 120°, 120°

Exercise 13.7 page 139

1. **a)** −4 and −3 **b)** −2 and −6
c) −1 and −12 **d)** 2 and 5
e) 2 and 3 **f)** 2 and 4

2. **a)** −5 and 2 **b)** −2 and 5 **c)** −7 and 2
d) −2 and 7 **e)** −5 and 3 **f)** −3 and 5

3. **a)** −3 and −2 **b)** −3 **c)** −8 and −3
d) 4 and 6 **e)** −4 and 3 **f)** −2 and 6

4. **a)** −2 and 4 **b)** −4 and 5 **c)** −6 and 5
d) −6 and 7 **e)** −7 and 9 **f)** −9 and 6

Exercise 13.8 page 139

1. **a)** −3 and 3 **b)** −4 and 4
c) −5 and 5 **d)** −11 and 11
e) −12 and 12 **f)** −15 and 15

2. **a)** −2.5 and 2.5 **b)** −2 and 2
c) −1.6 and 1.6 **d)** $-\frac{1}{2}$ and $\frac{1}{2}$
e) $-\frac{1}{3}$ and $\frac{1}{3}$ **f)** $-\frac{1}{20}$ and $\frac{1}{20}$

3. **a)** −4 and −1 **b)** −5 and −2
c) −4 and −2 **d)** 2 and 4
e) 2 and 5 **f)** −4 and 2

4. **a)** −2 and 5 **b)** −5 and 2 **c)** −3 and 6
d) −6 and 3 **e)** −4 and 6 **f)** −6 and 8

5. **a)** −4 and 3 **b)** −6 and −2 **c)** −9 and 4
d) −1 **e)** −2 **f)** −9 and −8

6. **a)** 0 and 8 **b)** 0 and 7 **c)** −3 and 0
d) −4 and 0 **e)** 0 and 9 **f)** 0 and 4

7. **a)** −1.5 and −1 **b)** −1 and 2.5
c) −1 and $\frac{1}{3}$ **d)** −5 and $-\frac{1}{2}$
e) 1.5 and 5 **f)** $-1\frac{1}{3}$ and $\frac{1}{2}$

8. **a)** −12 and 0 **b)** −9 and −3 **c)** −8 and 4
d) −7 and 2 **e)** −6 and 6 **f)** −10 and 10

Exercise 13.9 page 140

1. −4 and 3

2. −6 and 7

3. 2

4. 4

5. Height = 3 cm, base length = 12 cm

6. Height = 20 cm, base length = 2 cm

7. Base length = 6 cm, height = 5 cm

8. **a)** $9x + 14 = 50$ **b)** $x = 4$ **c)** 11 m × 6 m

Exercise 13.10 page 142

1. **a)** −3.14 and 4.14 **b)** −5.87 and 1.87
c) −6.14 and 1.14 **d)** −4.73 and −1.27
e) −6.89 and 1.89 **f)** 3.38 and 5.62

2. **a)** −5.30 and −1.70 **b)** −5.92 and 5.92
c) −3.79 and 0.79 **d)** −1.14 and 6.14
e) −4.77 and 3.77 **f)** −2.83 and 2.83

3. **a)** −0.73 and 2.73 **b)** −1.87 and 5.87
c) −1.79 and 2.79 **d)** −3.83 and 1.83
e) 0.38 and 2.62 **f)** 0.39 and 7.61

4. **a)** −0.85 and 2.35 **b)** −1.40 and 0.90
c) 0.14 and 1.46 **d)** −2 and −0.5
e) −0.39 and 1.72 **f)** −1.54 and 1.39

Exercise 13.11 page 144

1. **a)** $x < 4$

b) $x > 1$

c) $x \leqslant 3$

d) $x \geqslant 7$

e) $x < 2$

f) $x > 2$

2. a) $x < 7$

b) $x \geqslant -2$

c) $x > -3$

d) $x \geqslant -12$

e) $x > -24$

f) $x \geqslant -3$

3. a) $x < 2$

b) $x \leqslant 12$

c) $x \geqslant 2$

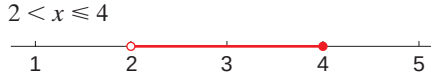
d) $x \geqslant -2$

e) $x > -0.5$

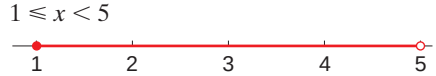
f) $x \geqslant 2$

Exercise 13.12 page 144

1. a) $2 < x \leqslant 4$



b) $1 \leqslant x < 5$

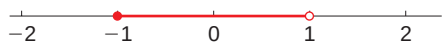


c) $3.5 \leqslant x < 5$

d) $2 \leqslant x < 4.2$

2. a) $1 < x \leqslant 5$

b) $-1 \leqslant x < 1$



c) $2 < x < 3$

d) No solution

Student assessment 1 page 145

1. **a)** 9 **b)** 11 **c)** −4 **d)** 6

2. **a)** 1.5 **b)** 7 **c)** 4 **d)** 3

3. **a)** −10 **b)** 12 **c)** 10 **d)** $11\frac{1}{4}$

4. **a)** 16 **b)** $-8\frac{2}{3}$ **c)** 2 **d)** 3.5

5. **a)** $x = 5$ $y = 2$ **b)** $x = 3\frac{1}{3}$ $y = 4\frac{1}{3}$
c) $x = 5$ $y = 4$ **d)** $x = 5$ $y = 1$

Student assessment 2 page 145

1. **a)** −6 **b)** 6 **c)** 4 **d)** 2.4

2. **a)** 0.5 **b)** 4 **c)** 9.5 **d)** 5

3. **a)** 6 **b)** 15 **c)** 22 **d)** 6

4. **a)** 8.5 **b)** $4\frac{1}{3}$ **c)** 8.5 **d)** 12

5. **a)** $x = 7$ $y = 4$ **b)** $p = 1$ $q = 2$
c) $x = 7$ $y = 1$ **d)** $m = 3$ $n = 5$

Student assessment 3 page 145

1. **a)** $4x + 40 = 180$ **b)** $x = 35$
c) 35°, 70°, 75°

2. 9

3. 30°, 30°, 30°, 30°, 30°, 30°, 45°, 45°, 45°, 45°

4. −5 and −1

5. 7.16 and 0.838 (3 s.f.)

6. $6 \leqslant x < 7$

7. None

Student assessment 4 page 146

1. **a)** $9x - 90 = 360$ **b)** $x = 50$
c) 50°, 60°, 100°, 150°

2. 2

3. 135°, 85°, 95°, 95°, 130°

4. −4 and 5

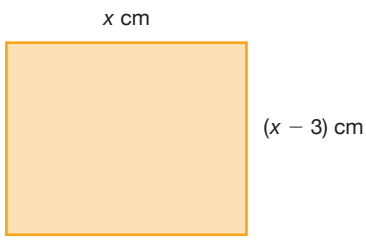
5. 2.77 and −1.27 (3 s.f.)

6. $3 < x \leqslant 5$

7. All values other than 0

Student assessment 5 page 147

1. **a)**



Perimeter = 54 cm

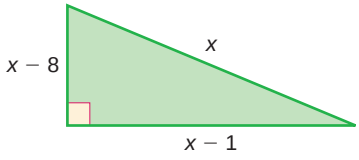
b) $4x - 6 = 54$
c) Length = 15 cm Width = 12 cm

2. **a)** $x, x - 8, x - 23$
b) $3x - 31 = 134$; 55, 47, 32

3. **b)** −0.317 and 6.32 (3 s.f.)

4. **a)** $x - y = 18$ $x + y + 70 + 40 = 360$
b) $x = 134$ $y = 116$

5. **a)**



c) 5 cm, 12 cm, 13 cm

Student assessment 6 page 148

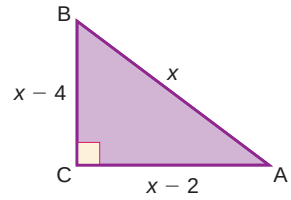
1. **a)** $x - y = 30$ $x + y + 40 = 180$
b) $x = 85$ $y = 55$

2. **b)** $5x + 100$ **c)** 92°
e) $92° + 82° + 72° + 62° + 52° = 360°$

3. **b)** $120 - 4x^2$
c) $4x^2 - 64 = 0$ $x = 4$

4. **b)** 0.44 and 4.56 (2 d.p.)

5. **a)**



c) 10 cm, 8 cm, 6 cm

14 Linear programming

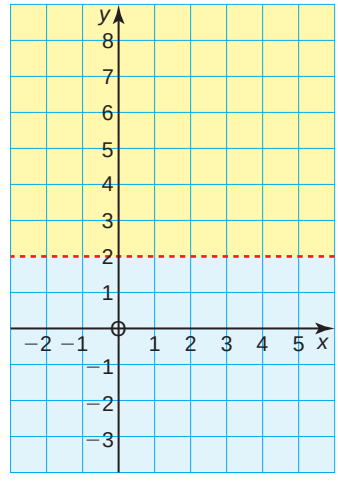
Exercise 14.1 page 149

1. **a)** $x < 2$ **b)** $y \geqslant 2$ **c)** $x \leqslant -2$
d) $y \leqslant 6$ **e)** $t > 0$ **f)** $p \leqslant -3$

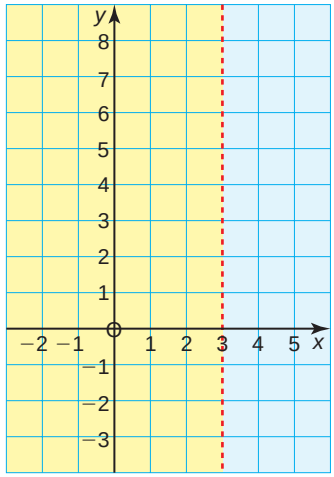
2. **a)** $2 < y \leqslant 4$ **b)** $1 \leqslant p < 5$
c) $5 \leqslant m < 7$ **d)** $3 < x < 4$

Exercise 14.2 page 150

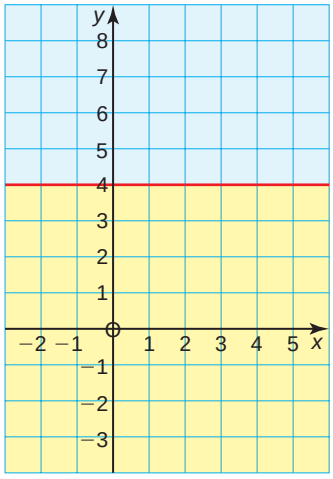
1. **a)**



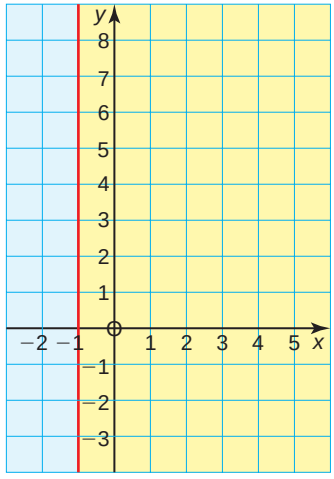
b)


y
x
−2 −1
1 2 3 4 5
8 7 6 5 4 3 2 1
−1 −2 −3

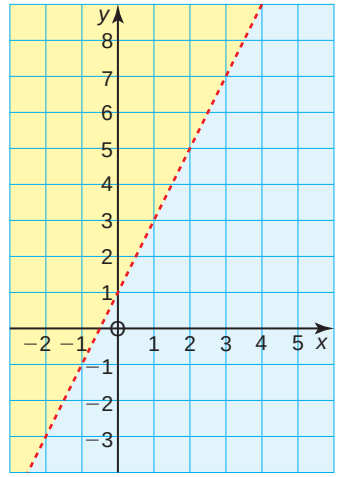
c)


y
x
−2 −1
1 2 3 4 5
8 7 6 5 4 3 2 1
−1 −2 −3

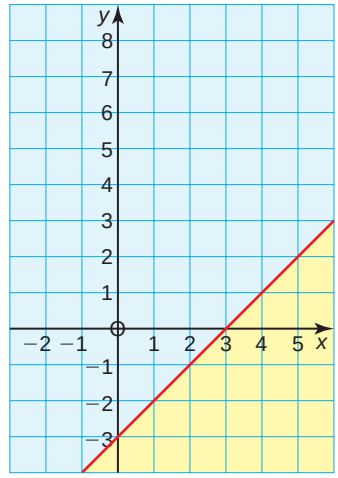
d)


y
x
−2 −1
1 2 3 4 5
8 7 6 5 4 3 2 1
−1 −2 −3

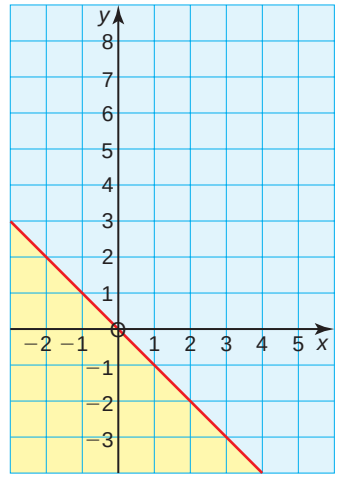
e)


y
x
−2 −1
1 2 3 4 5
8 7 6 5 4 3 2 1
−1 −2 −3

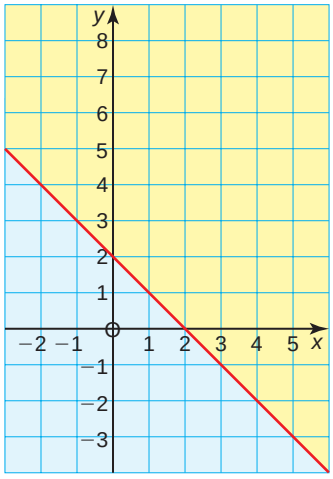
f)


y
x
−2 −1
1 2 3 4 5
8 7 6 5 4 3 2 1
−1 −2 −3

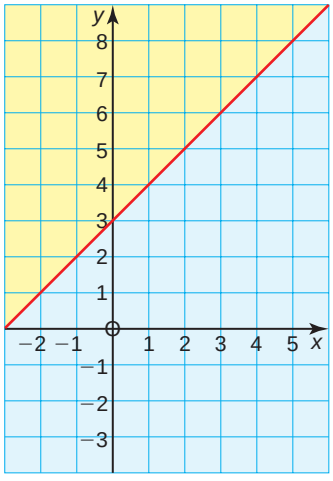
2. a)


y
x
−2 −1
1 2 3 4 5
8 7 6 5 4 3 2 1
−1 −2 −3

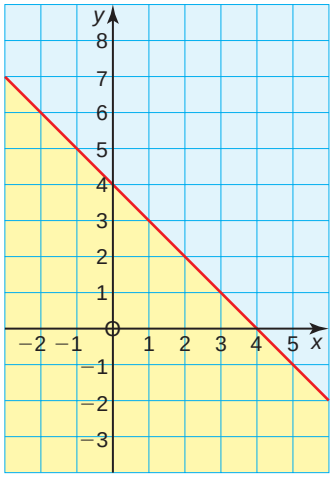
b)



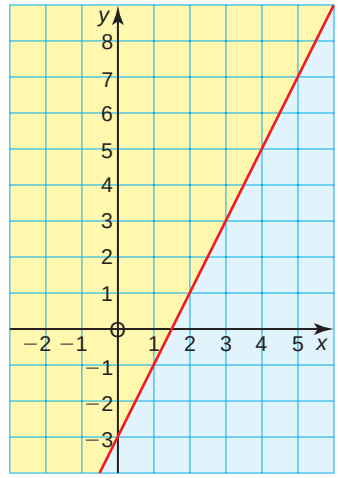
c)



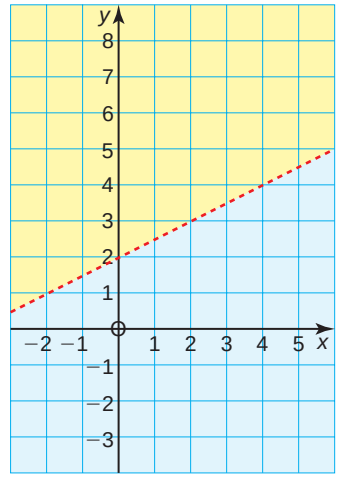
d)



e)

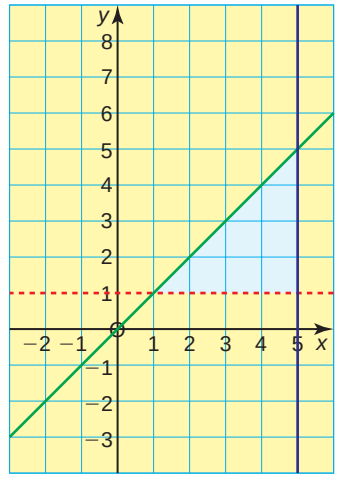


f)

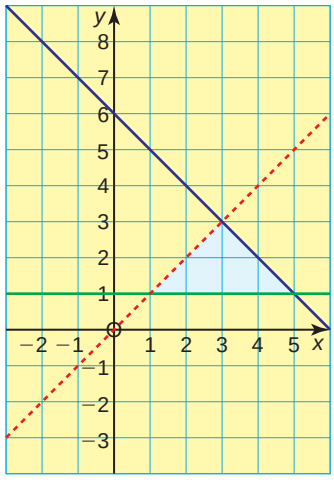


Exercise 14.3 page 152

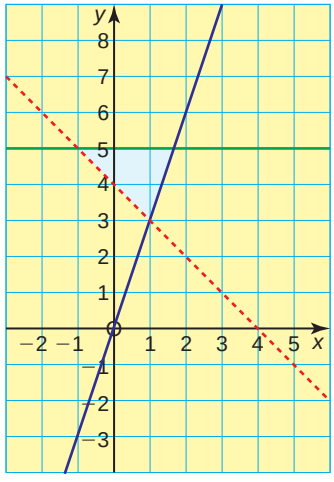
1.



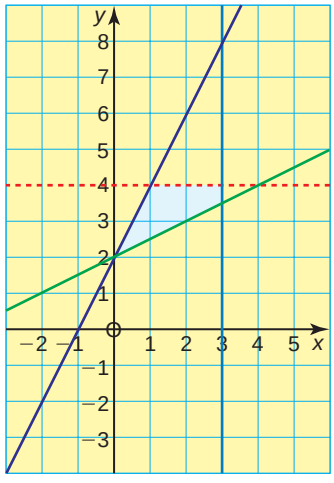
2.



3.



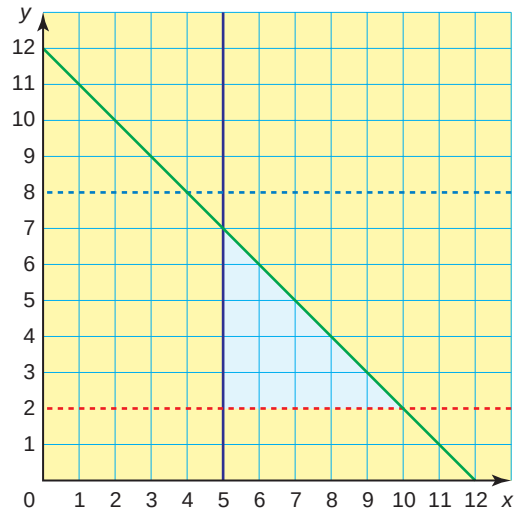
4.



Exercise 14.4 page 153

1. **a)** $x \geqslant 5 \quad 2 < y < 8 \quad x + y \leqslant 12$

b)



c) Any integer point in the unshaded region, e.g. (5, 3) meaning 5 car trips and 3 minibus trips

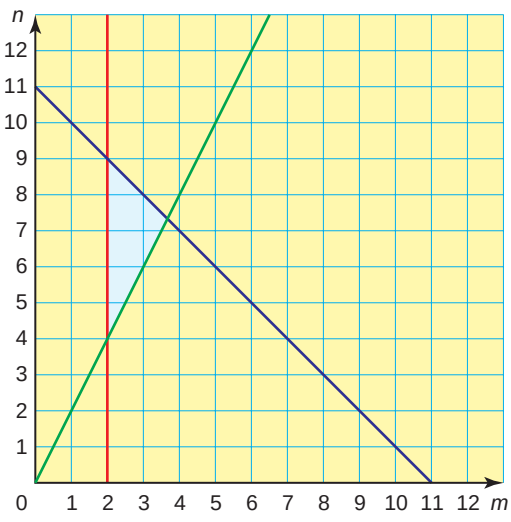
2. **a)** $p \geqslant 5 \quad q \geqslant 2 \quad p + q \leqslant 10$

b)

c) Any integer point in the unshaded region, e.g. (5, 2) meaning 5 loaves and 2 cakes

3. **a)** $m \geqslant 2 \quad n \geqslant 2m \quad m + n \leqslant 11$

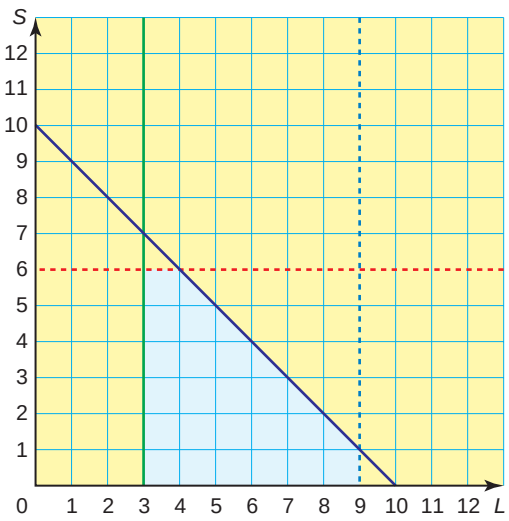
b)



c) Any integer point in the unshaded region, e.g. (2, 4) meaning 2 long curtains and 4 short ones

4. **a)** $3 \leqslant L < 9 \quad S < 6 \quad L + S \leqslant 10$

b)



c) Any of the points in the unshaded region, e.g. (3, 0) meaning 3 large oranges and no small ones.

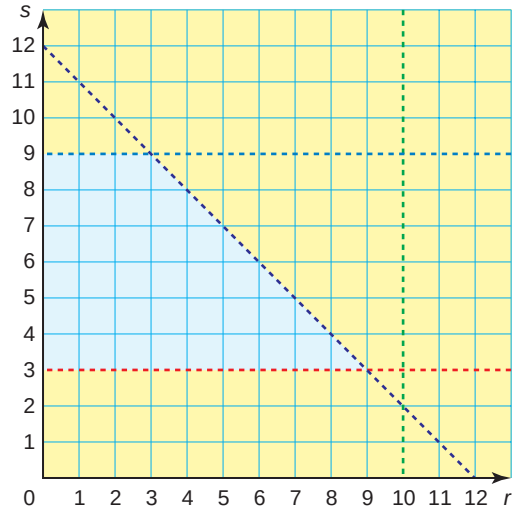
Student assessment 1 page 153

1. **a)** $x \leqslant 5$ **b)** $y \leqslant 3$

2. **a)** $2 < y \leqslant 4$ **b)** $4 < p \leqslant 8$

3. **a)** $r < 10 \quad 3 < s < 9 \quad s + r < 12$

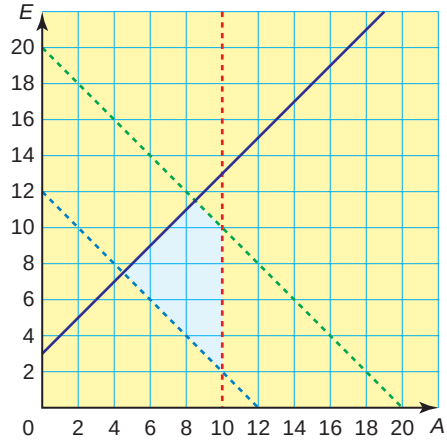
b)



c) Student's own answer

4. **a)** $12 < A + E < 20 \quad A < 10 \quad E \leqslant A + 3$

b)



c) Student's own answers

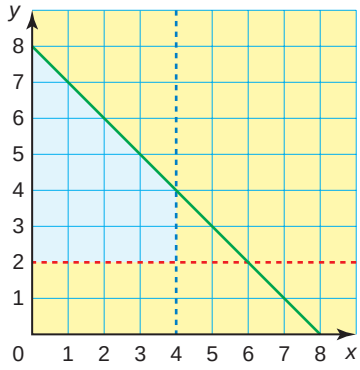
Student assessment 2 page 154

1. **a)** $x \leqslant 7$ **b)** $y \leqslant 9$

2. **a)** $1 \leqslant p < 4$ **b)** $4 < x \leqslant 7$

3. **a)** $x < 4 \quad y > 2 \quad x + y \leqslant 8$

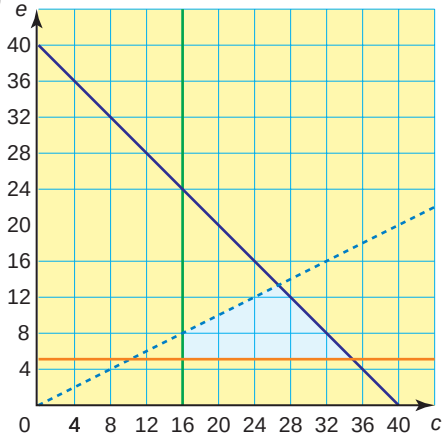
b)



c) Student's own answers

4. **a)** $c + e \leqslant 40 \quad c \geqslant 16 \quad e \geqslant 5 \quad c > 2e$

b)



c) Student's own answers

15 Sequences

Exercise 15.1 page 157

1. **a)** i) $3n + 2$
ii) 32

b) i) $4n - 4$
ii) 36

c) i) $n - 0.5$
ii) 9.5

d) i) $-3n + 9$
ii) -21

e) i) $3n - 10$
ii) 20

f) i) $-4n - 5$
ii) -45

2. **a)**

Position	1	2	5	12	50	n
Term	1	5	17	45	197	$4n - 3$

b)

Position	1	2	5	10	75	n
Term	5	11	29	59	449	$6n - 1$

c)

Position	1	3	8	50	100	n
Term	2	0	-5	-47	-97	$-n + 3$

d)

Position	1	2	3	10	100	n
Term	3	0	-3	-24	-294	$-3n + 6$

e)

Position	2	5	7	10	50	n
Term	1	10	16	25	145	$3n - 5$

f)

Position	1	2	5	20	50	n
Term	-5.5	-7	-11.5	-34	-79	$-1.5n - 4$

3. **a)** i) $+4$
ii) $4n + 1$
iii) 201

b) i) $+1$
ii) $n - 1$
iii) 49

c) i) $+3$
ii) $3n - 13$
iii) 137

d) i) $+0.5$
ii) $0.5n + 5.5$
iii) 30.5

e) i) $+4$
ii) $4n - 62$
iii) 138

f) i) -3
ii) $-3n + 75$
iii) -75

Exercise 15.2 page 161

1. $u_n = n^2 + 1$

2. $u_n = n^2 - 1$

3. $u_n = n^2 + 5$

4. $u_n = n^2 + 8$

5. $u_n = n^2 - 3$

6. $u_n = 2n^2 + 2$

7. $u_n = 2n^2 - 2$

8. $u_n = 3n^2 + 2$

9. $u_n = 4n^2 - 4$

10. $u_n = 5n^2 - 4$

Exercise 15.3 page 161

1. $u_n = n^3 + 10$

2. $u_n = n^3 - n$

3. $u_n = n^3 - 5$

4. $u_n = n^3 + n^2$

5. $u_n = n^3 + n^2 + 5n$

6. $u_n = n^3 + 3n^2 + 5n - 2$

7. $u_n = n^3 + n^2 - n$

8. $u_n = n^3 + 5n + 7$

Exercise 15.4 page 164

1. **a)** Geometric **b)** Geometric
c) Not geometric **d)** Geometric
e) Not geometric **f)** Not geometric

2 **a)** i) 3 **b)** i) $\frac{1}{5}$
ii) 162, 486 ii) $\frac{1}{25}, \frac{1}{125}$
iii) $u_n = 2(3)^{n-1}$ iii) $u_n = 25(\frac{1}{5})^{n-1}$
d) i) -3
ii) $-243, 729$
iii) $u_n = (-3)^n$

3 **a)** $-6, -12, -24$ **b)** 8

4 **a)** -4 **b)** $\frac{1}{4}$ **c)** $-65\,536$

5. \$38 203.20

6. **a)** Because $0.8^5 \neq 0$
b) \$3276.80
c) Because $0.8^n \neq 0$ where n is the number of year

Student assessment 1 page 165

1. **a)** i) 45, 54 ii) Terms increasing by 9
b) i) 30, 24 ii) Terms decreasing by 6
c) i) 2.25, 1.125 ii) Terms halving
d) i) $-12, -18$ ii) Terms decreasing by 6
e) i) 27, 8 ii) Descending order of cube numbers
f) i) 81, 243 ii) Terms multiplied by 3

2. **a)** $4n + 2$ **b)** $6n + 7$ **c)** $6n - 3$
d) $n^2 + 3$ **e)** $10n - 10$ **f)** $n^3 - 1$

3. **a)** i) $u_n = 4n - 3$
ii) 37
b) i) $u_n = -3n + 4$
ii) -26

4. **a)** $u_5 = 27, u_{100} = 597$
b) $u_5 = \frac{3}{2}, u_{100} = -46$

5. **a)**

Position	1	2	3	10	25	n
Term	17	14	11	-10	-55	$-3n + 20$

b)

Position	2	6	10	80	n
Term	-4	-2	0	35	$\frac{1}{2}n - 5$

Student assessment 2 page 166

1. **a)** \$515.46 **b)** 3 years

2. **a)** \$253.50 **b)** \$8.08

3. **a)** $-\frac{1}{3}$ **b)** 243 **c)** 10

4. $u_n = n^3 + 3n^2 + 4$

5. $u_n = n^3 + n^2 + 3n + 5$

16 Variation

Exercise 16.1 page 170

1. **a)** 3 **b)** 21 **c)** 27 **d)** 3 **e)** 10

2. **a)** 0.5 **b)** 8 **c)** 24.5 **d)** 8 **e)** 16

3. **a)** 24 **b)** $\frac{3}{8}$ **c)** $\frac{1}{9}$ **d)** 1

4. **a)** 0.25 **b)** 25 **c)** 4 **d)** $\frac{1}{16}$

Exercise 16.2 page 170

1. **a)** i) $y \propto x^3$ ii) $y = kx^3$
b) i) $y \propto \frac{1}{x^3}$ ii) $y = \frac{k}{x^3}$
c) i) $t \propto P$ ii) $t = kP$
d) i) $s \propto \frac{1}{t}$ ii) $s = \frac{k}{t}$
e) i) $A \propto r^2$ ii) $A = kr^2$
f) i) $T \propto \frac{1}{\sqrt{g}}$ ii) $T = \frac{k}{\sqrt{g}}$

2. 10.5

3. **a)** $\frac{1}{2}$ **b)** 2

4. 32

5. **a)** $\frac{1}{8}$ **b)** 0.4

6. 75

Exercise 16.3 page 171

1. **a)** $h = kt^2$ **b)** 5 **c)** 45 m **d)** 6 s

2. **a)** $v = k\sqrt{e}$ **b)** 3 **c)** 49 J

3. **a)** $l = km^{\frac{1}{3}}$ **b)** 3 **c)** $l = 6$ cm

4. **a)** $P = kI^2$ **b)** 5 amps

Student assessment 1 page 172

1. **a)** 1.5 **b)** 15 **c)** 3 **d)** 12

2. **a)** 10 **b)** 2.5 **c)** 1 **d)** 20

3. **a)** $\frac{1}{3}$ **b)** 72 **c)** $\frac{1}{3}$ **d)** 12

4. **a)** 5 **b)** $\frac{5}{4}$ **c)** $\frac{1}{2}$ **d)** 1

5. **a)** $\frac{1}{3}$ **b)** $\frac{4}{3}$ **c)** ± 2 **d)** ± 1

Student assessment 2 page 173

1. **a)**

x	1	2	4	8	16	32
y	32	16	8	4	2	1

b) 1.6

2. **a)**

x	1	2	4	5	10
y	5	10	20	25	50

b)

x	1	2	4	5	10
y	20	10	5	4	2

c)

x	4	16	25	36	64
y	4	8	10	12	16

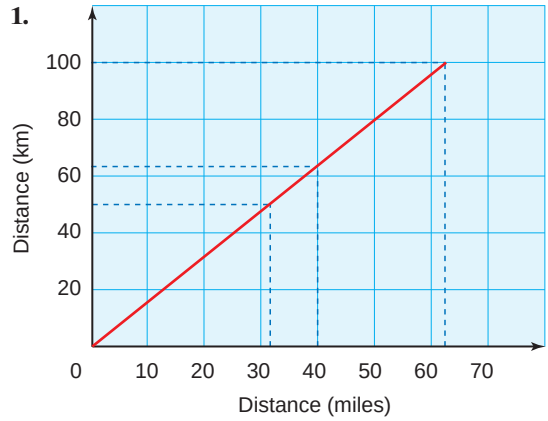
3. **a)** 0.8 **b)** 0.8

4. **a)** 1.6 (1 d.p.) **b)** 2

17 Graphs in practical situations

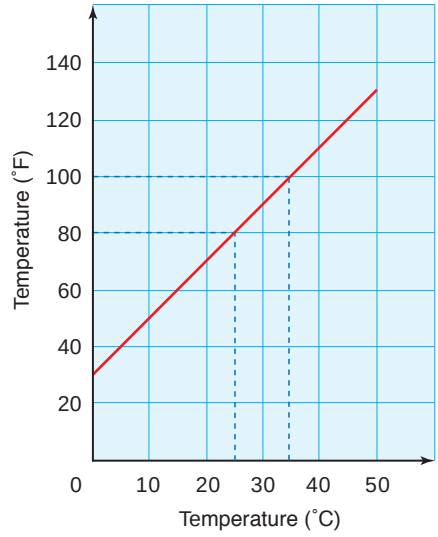
Exercise 17.1 page 175

1.



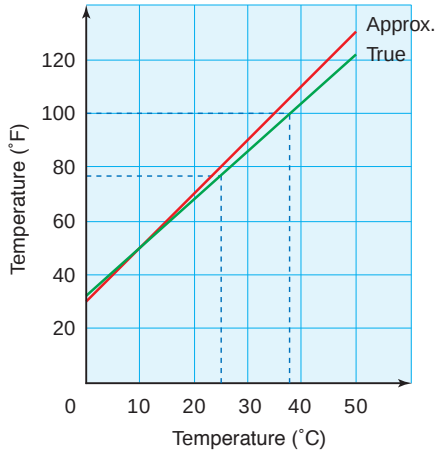
a) 50 km = 31 miles
b) 40 miles = 64 km, therefore 80 miles = 128 km
c) 100 km/h = 62 mph
d) 40 mph = 64 km/h

2.



a) 25 °C = 80 °F
b) 100 °F = 35 °C
c) 0 °C = 30 °F
d) 100 °F = 35 °C, therefore 200 °F = 70 °C

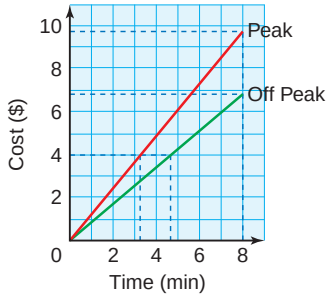
3.



i) a) 25 °C = 77 °F **b)** 100 °F = 38 °C
c) 0 °C = 32 °F
d) 100 °F = 38 °C, therefore 200 °F = 76 °C

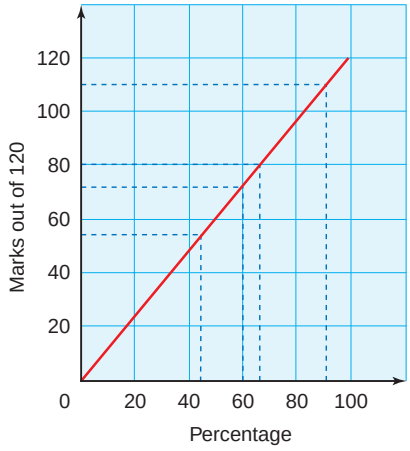
ii) The rough conversion is most useful at lower temperatures (i.e. between 0 and 20 °C).

4. a)



b) 8 min = $6.80 **c)** 8 min = $9.60
d) Extra time = 1 min 20 s

5.



a) 80 = 67% **b)** 110 = 92%
c) 54 = 45% **d)** 72 = 60%

Exercise 17.2 page 176

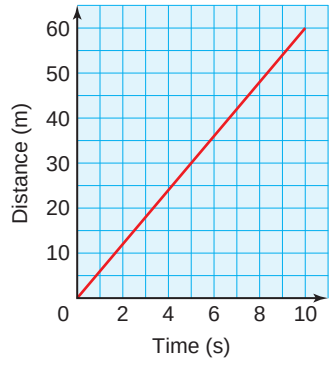
1. **a)** 6 m/s **b)** 4 m/s **c)** 39 km/h
d) 20 km/h **e)** 160 km/h **f)** 50 km/h

2. **a)** 400 m **b)** 182 m **c)** 210 km
d) 255 km **e)** 10 km **f)** 79.2 km

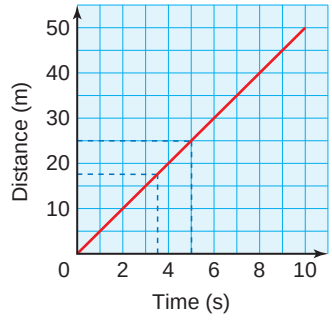
3. **a)** 5 s **b)** 50 s **c)** 4 min
d) 1 min 11.4 s
e) 5 s
f) 4 min

Exercise 17.3 page 176

1.



2.



a) 5 s **b)** 17.5 m

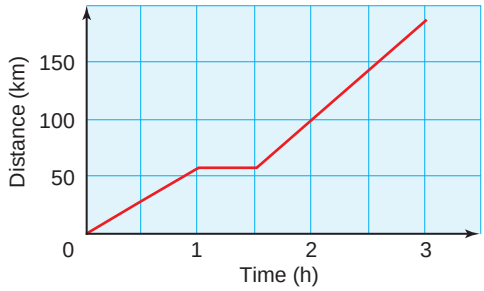
3. **a)** Speed A = 40 m/s Speed B = $13\frac{1}{3}$ m/s
b) Distance apart = $453\frac{1}{3}$ m

4. **a)** $\frac{2}{3}$ m/s **b)** 6 m/s, $\frac{2}{3}$ m/s **c)** 1 m/s
d) $\frac{1}{2}$ m **e)** $7\frac{1}{3}$ m

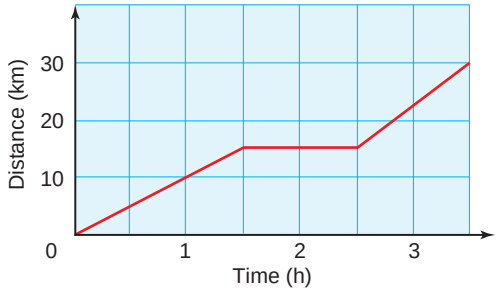
Exercise 17.4 page 178

1. **a)** 45 km/h **b)** 20 km/h
c) Paul has arrived at the restaurant and is waiting for Helena.

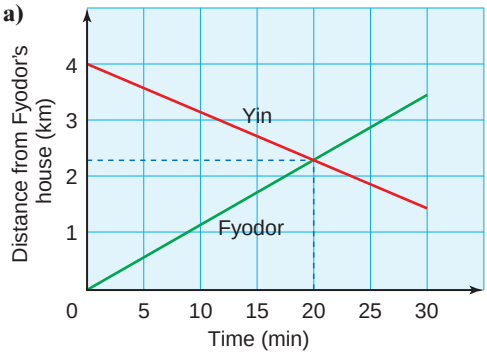
2.



3.

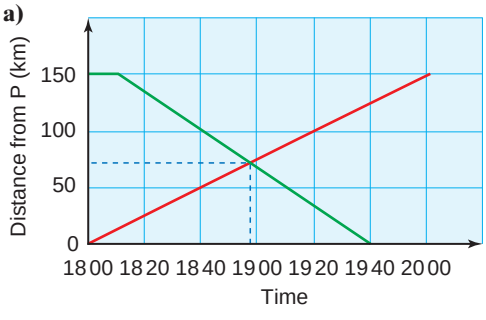


4. **a)**



b) After 20 min **c)** Distance = $2\frac{1}{3}$ km

5. **a)**



b) Time ≈ 18 57
c) Distance from Q ≈ 79 km
d) The 18 10 train from station Q arrives first.

6. **a)** a: $133\frac{1}{3}$ km/h b: 0 km/h c: 200 km/h
b) d: 100 km/h e: 200 km/h

Exercise 17.5 page 180

1. Acceleration is 0.375 m/s^2

2. Acceleration is 0.2 m/s^2

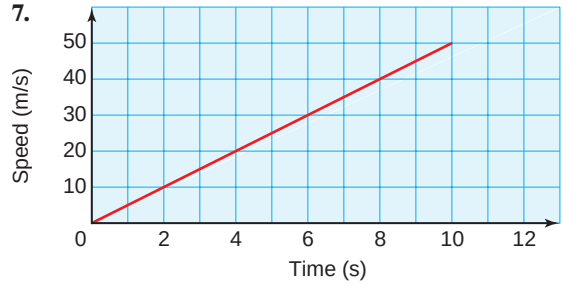
3. Acceleration is 7 m/s^2

4. Acceleration is 1.75 m/s^2

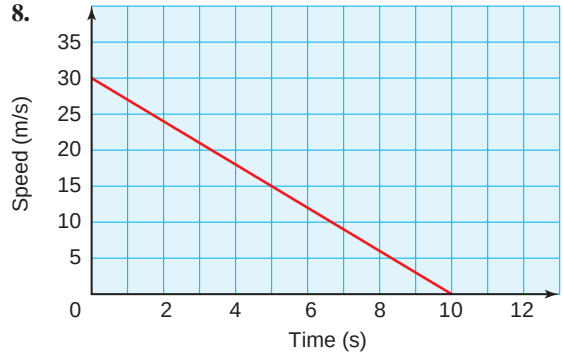
5. Deceleration is 0.25 m/s^2

6. Deceleration is 1 m/s^2

7.



8.



Exercise 17.6 page 180

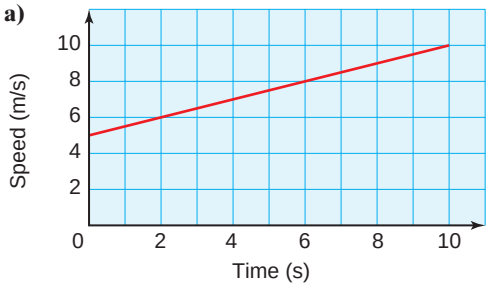
1. **a)** 1.5 m/s^2 **b)** 0 m/s^2 **c)** 0.5 m/s^2

2. **a)** Cheetah **b)** 7.5 m/s^2 **c)** 5 m/s^2
d) 15 m/s^2

3. **a)** 0.5 m/s^2
b) 0.25 m/s^2
c) 0.104 m/s^2 (3 s.f.)
d) Travelling at a constant speed of 30 m/s
e) Stationary

Exercise 17.7 page 182

1. a)

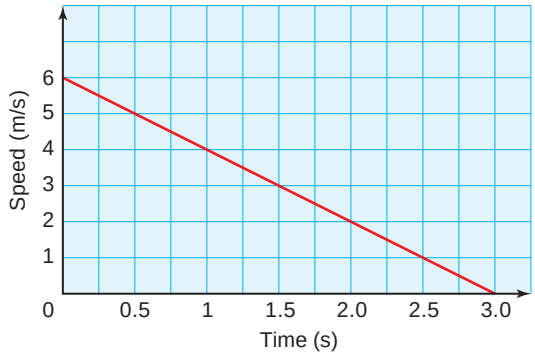


b) 0.5 m/s^2 **c)** 75 m

2. a)

Time (s)	0	0.5	1	1.5	2	2.5	3
Speed (m/s)	6	5	4	3	2	1	0

b)



c) 9 m

3. a) 1.5 m/s^2 **b)** 2400 m **c)** 40 s

4. a) 390 m **b)** 240 m

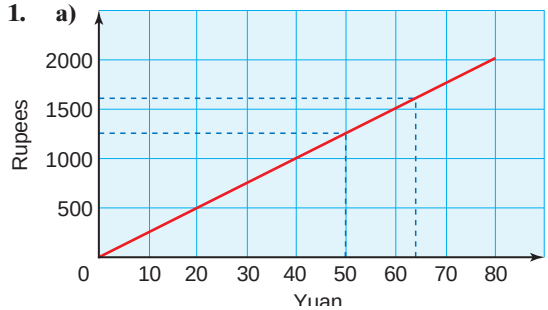
5. 21.45 km

6. 720 m

7. a) 0.37 m/s^2 **b)** 2.16 m/s^2 **c)** 208 m
d) 204 m **e)** 4 m

Student assessment 1 page 184

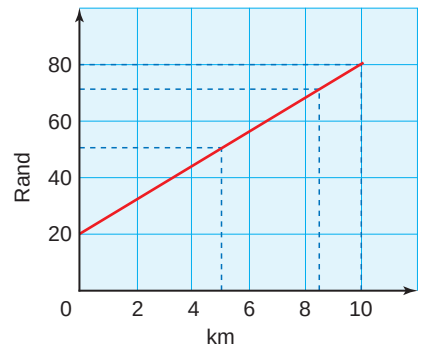
1. a)



b) 50 yuan = 1250 rupees

c) 1600 rupees = 64 yuan

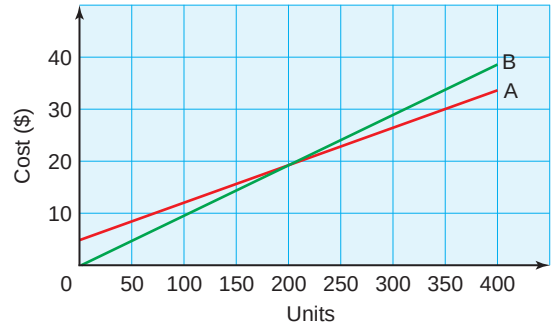
2. a)



i) 5 km: 50 rand ii) 8.5 km: 71 rand

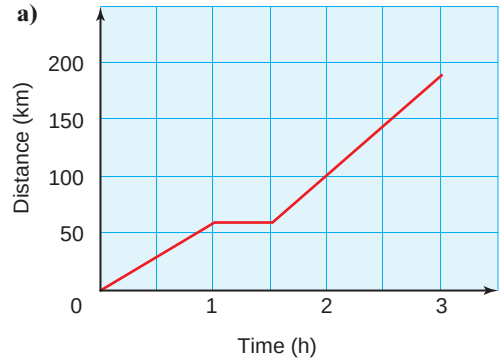
b) 80 rand: 10 km

3. a)



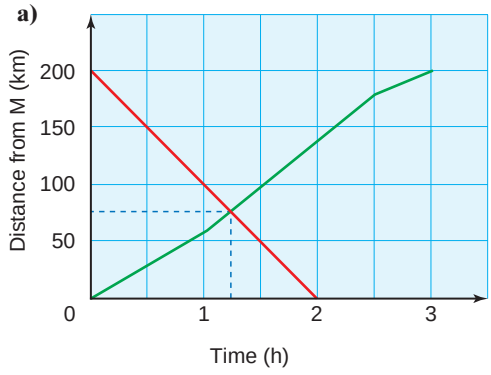
b) If the customer uses under 200 units/ quarter then he or she should use account type B.

4. **a)**



b) 180 km

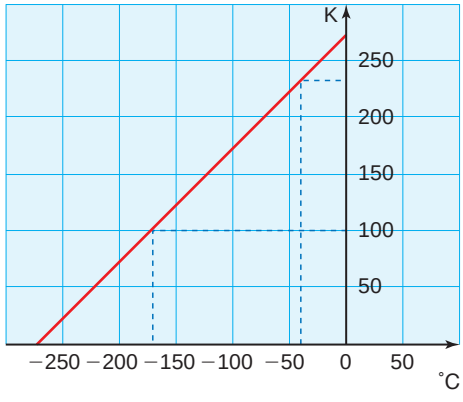
5. **a)**



b) Distance from M ≈ 77 km
c) Time ≈ 1 h 13 min after start

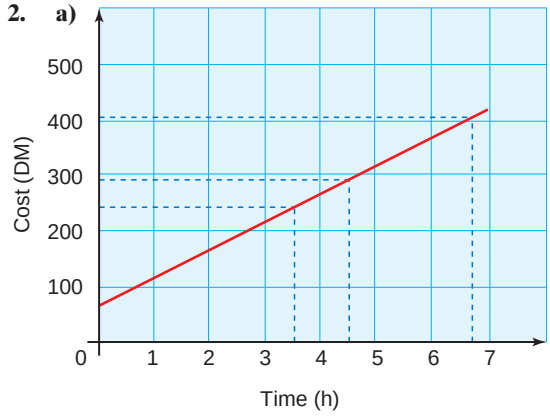
Student assessment 2 page 185

1.



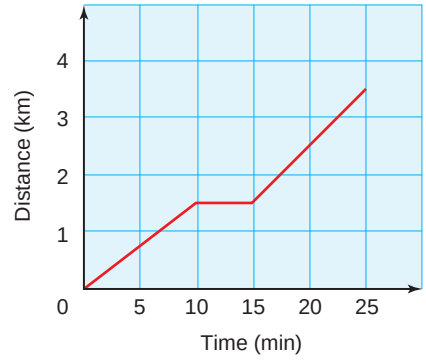
a) −40 °C = 233 K
b) 100 K = −173 °C

2. **a)**



b) $295 **c)** ≈$408 **d)** $3\frac{1}{2}$ h

3. **a)**



b) 25 min

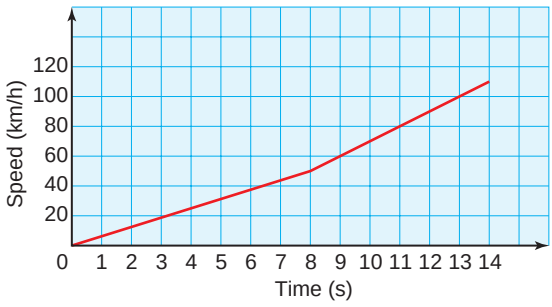
4. **a)** B, C
b) B because it illustrates going back in time,
C because it illustrates infinite speed

Student assessment 3 page 186

1. **a)** 2 m/s^2 **b)** 225 m **c)** 10.6 s (3 s.f.)

2. **a)** 4 m/s^2 **b)** 3 m/s^2 **c)** 102 m **d)** 9.83 s

3. a)



b) 189 m (3 s.f.)

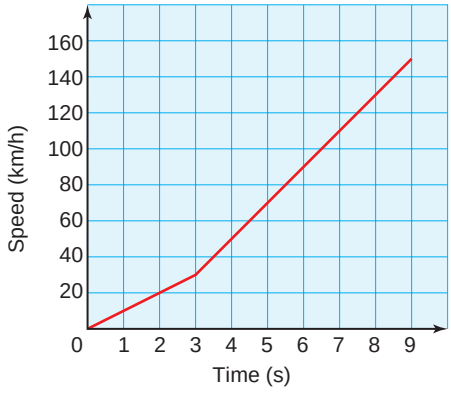
4. a) 1 m/s^2
b) 750 m
c) 1237.5 m

Student assessment 4 page 187

1. a) 2.5 m/s^2
b) 180 m
c) 3.5 s (1 d.p.)

2. a) 0.278 m/s^2 (3 s.f.)
b) 93.8 m (3 s.f.)
c) 97.2 m (3 s.f.)

3. a)



b) 162.5 m

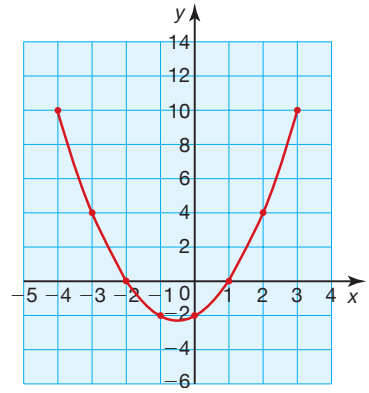
4. a) 3.33 m/s^2 (3 s.f.)
b) 240 m
c) 212.5 m

18 Graphs of functions

Exercise 18.1 page 190

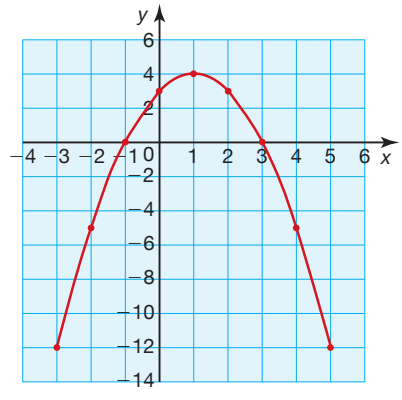
1.

x	−4	−3	−2	−1	0	1	2	3
y	10	4	0	−2	−2	0	4	10



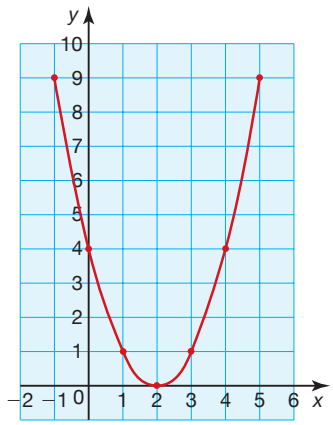
2.

x	−3	−2	−1	0	1	2	3	4	5
y	−12	−5	0	3	4	3	0	−5	−12



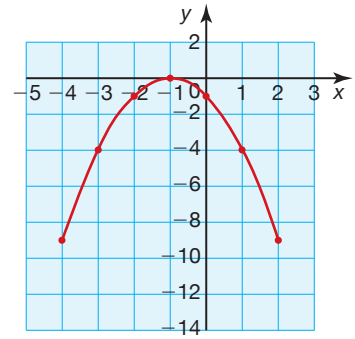
3.

x	−1	0	1	2	3	4	5
y	9	4	1	0	1	4	9



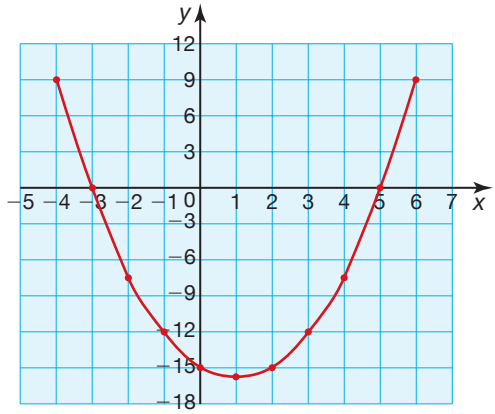
4.

x	−4	−3	−2	−1	0	1	2
y	−9	−4	−1	0	−1	−4	−9



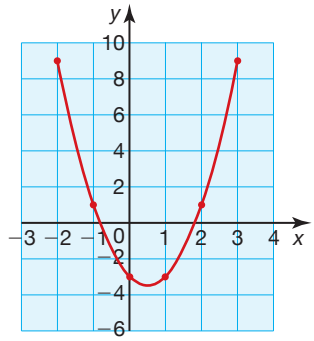
5.

x	−4	−3	−2	−1	0	1	2	3	4	5	6
y	9	0	−7	−12	−15	−16	−15	−12	−7	0	9



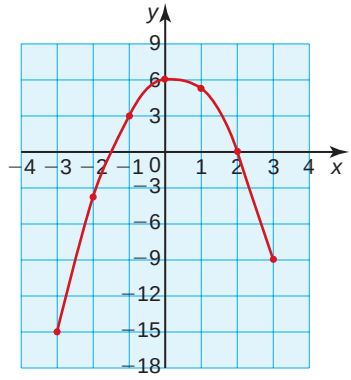
6.

x	−2	−1	0	1	2	3
y	9	1	−3	−3	1	9



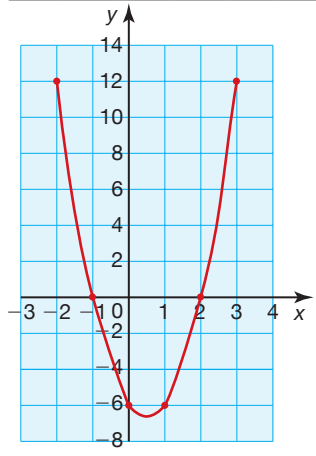
7.

x	−3	−2	−1	0	1	2	3
y	−15	−4	3	6	5	0	−9



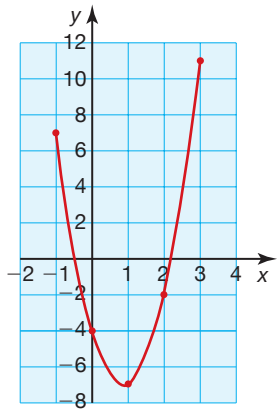
8.

x	−2	−1	0	1	2	3
y	12	0	−6	−6	0	12



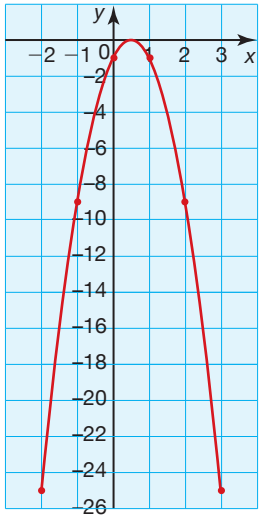
9.

x	−1	0	1	2	3
y	7	−4	−7	−2	11



10.

x	−2	−1	0	1	2	3
y	−25	−9	−1	−1	−9	−25



Exercise 18.2 page 191

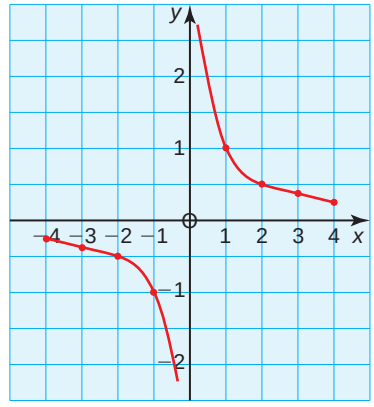
1. −2 and 3
2. −1 and 1
3. 3
4. −4 and 3
5. 2
6. 0.5 and 3
7. 1
8. $-\frac{1}{3}$ and 2

Exercise 18.3 page 192

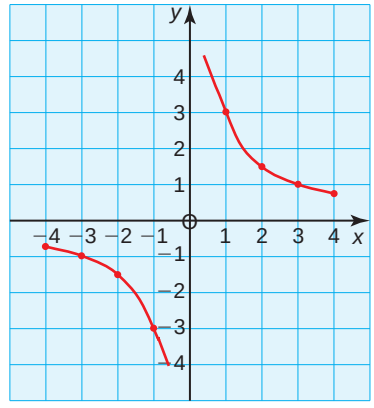
1. −1.6 and 2.6
2. No solution
3. 2 and 4
4. −3.5 and 2.5
5. 0.3 and 3.7
6. 0 and 3.5
7. −0.2 and 2.2
8. $-\frac{1}{3}$ and 2

Exercise 18.4 page 193

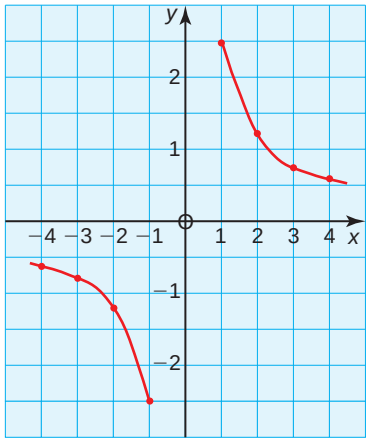
1.



2.



3.

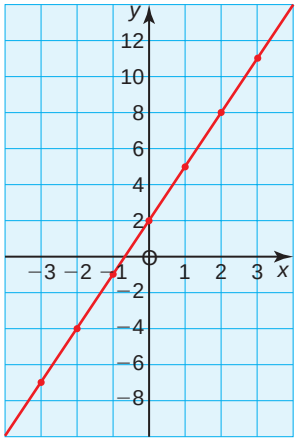


Exercise 18.5 page 194

1. i)

x	−3	−2	−1	0	1	2	3
$f(x)$	−7	−4	−1	2	5	8	11

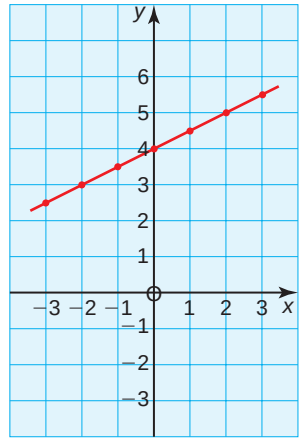
ii)



2. i)

x	−3	−2	−1	0	1	2	3
$f(x)$	2.5	3	3.5	4	4.5	5	5.5

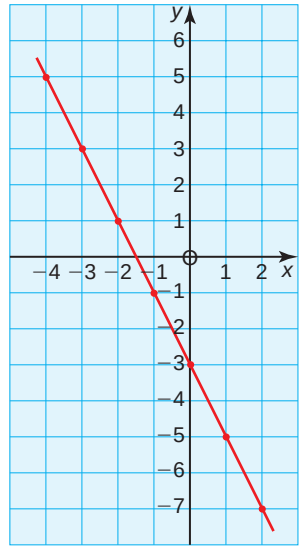
ii)



3. i)

x	−4	−3	−2	−1	0	1	2
$f(x)$	5	3	1	−1	−3	−5	−7

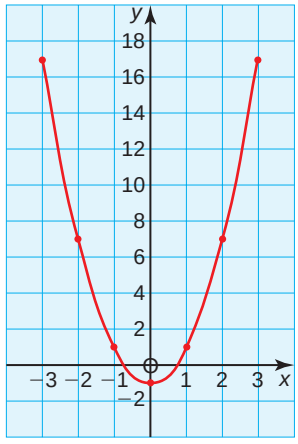
ii)



4. i)

x	−3	−2	−1	0	1	2	3
$f(x)$	17	7	1	−1	1	7	17

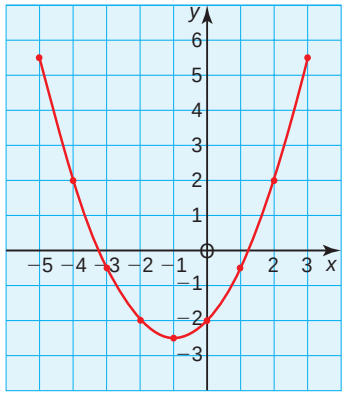
ii)



5. i)

x	−5	−4	−3	−2	−1	0	1	2	3
$f(x)$	5.5	2	−0.5	−2	−2.5	−2	−0.5	2	5.5

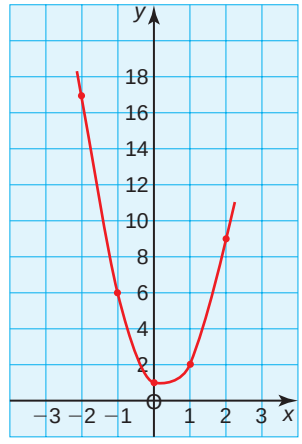
ii)



6. i)

x	−2	−1	0	1	2
$f(x)$	17	6	1	2	9

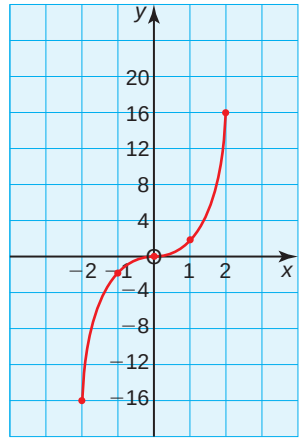
ii)



7. i)

x	−2	−1	0	1	2
$f(x)$	−16	−2	0	2	16

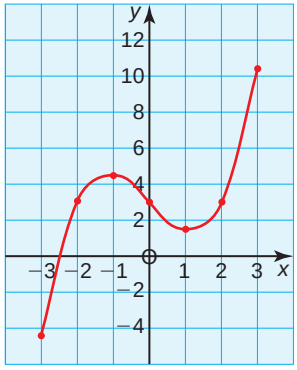
ii)



8. i)

x	−3	−2	−1	0	1	2	3
$f(x)$	−4.5	3	4.5	3	1.5	3	10.5

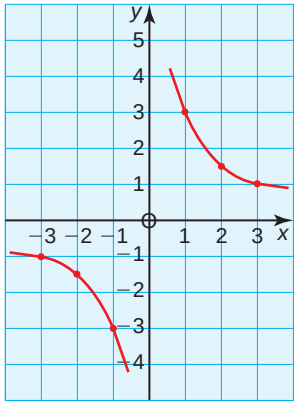
ii)



9. i)

x	−3	−2	−1	0	1	2	3
$f(x)$	−1	−1.5	−3	–	3	1.5	1

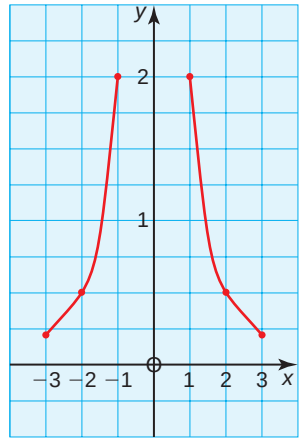
ii)



10. i)

x	−3	−2	−1	0	1	2	3
$f(x)$	0.22	0.5	2	–	2	0.5	0.22

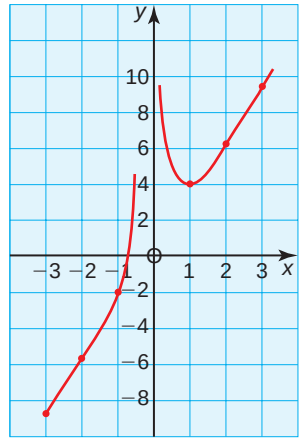
ii)



11. i)

x	−3	−2	−1	0	1	2	3
$f(x)$	−8.88	−5.75	−2	–	4	6.25	9.11

ii)

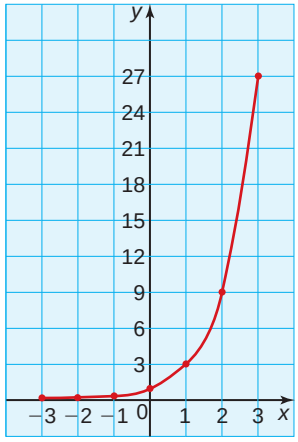


Exercise 18.6 page 195

1. i)

x	−3	−2	−1	0	1	2	3
$f(x)$	0.04	0.11	0.33	1	3	9	27

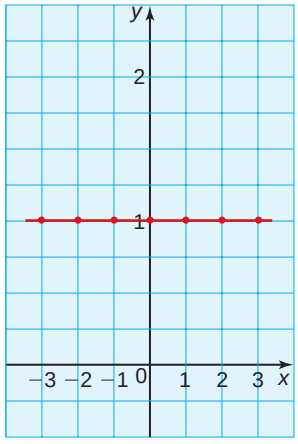
ii)



2. i)

x	−3	−2	−1	0	1	2	3
$f(x)$	1	1	1	1	1	1	1

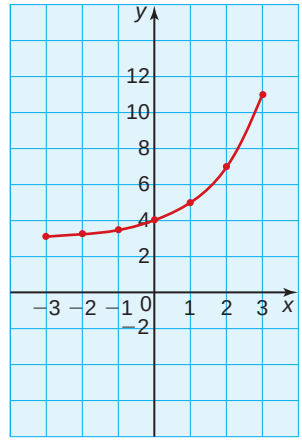
ii)



3. i)

x	−3	−2	−1	0	1	2	3
$f(x)$	3.125	3.25	3.5	4	5	7	11

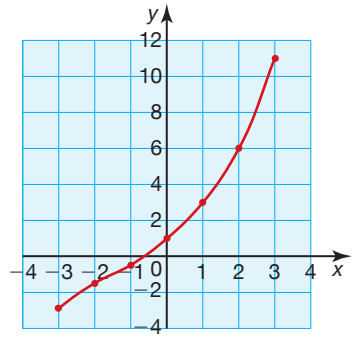
ii)



4. i)

x	−3	−2	−1	0	1	2	3
$f(x)$	−2.875	−1.75	−0.5	1	3	6	11

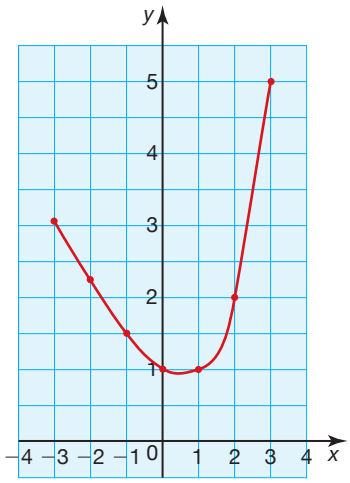
ii)



5. i)

x	−3	−2	−1	0	1	2	3
$f(x)$	3.125	2.25	1.5	1	1	2	5

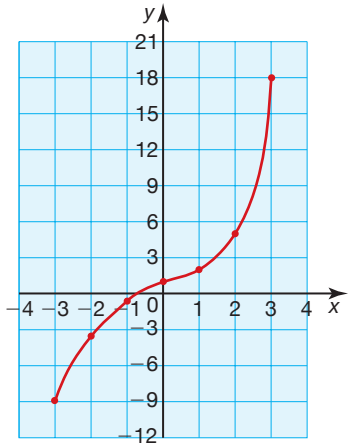
ii)



6. i)

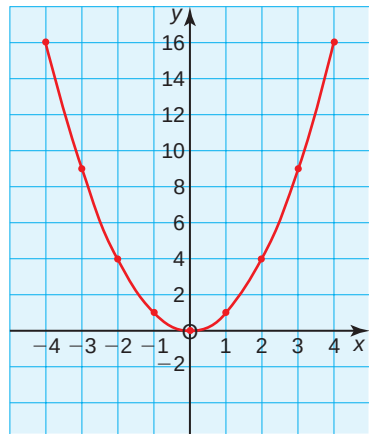
x	−3	−2	−1	0	1	2	3
$f(x)$	−8.96	−3.89	−0.67	1	2	5	18

ii)



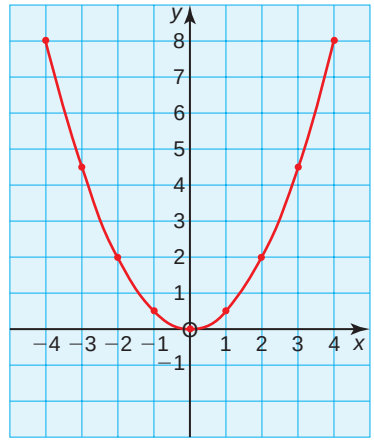
Exercise 18.7 page 197

1. i)



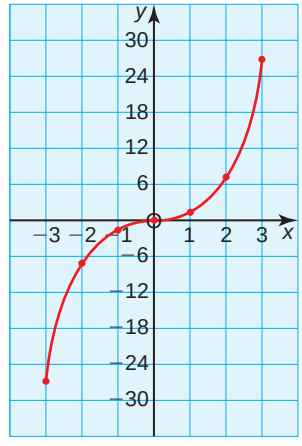
ii) 2

2. i)



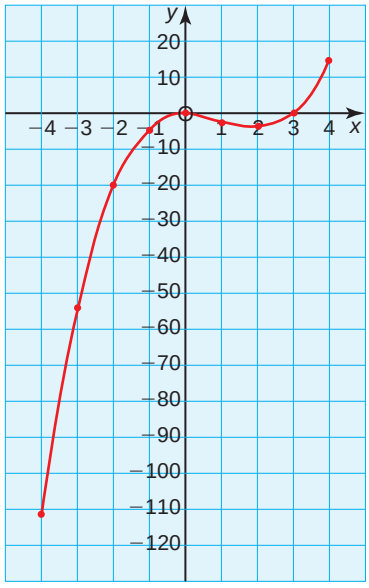
ii) −2

3. i)



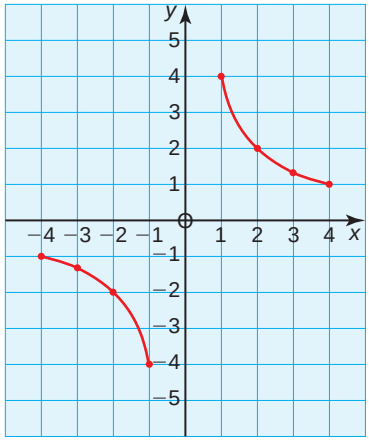
ii) 3

4. i)



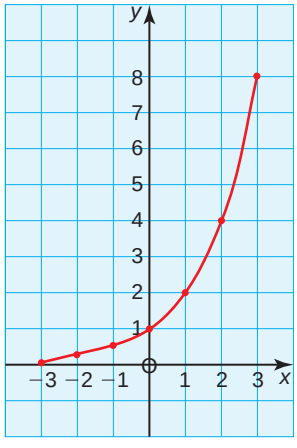
ii) 24

5. i)



ii) −4

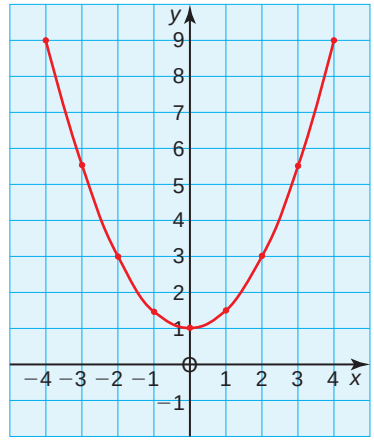
6. i)



ii) 0.7

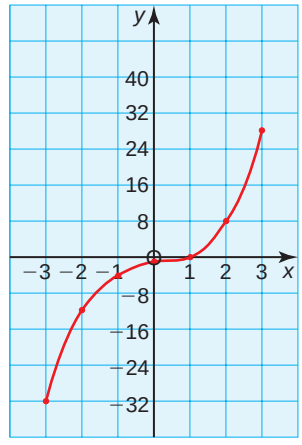
Exercise 18.8 page 198

1. **a)**



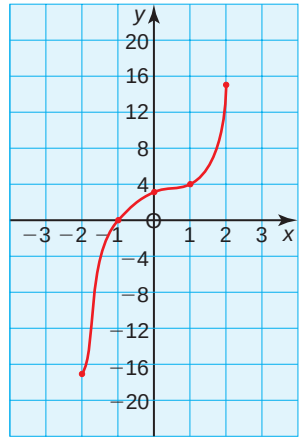
b) $x = \pm 2.8$

2. **a)**



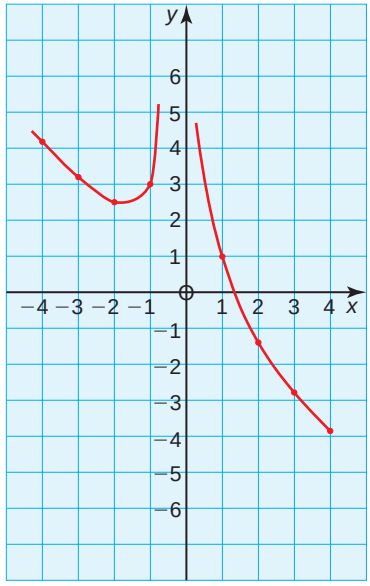
b) $x = 1.7$

3. **a)**



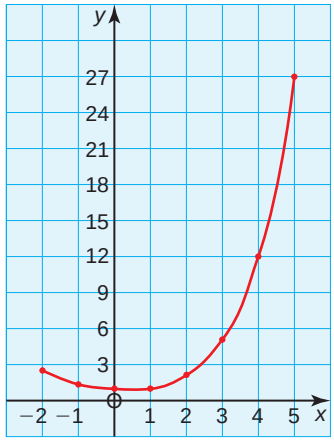
b) $x = 1.5$

4. **a)**



b) $x = -0.4, 0.5, 2.4$

5. **a)**



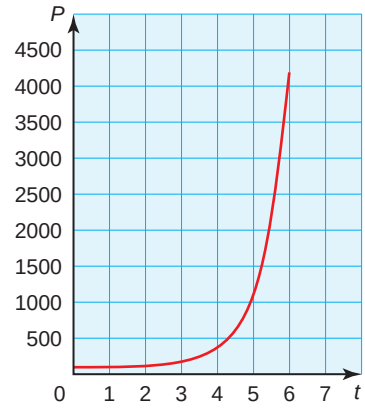
b) $x = -0.7, 3$

6. **a)** 7 cm **b)** 0 cm **c)** 5 hours
e) Approx. $5\frac{1}{2}$ hours

7. **a)** Approx. 2.5 **b)** $-\frac{1}{2}$

8. **a)** Approx. 4.3 **b)** Approx. 3.3
c) Approx. 2.3

9. **a)** 101
b) 1120
c)

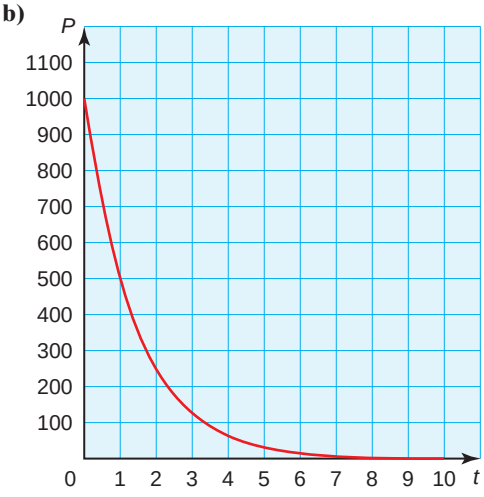


d) Approx. 4.5 hours

10. **a)**

t	0	1	2	3	4	5	6	7	8	9	10
P	1000	500	250	125	63	31	16	8	4	2	1

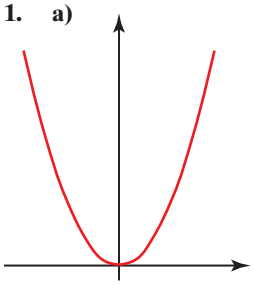
b)



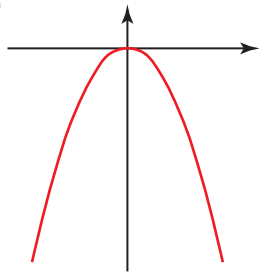
c) Approx. 90 insects

Student assessment 1 page 200

1. a)



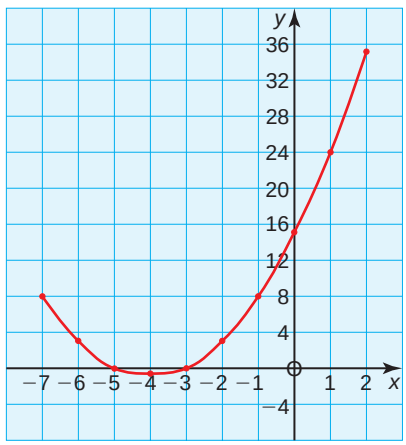
b)



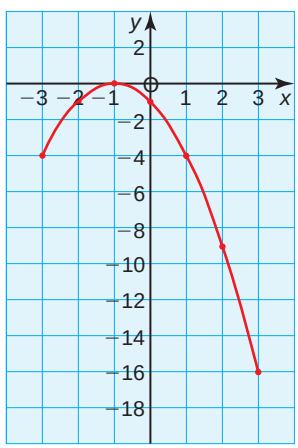
2. a)

x	−7	−6	−5	−4	−3	−2	−1	0	1	2
y	8	3	0	−1	0	3	8	15	24	35

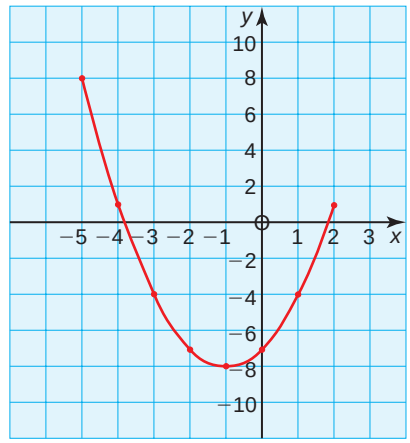
b)



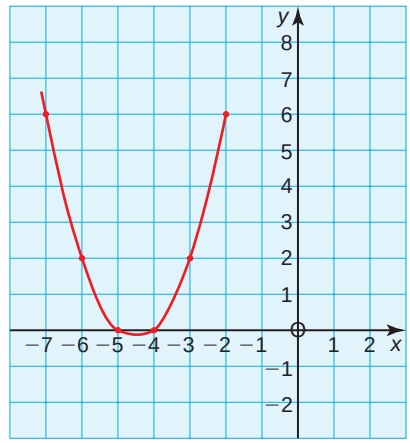
3. a)



b)

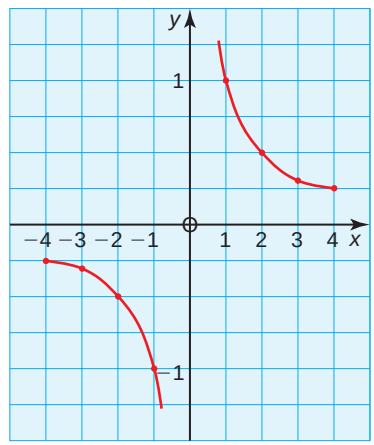


4. a)



b) $x = -7$ and -2

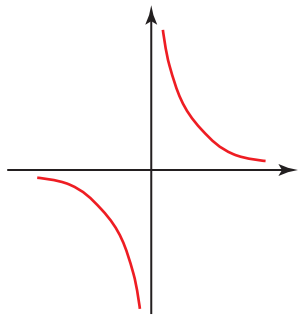
5. a)



b) $x = 0.4$ and 2.6

Student assessment 2 page 201

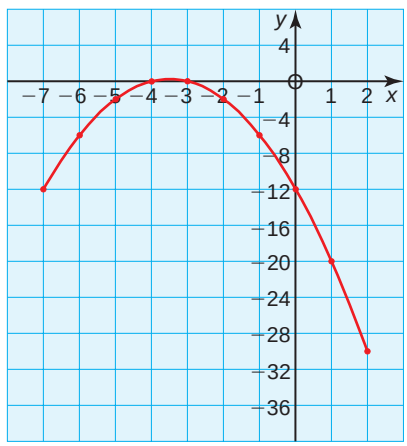
1.



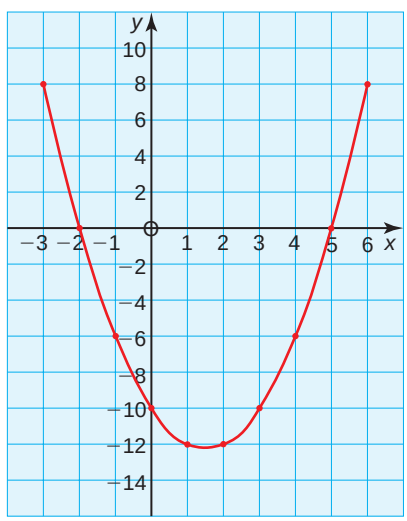
2. a)

x	−7	−6	−5	−4	−3	−2	−1	0	1	2
y	−12	−6	−2	0	0	−2	−6	−12	−20	−30

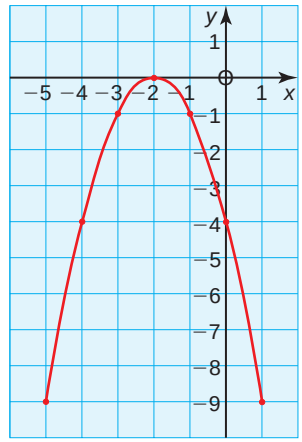
b)



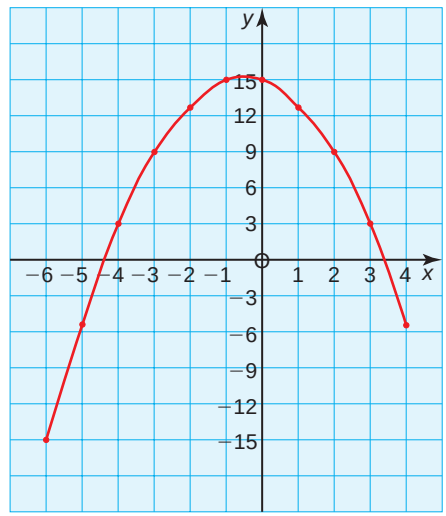
3. a)



b)

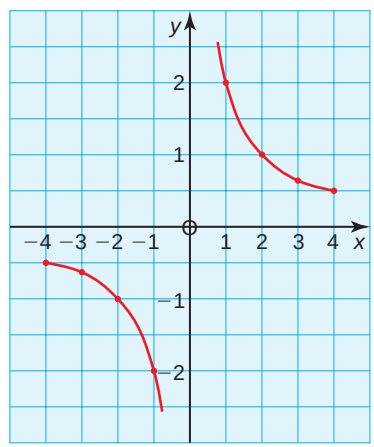


4. a)



b) i) $x = -3.7$ and 2.7 ii) $x = -1.8$ and 2.8

5. a)



b) $x = -2$ and 1

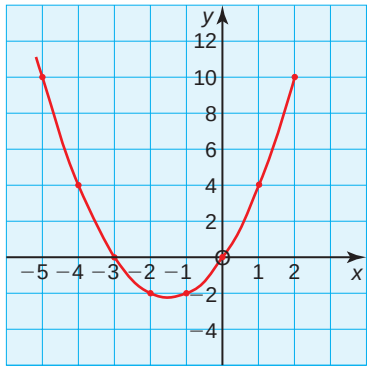
Student assessment 3 page 201

1. **a)** i) Linear ii) Quadratic
b) Student's own answer

2. **a)** i)

x	−5	−4	−3	−2	−1	0	1	2
f(x)	10	4	0	−2	−2	0	4	10

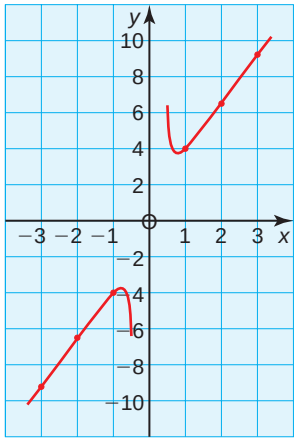
ii)



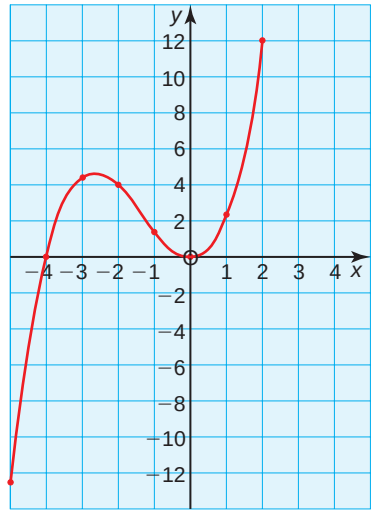
b) i)

x	−3	−2	−1	0	1	2	3
f(x)	−9.3	−6.5	−4	–	4	6.5	9.3

ii)

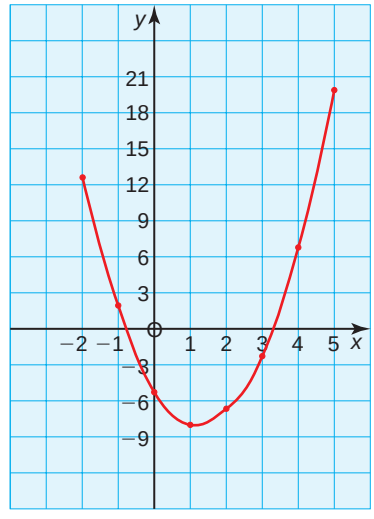


3. **a)**



b) i) 5.5 ii) −2.5

4. **a)**



b) $x = -0.8$ and 3.3 **c)** $x = -1.6$ and 3.1

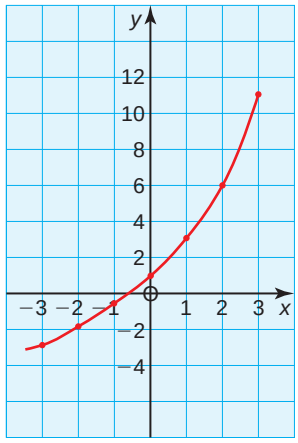
Student assessment 4 page 202

1. **a)** i) Reciprocal ii) Exponential
b) Student's own answer

2. **a)** i)

x	−3	−2	−1	0	1	2	3
f(x)	−2.9	−1.8	−0.5	1	3	6	11

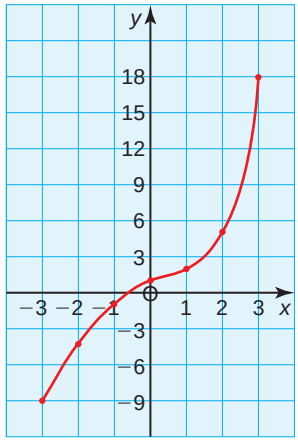
ii)



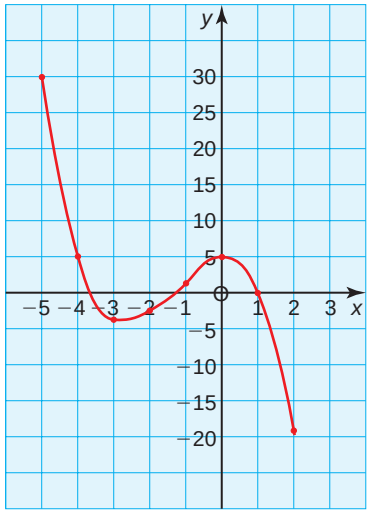
b) i)

x	−3	−2	−1	0	1	2	3
$f(x)$	−9.0	−3.9	−0.7	1	2	5	18

ii)



3. a)

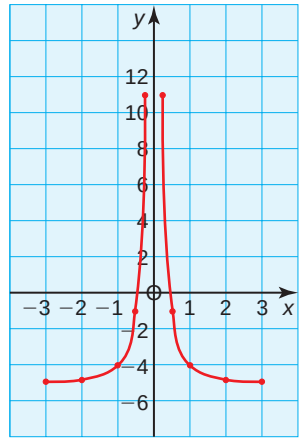


b) i) 0 ii) 4

4. a)

x	−3	−2	−1	−0.5	−0.25	0	0.25	0.5	1	2	3
y	−4.9	−4.8	−4	−1	11	–	11	−1	−4	−4.8	−4.9

b)



c) $x = \pm 0.4$ **d)** $x = \pm 0.4$ and ± 2.6

19 Functions

Exercise 19.1 page 204

1. **a)** 6 **b)** 10 **c)** 3 **d)** 5
e) 2 **f)** −2 **g)** −10 **h)** 1

2. **a)** 10 **b)** 22 **c)** 8 **d)** −4
e) −5 **f)** −18 **g)** −23 **h)** −6

3. **a)** 2 **b)** −28 **c)** −20.5 **d)** −14
e) 1.5 **f)** 12 **g)** 34.5 **h)** 13.5

4. **a)** −19 **b)** −26.5 **c)** −7 **d)** −8.2
e) 20 **f)** 8 **g)** −1 **h)** 3.5

Exercise 19.2 page 204

1. **a)** 2 **b)** 6.5 **c)** 2.375 **d)** 0.5
e) 0.125 **f)** −4 **g)** −2.5 **h)** −0.7

2. **a)** 4 **b)** 9 **c)** −1 **d)** −6
e) −3.5 **f)** −16 **g)** $-\frac{4}{3}$ **h)** $-\frac{7}{6}$

3. **a)** 0.5 **b)** 2 **c)** −4 **d)** −0.25
e) 5 **f)** 2.75 **g)** 35 **h)** 4.25

4. **a)** 4 **b)** 1.5 **c)** 2.75 **d)** 0.25
e) −3.5 **f)** 0.5 **g)** 0.375 **h)** 0.875

Exercise 19.3 page 204

1. **a)** 19 **b)** 52 **c)** 4 **d)** 3
e) 4 **f)** 3.25 **g)** 12 **h)** 5

2. **a)** 70 **b)** 187 **c)** -2 **d)** -5
e) 7 **f)** 4 **g)** -4.25 **h)** $-4\frac{2}{3}$

3. **a)** -14 **b)** 3.5 **c)** 4 **d)** -0.5
e) -28 **f)** 2 **g)** -6 **h)** -68

4. **a)** -5 **b)** 32.5 **c)** 0
d) -6.875 **e)** -7.5 **f)** 15
g) -6.25 **h)** 8.125

5. **a)** 0 **b)** 24 **c)** 0
d) 10.5 **e)** -13.5 **f)** $-\frac{25}{6}$
g) -97.5 **h)** 21.9 (3 s.f.)

6. **a)** 9 **b)** 0 **c)** -5
d) Infinite **e)** 0 **f)** 11.2
g) $-\frac{11}{6}$ **h)** 9

Exercise 19.4 page 205

1. **a)** $2x + 3$ **b)** $4x - 5$ **c)** $2x^2 + 1$
d) $x + 1$ **e)** $\frac{x}{2} + 3$ **f)** $x + 1$

2. **a)** $12x^2 - 4$ **b)** $\frac{3x^2}{16} - 4$ **c)** $6x - 4$
d) $27x^2$ **e)** $3x^2 - 6x - 1$
f) $12x^2 + 24x + 8$

3. **a)** $4x^2 + 3x + 2$ **b)** $16x^2 + 6x$
c) $4x^2 + 19x$ **d)** $4x^2 - 5x$
e) $\frac{2x^2 + 3x - 4}{2}$ **f)** $36x^2 + 57x + 20$

Exercise 19.5 page 206

1. **a)** $x - 3$ **b)** $x - 6$ **c)** $x + 5$
d) x **e)** $\frac{x}{2}$ **f)** $3x$

2. **a)** $\frac{x}{4}$ **b)** $\frac{x - 5}{2}$ **c)** $\frac{x + 6}{3}$
d) $2x - 4$ **e)** $\frac{4x + 2}{3}$ **f)** $\frac{5x - 7}{8}$

3. **a)** $2(x - 3)$ **b)** $4(x + 2)$ **c)** $\frac{x + 24}{12}$
d) $\frac{x - 18}{6}$ **e)** $\frac{x + 4}{6}$ **f)** $\frac{3x + 10}{8}$

Exercise 19.6 page 206

1. **a)** 6 **b)** 4 **c)** -1
2. **a)** 2 **b)** -0.5 **c)** -6
3. **a)** 3 **b)** 1.5 **c)** 2
4. **a)** 4 **b)** -2 **c)** -11
5. **a)** 4.5 **b)** 6 **c)** 0
6. **a)** 8 **b)** -2 **c)** 0.5

Exercise 19.7 page 207

1. **a)** $x + 2$ **b)** $x + 3$ **c)** $2x$ **d)** x
2. **a)** $2x + 8$ **b)** $6x + 1$ **c)** $8x + 2$
d) $-x + 2$
3. **a)** $\frac{2x - 1}{2}$ **b)** $\frac{3x - 5}{2}$ **c)** $\frac{x + 7}{3}$
d) $\frac{8x - 13}{20}$
4. **a)** 1 **b)** 8 **c)** 39 **d)** -2
5. **a)** 50 **b)** -2.9 **c)** 10 **d)** -1.6

Student assessment 1 page 208

1. **a)** 9 **b)** -1 **c)** -16
2. **a)** 5 **b)** -1 **c)** -5.5
3. **a)** -6 **b)** 0 **c)** 15
4. **a)** $-x + 4$ **b)** $\frac{2x + 18}{3}$
5. **a)** 5 **b)** 2
6. $-4x + 14$

Student assessment 2 page 208

1. **a)** 13 **b)** -2 **c)** 1
2. **a)** -2 **b)** 1 **c)** -1
3. **a)** -2 **b)** 0 **c)** 18
4. **a)** $\frac{x - 9}{-3}$ **b)** $4x + 2$
5. **a)** 1 **b)** 2
6. $32x - 6$

Topic 2 Mathematical investigations and ICT

House of cards page 209

1. 155

2. 8475

3. The formula can be proved using the method of differences (see Chapter 15) as follows:

Height of house	1	2	3	4	5
Number of cards	2	7	15	26	40
1st difference	5	8	11	14	
2nd difference	3	3	3		

Comparing with the algebraic table below:

Position	1	2	3	4	5
Term	$a + b + c$	$4a + 2b + c$	$9a + 3b + c$	$16a + 4b + c$	$25a + 5b + c$
1st difference	$3a + b$	$5a + b$	$7a + b$	$9a + b$	
2nd difference	$2a$	$2a$	$2a$		

It can be deduced that:

$2a = 3$ therefore $a = \frac{3}{2}$

$3a + b = 5$ therefore $b = \frac{1}{2}$

$a + b + c = 2$ therefore $c = 0$.

This produces the rule $c = \frac{3}{2}n^2 + \frac{1}{2}n$ which factorises to $c = \frac{1}{2}n(3n + 1)$.

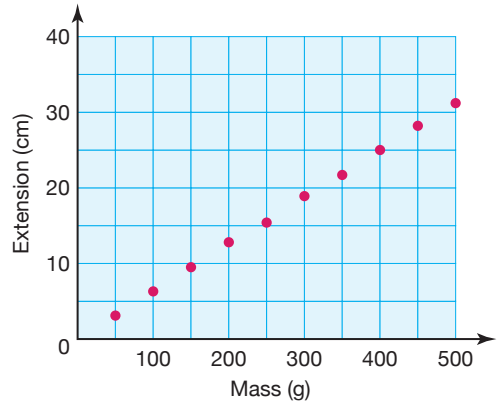
Chequered boards page 209

1. Student's own diagrams and results, presented in a logical table.

2. Where either m or n is even, the number of black and white squares is given by $\frac{mn}{2}$.
Where both m and n are odd, the number of black and white squares differ by one. The number of black squares is $\frac{mn-1}{2}$ and the number of white squares is $\frac{mn+1}{2}$, assuming that the bottom right-hand corner is white.

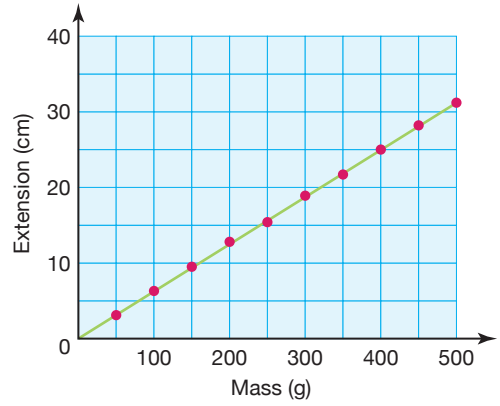
Modelling: Stretching a spring page 210

1.



2. Linear

3.



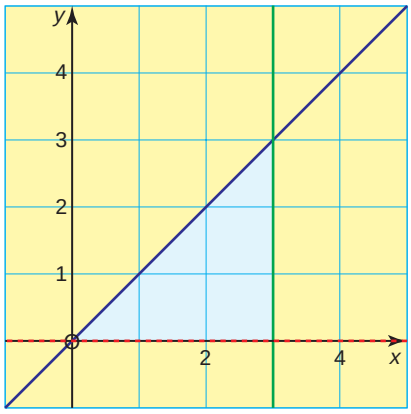
4. $y \approx 0.06x$

5. 16.5 cm

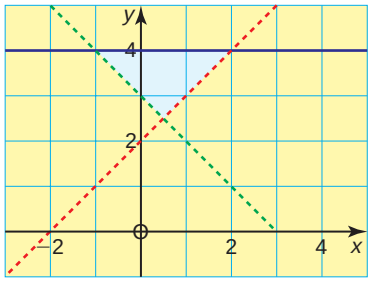
6. The spring is likely to snap (or exceed its elastic limit).

ICT activity 1 page 211

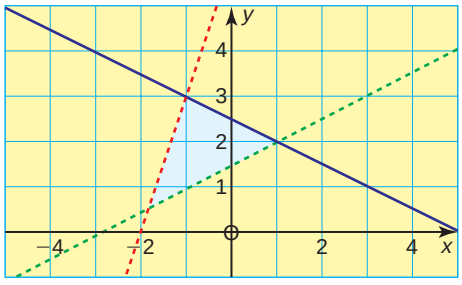
1.



2.



3.



ICT activity 2 page 211

1. $x \approx 2.7$

2. $x \approx 2.6$

3. $x \approx 2.1$

4. $x \approx 0.6$

20 Geometrical vocabulary

Exercise 20.1 page 216

1. **a)** and **f)** – hypotenuse right-angle
b) and **c)** – ASA
d) and **e)** – SAS
g) and **h)** – SSS

Exercise 20.2 page 218

1.

	Rectangle	Square	Parallelogram	Kite	Rhombus	Equilateral triangle
Opposite sides equal in length	Yes	Yes	Yes	No	Yes	n/a
All sides equal in length	No	Yes	No	No	Yes	Yes
All angles right angles	Yes	Yes	No	No	No	No
Both pairs of opposite sides parallel	Yes	Yes	Yes	No	Yes	n/a
Diagonals equal in length	Yes	Yes	No	No	No	n/a
Diagonals intersect at right angles	No	Yes	No	Yes	Yes	n/a
All angles equal	Yes	Yes	No	No	No	Yes

Exercise 20.3 page 219

Student's own diagrams

Student assessment 1 page 220

1. **a)** Acute **b)** Obtuse **c)** Reflex **d)** Right angle

2. Student's own diagram

3. **a)** Obtuse scalene triangle
b) Right-angled scalene

4. Student's own diagram

5. All sides equal in length
Both pairs of opposite sides are parallel
Diagonals intersect at right angles

6. Student's own diagram

21 Geometrical constructions and scale drawings

Exercise 21.1 page 222

1. Student's own construction

2. Student's own construction

3. Student's own construction

4. **a)** Student's own construction attempt
b) It is not possible as AC + BC < AB

Exercise 21.2 page 223

1. Student's own construction

2. Student's own construction

Exercise 21.3 page 226

1. Student's own construction

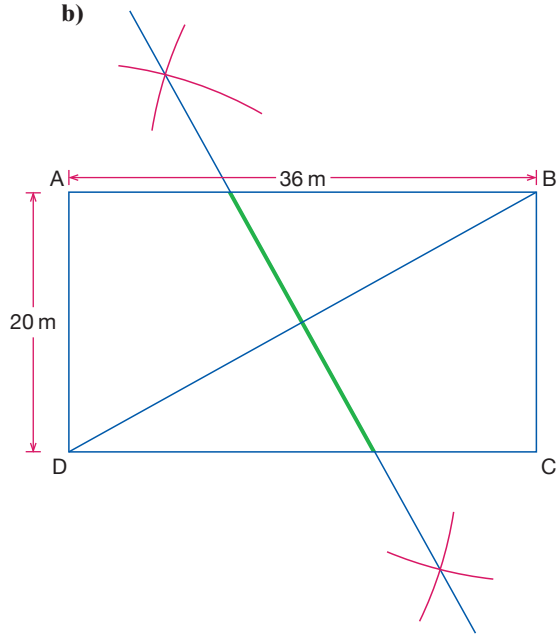
2. Student's own construction

Exercise 21.4 page 228

1. **a)** 300 m **b)** 250 m **c)** 300 m **d)** 416 m

2. **a)** 10 cm **b)** 8 cm **c)** 6 cm **d)** 6.8 cm

3. **a)** Student's own construction
b)



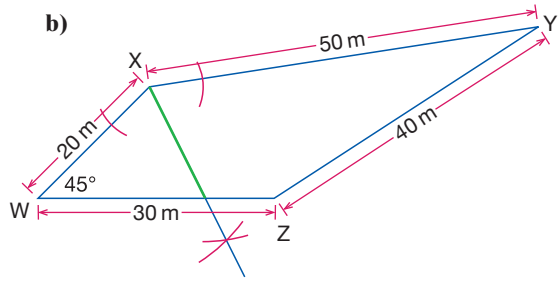
c) Approx. 23 m

4. **a)** Student's own construction
b) Approx. 24 m^2

5. **a)** Student's own construction
b) Approx. 12 km

6. **a)** Student's own construction

b)



c) Approx. 16 m

Student assessment 1 page 229

1. Student's own construction

2. Student's own construction

3. **a)** Student's own construction
b) Approx. 53 m^2

Student assessment 2 page 230

1. Student's own construction

2. **a)** 22 cm **b)** 24.4 cm

3. **a)** Student's own construction
b) Approx. 23 cm^2

22 Similarity

Exercise 22.1 page 231

1. **a)** Interior angles are the same, i.e. 60°, 30° and 90°.
b) $\frac{5}{8}$
c) $x = 6.25$ cm $y = 3.75$ cm

2. A, C and F are similar. B and D are similar.

3. **a)** 6 cm **b)** 9 cm

4. $p = 4.8$ cm $q = 4.5$ cm $r = 7.5$ cm

5. $e = 10$ cm $f = 2\frac{2}{3}$ cm

6. **a)** 10 cm^2 **b)** 1.6 **c)** 25.6 cm^2

7. **a)** 10 cm **b)** 2.5 **c)** 150 cm^2

8. **a)** $33\frac{1}{3}$ cm^2 **b)** $6\frac{2}{3}$ cm

9. 9.6 cm

10. No, as the corresponding angles may not be the same.

11. No, as, despite the corresponding angles being the same, the slanting side lengths may not be in the same ratio as the horizontal sides.

Exercise 22.2 page 234

1. 50 cm^2

2. 10 cm^2

3. **a)** i) 456 cm^2 (3 s.f.) ii) 90 cm^2 iii) 40 cm^2
b) Triangle I

4. 43.56 cm^2

5. 56.25 cm^2

6. 18.1 cm^2 (3 s.f.)

Exercise 22.3 page 235

1. **a)** i) 8 m ii) 4.8 m **b)** $2\sqrt{2}$ m
c) $25\sqrt{2}$ cm^2

2. **a)** 220 cm^2 **b)** 1375 cm^2 **c)** 200 cm^3
d) 3125 cm^3

3. **a)** 54 cm^2 **b)** 3 **c)** 729 cm^3

4. **a)** $1 : n^2$ **b)** $1 : n^3$

5. 112 cm^3

6. 0.64 litres

Exercise 22.4 page 236

1. 20 cm

2. **a)** 1 : 8 **b)** 1 : 7

3. 16 cm^3

4. **a)** Not similar. Student's own reasons
b) 1 : 3

5. **a)** 16 cm^2 **b)** 64 cm^2 **c)** 144 cm^2

6. **a)** 30 km^2 **b)** 6 cm^2

7. **a)** 10 cm × 20 cm × 30 cm **b)** 100 g

Student assessment 1 page 237

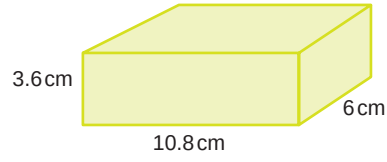
1. A and C

2. $1 : \left(\frac{H}{h}\right)^2$

3. **a)** Yes. Student's own explanation
b) 5 cm **c)** 8 cm **d)** 6 cm

4. 15 m

5. **a)**



b) 233.28 cm^3 **c)** 250.56 cm^2

6. **a)** 12.8 cm^3 **b)** 880 cm^2 **c)** 35.2 cm^2

7. 1250 cm^2

8. 27 000 cm^3

Student assessment 2 page 239

1. **a)** 5 **b)** $x = 4.5$ $y = 7.5$

2. **a)** $1 : \left(\frac{H}{h}\right)^2$ **b)** $1 : \left(\frac{H}{h}\right)^3$

3. $x = \sqrt{41}$ cm $y = \frac{3\sqrt{41}}{5}$ cm $z = 6.4$ cm

4. 156 cm^2 (3 s.f.)

5. **a)** 1000 cm^3 **b)** 600 cm^2

6. 18.75 cm^3

7. **a)** 4 m by 3 m **b)** 12 m^2

8. 3200 cm^3

23 Symmetry

Exercise 23.1 page 241

1. **a)** i) Student's planes ii) 3
b) i) Student's planes ii) 2
c) i) Student's planes ii) 4
d) i) Student's planes ii) 4
e) i) Student's planes ii) Infinite
f) i) Student's planes ii) Infinite
g) i) Student's planes ii) Infinite
h) i) Student's planes ii) 9

2. **a)** 2 **b)** 2 **c)** 3
d) 4 **e)** Infinite **f)** Infinite
g) Infinite **h)** 4

Exercise 23.2 page 242

1. **a)** Isosceles **b)** Perpendicular bisector
c) 50° **d)** 50°
e) 3.58 cm (3 s.f.) **f)** 9.33 cm (3 s.f.)

2. **a)** True **b)** True **c)** False **d)** True

3. **a)** False **b)** True

Exercise 23.3 page 243

1. **a)** 70° **b)** 72° **c)** 21°

2. **a)** 10.9 cm (3 s.f.)
b) 9.33 cm (3 s.f.)
c) 3.48 cm (3 s.f.)

Student assessment 1 page 243

1. **a)b)c)** Student's own diagrams

2. **a)b)** Student's own diagrams

3. Proof

4. 50°

5. 11.8 mm (3 s.f.)

Student assessment 2 page 244

1. **a)b)c)** Student's own diagrams

2. **a)b)c)** Student's own diagrams

3. Proof

4. **a)** 24° **b)** 9.14 cm (3 s.f.)

5. 88.0 cm (3 s.f.)

24 Angle properties

Exercise 24.1 page 247

1. $p = 54°$ $q = 63°$

2. $a = 55°$ $b = 80°$ $c = 100°$

3. $v = 120°$ $w = 60°$ $x = 120°$ $y = 60°$
$z = 60°$

4. $a = 50°$ $b = 130°$ $c = 45°$ $d = 135°$

5. $p = 45°$ $q = 135°$ $r = 45°$ $s = 45°$
$t = 135°$

6. $d = 70°$ $e = 30°$

7. $a = 37°$

8. $a = 36°$

Exercise 24.2 page 248

1. **a)** 70° **b)** 55° **c)** 60° **d)** 73°
e) 45° **f)** 110°

2. **a)** $a = 30°$ $b = 45°$
b) $x = 50°$ $y = 80°$ $z = 70°$
c) $p = 130°$ $q = 15°$ $r = 60°$
d) $d = 35°$ $e = 55°$ $f = 55°$
e) $a = 27.5°$ $b = 27.5°$ $c = 55°$
$d = 27.5°$ $e = 97.5°$
f) $p = 45°$ $q = 45°$ $r = 67.5°$
$s = 112.5°$

Exercise 24.3 page 250

1. $a = 115°$

2. $x = 40°$ $y = 140°$ $z = 140°$

3. $m = 75°$ $n = 75°$

4. $s = 65°$ $t = 115°$ $u = 115°$

5. $h = 120°$ $i = 60°$ $j = 120°$ $k = 60°$

6. $a = 80°$ $b = 20°$ $c = 20°$ $d = 20°$
$e = 140°$

7. $p = 40°$ $q = 130°$ $r = 50°$

8. $p = 75°$ $q = 30°$ $r = 50°$ $s = 80°$
$t = 70°$ $u = 70°$ $v = 40°$

Exercise 24.4 page 253

1. **a)** 720° **b)** 1260° **c)** 900°

2. **a)** 135° **b)** 90° **c)** 144° **d)** 150°

3. **a)** 72° **b)** 30° **c)** 51.4° (1 d.p.)

4. **a)** 18 **b)** 10 **c)** 36 **d)** 8
e) 20 **f)** 120

5. **a)** 5 **b)** 12 **c)** 20 **d)** 15
e) 40 **f)** 360

6. 12

Exercise 24.5 page 254

1. 60° **2.** 135°

3. 20° **4.** 32°

5. 110° **6.** 22.5°

Exercise 24.6 page 255

1. 35° **2.** 60°

3. 40° **4.** 45°

5. 24° **6.** 26°

7. 13 cm **8.** 8 cm

9. 17.7 cm (1 d.p.)

Exercise 24.7 page 256

1. **a)** $x = 54°$ **b)** 54°, 108°, 162°, 54°, 162°

2. 125°, 145°

3. 64°

4. $a = 135°$, $b = 125°$, $c = 130°$, $d = 110°$, $e = 85°$

Exercise 24.8 page 257

1. 55°

2. 80°

3. 90°

4. 100°

5. 80°

6. 20°

7. $x = 54°$, $y = 18°$

8. $x = 50°$, $y = 25°$

Exercise 24.9 page 258

1. $a = 72°$

2. $b = 33°$, $c = 66°$

3. $d = 48°$, $e = 32°$

4. $f = 30°$, $g = 120°$, $h = 120°$, $i = 30°$, $j = 30°$

5. $k = 55°$, $l = 55°$, $m = 55°$, $n = 55°$

6. $p = 65°$, $q = 40°$

Exercise 24.10 page 259

1. $a = 80°$, $b = 65°$

2. $c = 110°$, $d = 98°$, $e = 70°$

3. $f = 115°$, $g = 75°$

4. $i = 98°$, $j = 90°$, $k = 90°$

5. $l = 95°$, $m = 55°$, $n = 85°$, $p = 95°$, $q = 55°$

6. $r = 70°$, $s = 120°$, $t = 60°$, $u = 110°$

Student assessment 1 page 259

1. **a)** $p = 135°$ $q = 135°$ $r = 45°$
b) $a = 120°$ $b = 60°$ $c = 60°$ $d = 60°$

2. **a)** $m = 50°$ $n = 90°$ $p = 40°$ $q = 140°$
b) $w = 55°$ $x = 70°$ $y = 55°$ $z = 55°$
c) $a = 70°$ $b = 110°$ $c = 110°$ $d = 70°$
$e = 30°$

3. 162°

4. 1260°

5. 360°

6. 72°

7. **a)** 90°
b) 6.5 cm

8. 58°

9. 30°

10. 25°

11. 152°

Student assessment 2 page 261

1. **a)** $p = 130°$ $q = 130°$ $r = 50°$
b) $a = 50°$ $b = 50°$ $c = 50°$ $d = 50°$

2. **a)** $m = 50°$ $n = 70°$ $p = 70°$ $q = 50°$
b) $w = 85°$ $x = 50°$ $y = 50°$ $z = 45°$
c) $a = 130°$ $b = 25°$ $c = 25°$ $d = 50°$
d) $a = 40°$ $b = 100°$ $c = 40°$ $d = 70°$
$e = 70°$

3. 165°

4. 1800°

5. 30°

6. **a)** 90°
b) 13 cm

7. 28°

8. 125°

9. 45°

10. 42°

11. 38°

Student assessment 3 page 262

1. **a)** ii) ∠OBA ∠OBC
b) i) ∠DAB + ∠DCB ∠ADC + ∠CBA
c) iii) ∠DAC = ∠DBC ∠ADB = ∠ACB
d) ii) ∠CAB iii) ∠ACB = ∠ABC

2. **a)** 42° **b)** 21°

3. ∠DAB = 117° ∠ABC = 92°

4. ∠BDC = 25° ∠DAB = 115°

5. ∠OQR = 15° ∠ORQ = ∠OPQ = 90°
∠RPQ = ∠PRQ = 75°
∠OPR = ∠ORP = 15° ∠ROP = 150°

6. 35°, 54°, 91° respectively

7. 95°, 85°, 85°, 92° respectively

Student assessment 4 page 264

1. **a)** i) ∠DAB + ∠DCB ∠CDA + ∠CBA
b) i) ∠AOC + ∠CBA ∠OCB + ∠OAB
ii) ∠OAB, ∠OCB
iii)∠ABO = ∠CBO ∠AOB = ∠COB
c) iii) ∠TPS = ∠TQS = ∠TRS
∠PTR = ∠PSR ∠PTQ = ∠PSQ
∠QTR = ∠QSR
d) ii) ∠OAX, ∠OCY
iii) ∠DCB = ∠BAD ∠CDA = ∠CBA

2. 36°

3. ∠ABC = 50° ∠OAB = 25° ∠CAO = 40°

4. ∠BCD = 40° = ∠BAD
∠ABC = 65° = ∠ADC

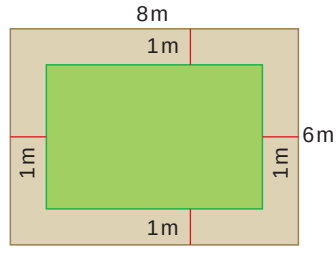
5. 28° and 56° respectively

6. 67.5°

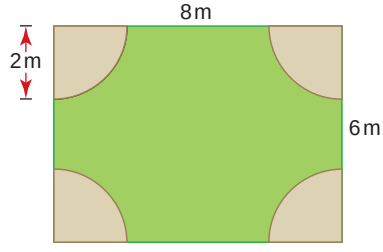
25 Loci

Exercise 25.1 page 266

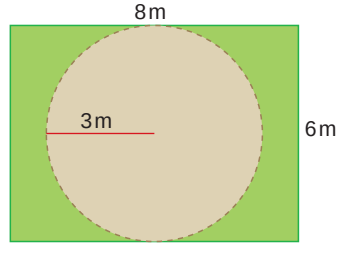
1.



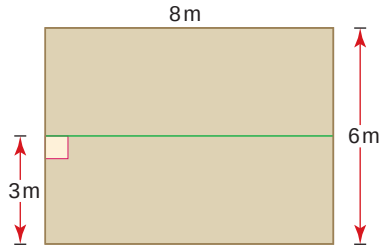
2.



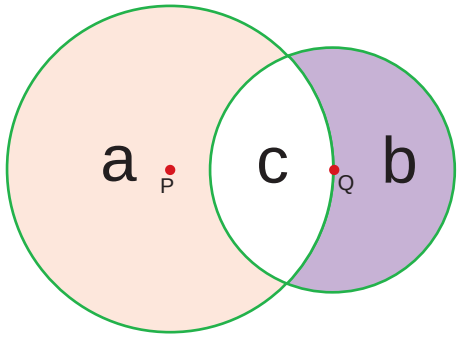
3.



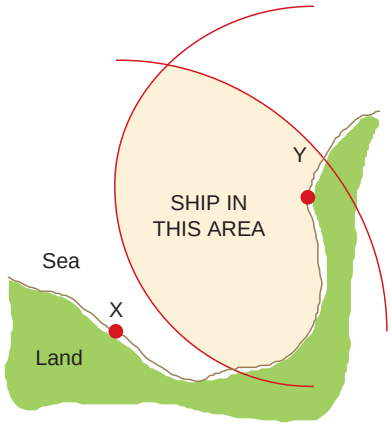
4.



5. **a)** **b)** **c)** **d)**



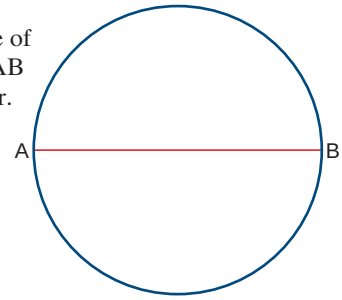
6.



7. **a)** Student's own diagrams. L, M and N will all lie on the circumference of a circle, the centre of the circle being the point equidistant from L, M and N.

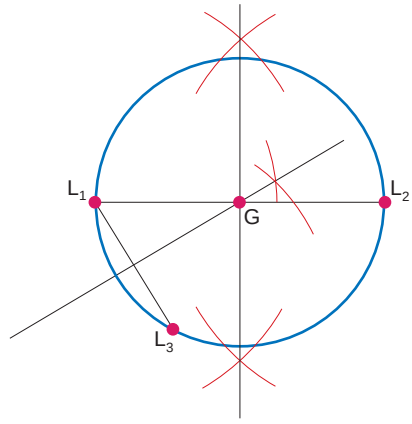
b) There would be no point equidistant from all three (except in the infinite!).

8. C is on the circumference of a circle with AB as its diameter.



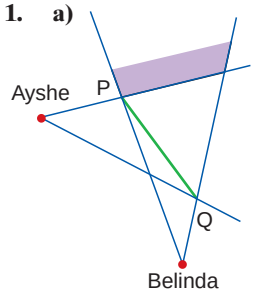
9. Student's own construction

10.

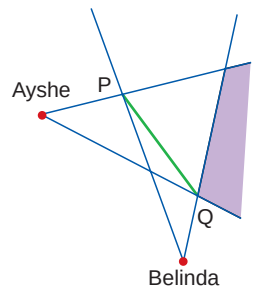


Exercise 25.2 page 267

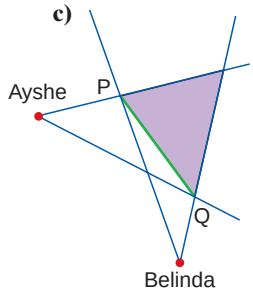
1. **a)**



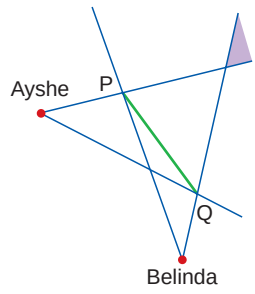
b)



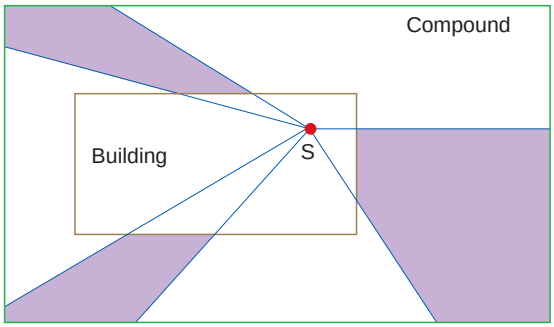
c)



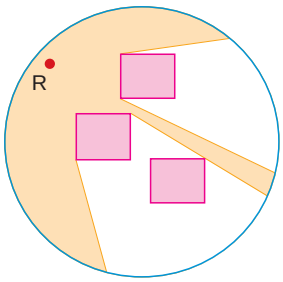
d)



2.



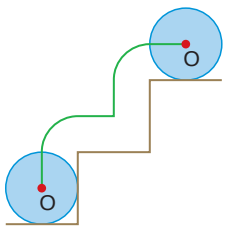
3.



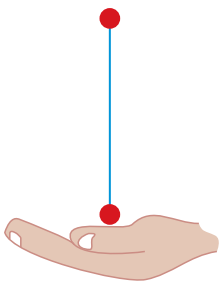
Exercise 25.3 page 268

1.

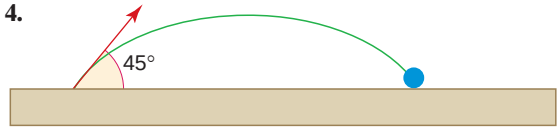
2.



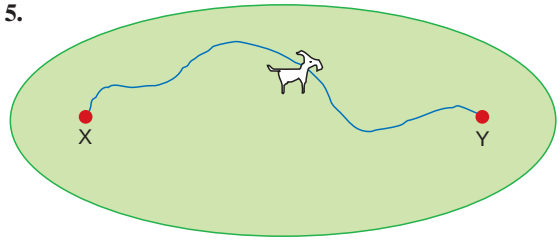
3.



4.

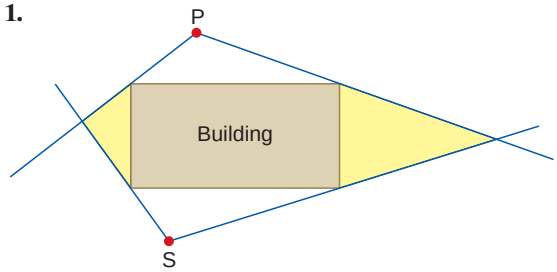


5.

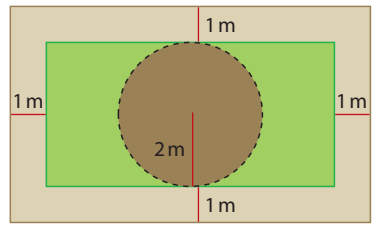


Student assessment 1 page 269

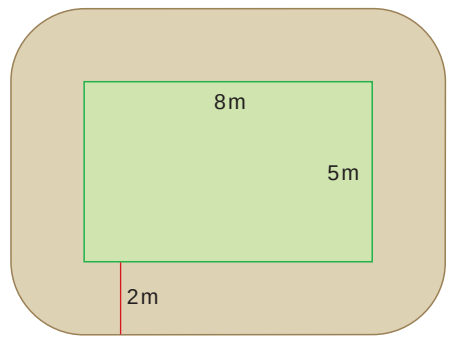
1.



2. **a)** **b)**

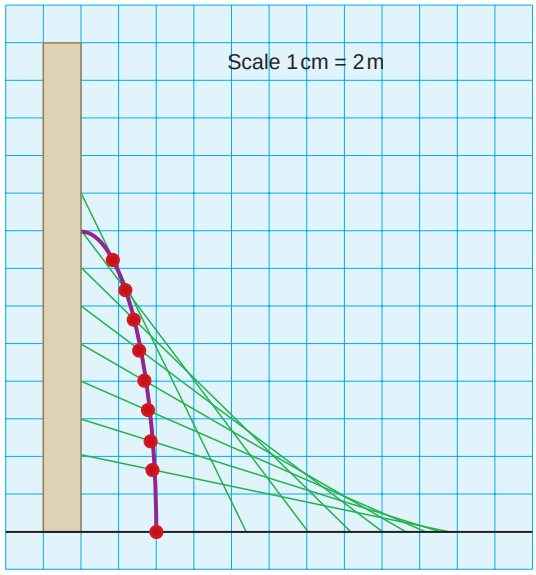


3.

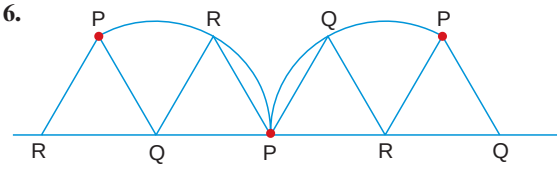


4. Ship's path is perpendicular bisector of AB.

5.

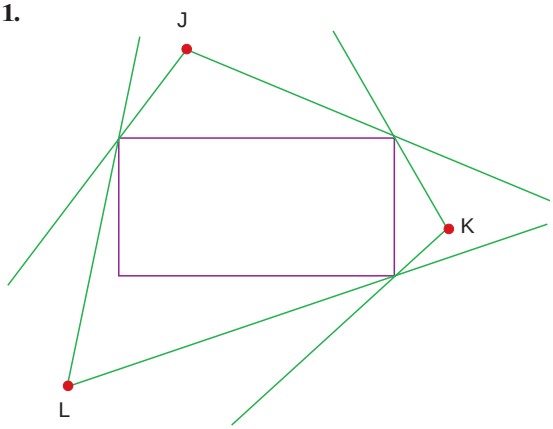


6.



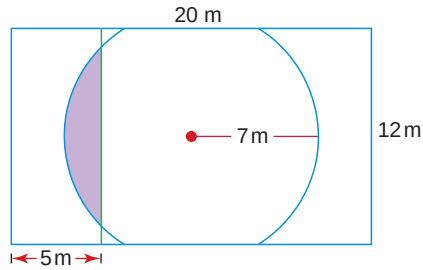
Student assessment 2 page 270

1.

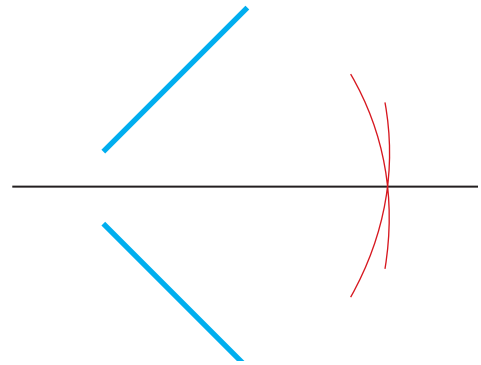


None of the friends can see each other, as shown above.

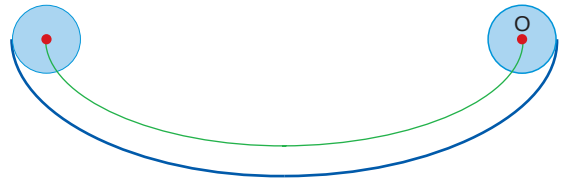
2.



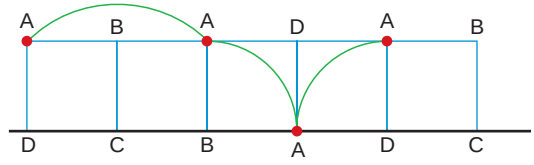
3.



4.



5.



Topic 3 Mathematical investigations and ICT

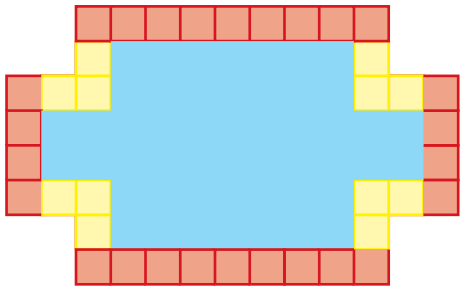
Fountain borders page 271

1. Student's results

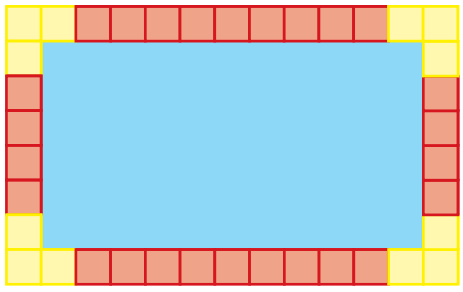
2. $T = 2(m + n + 2)$

3. There are many ways to prove the algebraic rule, for example:

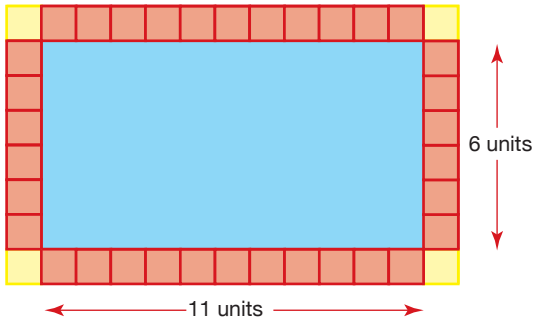
The original pool considered:



has the same number of tiles as a rectangular pool of dimensions 11 × 6 units:



In the diagram below it can be seen that the number of tiles along the length and width of the pool is twice the length and width. This leaves the four tiles needed for the corners.



Hence $T = 2m + 2n + 4$ which factorises to $T = 2(m + n + 2)$.

Tiled walls page 272

1. Student's diagrams

2. Student's ordered table of results

3. $c = (l - 1)(w - 1)$

4. $t = 2(l - 1) + 2(w - 1)$

ICT activity 1 page 272

2. The ratios are the same.

3. **a)** The ratios remain the same.
b) Ratios are still equal to each other (but probably of a different value from Q.1 (d)).

4. The ratios change as ED no longer parallel to AB.

ICT activity 2 page 273

Student's own demonstrations

26 Measures

Exercise 26.1 page 276

1. **a)** 100 **b)** 1000 **c)** $\frac{1}{1000}$ **d)** $\frac{1}{1000}$
e) Millilitre

2. **a)** m, cm **b)** cm **c)** g **d)** ml
e) cm, m **f)** tonne **g)** litres **h)** km
i) tonne **j)** litres

Exercise 26.2 page 277

1. **a)** mm **b)** m **c)** mm **d)** m **e)** m

2. **a)** 85 **b)** 230 **c)** 830 **d)** 50 **e)** 4

3. **a)** 5.6 **b)** 6400 **c)** 0.96 **d)** 4
e) 0.012

4. **a)** 1.15 **b)** 250 **c)** 0.5 **d)** 0.07
e) 0.008

Exercise 26.3 page 278

1. **a)** 3800 **b)** 28.5 **c)** 4280 **d)** 0.32
e) 500

Exercise 26.4 page 278

1. **a)** 4500 ml **b)** 1530 ml **c)** 7050 ml
d) 1000 ml

2. **a)** 1.2 litres **b)** 1.34 litres **c)** 1.4 litres
d) 1.4 litres

Exercise 26.5 page 279

1. **a)** 100 000 cm^2 **b)** 2 000 000 mm^2
c) 5 000 000 m^2 **d)** 3 200 000 m^2
e) 830 mm^2

2. **a)** 0.05 m^2 **b)** 150 cm^2
c) 0.001 km^2 **d)** 0.04 m^2
e) 0.000 25 km^2

3. **a)** 2 500 000 cm^3 **b)** 3400 mm^3
c) 2 000 000 000 m^3 **d)** 200 000 cm^3
e) 30 000 000 mm^3

4. **a)** 0.15 m^3 **b)** 24 cm^3
c) 0.000 85 km^3 **d)** 0.3 cm^3
e) 0.000 015 m^3

Student assessment 1 page 280

1. **a)** 26 **b)** 88 **c)** 6.8 **d)** 875

2. **a)** 4200 **b)** 3.94 **c)** 4100 **d)** 720

3. **a)** 1.8 **b)** 3200 **c)** 83 **d)** 250

4. **a)** 5600 mm^2
b) 2 050 000 cm^3

5. **a)** 0.008 67 m^3
b) 0.444 km^3

Student assessment 2 page 280

1. **a)** 310 **b)** 6400 **c)** 4 **d)** 46

2. **a)** 0.0036 **b)** 0.55 **c)** 6.5 **d)** 1.51

3. **a)** 3.4 **b)** 6700 **c)** 730 **d)** 300

4. **a)** 30 000 mm^2 **b)** 5000 m^2

5. **a)** 0.1004 m^3 **b)** 0.000 005 005 km^3

27 Perimeter, area and volume

Exercise 27.1 page 282

1. **a)** 6 cm^2 **b)** 32.5 cm^2 **c)** 20 cm^2
d) 60 cm^2 **e)** 108 cm^2 **f)** 55 cm^2

2. **a)** 64 cm^2 **b)** 1168 mm^2 **c)** 300 cm^2
d) 937.5 mm^2

Exercise 27.2 page 284

1. 58.5 cm^2

2. 84 cm^2

3. 118.7 cm^2

4. 157.5 cm^2

Exercise 27.3 page 285

1. 4

2. 3

3. 23.5 m^2

4. **a)** 16 m^2, 24 m^2 **b)** 100 m^2 **c)** 15

Exercise 27.4 page 287

1. **a)** 25.1 cm **b)** 22.0 cm **c)** 28.9 cm
d) 1.57 m

2. **a)** 50.3 cm^2 **b)** 38.5 cm^2 **c)** 66.5 cm^2
d) 0.196 m^2

3. **a)** 2.39 cm (3 s.f.) **b)** 0.5 cm
c) 0.637 m (3 s.f.) **d)** 1.27 mm (3 s.f.)

4. **a)** 4.51 cm (3 s.f.) **b)** 6 cm
c) 3.23 m (3 s.f.) **d)** 4.31 mm (3 s.f.)

Exercise 27.5 page 287

1. **a)** 1.57 m (2 d.p.) **b)** 637 times (3 s.f.)

2. 188 m (3 s.f.)

3. 264 mm^2 (3 s.f.)

4. **a)** 144 cm^2 **b)** 28.3 cm^2 (3 s.f.)
c) 30.9 cm^2 (3 s.f.)

5. **a)** 57.1 m (3 s.f.) **b)** 178.3 m^2 (3 s.f.)

Exercise 27.6 page 289

1. **a)** 460 cm^2 **b)** 208 cm^2 **c)** 147.78 cm^2
d) 33.52 cm^2

2. **a)** 2 cm **b)** 4 cm **c)** 6 cm **d)** 5 cm

3. **a)** 101 cm^2 (3 s.f.) **b)** 276 cm^2 (3 s.f.)
c) 279 cm^2 (3 s.f.) **d)** 25.6 cm^2 (3 s.f.)

4. **a)** 1.2 cm **b)** 0.5 cm **c)** 1.7 cm
d) 7.0 cm

Exercise 27.7 page 289

1. **a)** 24 cm^2 **b)** 2 cm

2. **a)** 216 cm^2 **b)** 15.2 cm (3 s.f.)

3. **a)** 94.2 cm^2 (3 s.f.) **b)** 14 cm

4. 4.4 cm

Exercise 27.8 page 290

1. **a)** 24 cm^3 **b)** 18 cm^3 **c)** 27.6 cm^3
d) 8.82 cm^3

2. **a)** 452 cm^3 (3 s.f.) **b)** 277 cm^3 (3 s.f.)
c) 196 cm^3 (3 s.f.) **d)** 0.481 cm^3 (3 s.f.)

3. **a)** 108 cm^3 **b)** 140 cm^3 **c)** 42 cm^3
d) 6.2 cm^3

4. **a)** 70 cm^3 **b)** 96 cm^3 **c)** 380 cm^3
d) 137.5 cm^3

Exercise 27.9 page 291

1. **a)** 16 cm **b)** 4096 cm^3 **c)** 3217 cm^3
d) 21.5% (3 s.f.)

2. **a)** 42 cm^2 **b)** 840 cm^3

3. 6.3 cm

4. 2.90 m^3 (3 s.f.)

Exercise 27.10 page 293

1. **a)** 6.28 cm **b)** 2.09 cm **c)** 11.5 cm
d) 23.6 cm

2. **a)** 32.7° (1 d.p.) **b)** 229.2° (1 d.p.)
c) 57.3° (1 d.p.) **d)** 114.6° (1 d.p.)

3. **a)** 12.2 cm (3 s.f.) **b)** 4.58 cm (3 s.f.)
c) 18.6 cm (3 s.f.) **d)** 4.01 cm (3 s.f.)

Exercise 27.11 page 293

1. **a)** 48.8 cm (3 s.f.) **b)** 105 cm (3 s.f.)

2. **a)** 3.67 cm (3 s.f.) **b)** 49.7 cm (3 s.f.)
c) 68.8° (1 d.p.)

3. **a)** 12 cm **b)** 54 cm **c)** 47.7° (1 d.p.)

Exercise 27.12 page 294

1. **a)** 33.5 cm^2 (3 s.f.) **b)** 205 cm^2 (3 s.f.)
c) 5.65 cm^2 (3 s.f.) **d)** 44.7 cm^2 (3 s.f.)

2. **a)** 18.5 cm (3 s.f.) **b)** 20.0 cm (3 s.f.)
c) 1.75 cm (3 s.f.) **d)** 12.4 cm (3 s.f.)

3. **a)** 48° **b)** 34°
c) 20° **d)** 127°

Exercise 27.13 page 295

1. 79.2 m^2 (3 s.f.)

2. **a)** 118 cm^2 (3 s.f.) **b)** 39.3 cm^2 (3 s.f.)
c) 8.66 cm (3 s.f.)

3. **a)** 4.19 cm (3 s.f.) **b)** 114 cm^2 (3 s.f.)
c) 62.8 cm^3 (3 s.f.)

4. **a)** 9.40 cm (3 s.f.) **b)** 0.60 cm (2 s.f.)
c) 34.9 cm^2 (3 s.f.)
d) Length = 17.1 cm (3 s.f.)
Width = 10.6 cm (3 s.f.)
e) 41.7 cm^2 (3 s.f.)

5. **a)** 20° **b)** 0.64 cm (2 s.f.)
c) Length = 13.7 cm (3 s.f.)
Width = 11.3 cm (3 s.f.)
d) 5.43 cm^2 (3 s.f.)

Exercise 27.14 page 296

1. **a)** 905 cm^3 (3 s.f.) **b)** 3590 cm^3 (3 s.f.)
c) 2310 cm^3 (3 s.f.) **d)** 1.44 cm^3 (3 s.f.)

2. **a)** 3.1 cm **b)** 5.6 cm
c) 36.3 cm **d)** 0.6 cm

Exercise 27.15 page 297

1. 6.3 cm

2. 86.7 cm^3 (3 s.f.)

3. 11.9 cm (1 d.p.)

4. **a)** 4190 cm^3 (3 s.f.) **b)** 8000 cm^3 **c)** 48%

5. 10.0 cm

6. A = 4.1 cm, B = 3.6 cm, C = 3.1 cm

7. 3 : 2

Exercise 27.16 page 297

1. **a)** 452 cm^2 (3 s.f.) **b)** 254 cm^2 (3 s.f.)
c) 1890 cm^2 (3 s.f.) **d)** 4 cm^2

2. **a)** 1.99 cm (3 s.f.) **b)** 1.15 cm (3 s.f.)
c) 3.09 mm (3 s.f.) **d)** 0.5 cm

3. 1 : 4

4. **a)** 157 cm^2 (3 s.f.) **b)** 15 cm
c) 707 cm^2 (3 s.f.)

5. **a)** 804.2 cm^2 **b)** 5.9 cm (1 d.p.)

Exercise 27.17 page 299

1. 40 cm^3

2. 133 cm^3 (3 s.f.)

3. 64 cm^3

4. 70 cm^3

Exercise 27.18 page 299

1. 7 cm

2. 5 cm

3. **a)** 8 cm **b)** 384 cm^3 **c)** 378 cm^3

4. **a)** 3.6 cm **b)** 21.7 cm^3 (3 s.f.)
c) 88.7 cm^3 (3 s.f.)

Exercise 27.19 page 300

1. 6.93 cm^2 (3 s.f.)

2. 189 cm^2 (3 s.f.)

3. 73.3 cm^2 (3 s.f.)

4. 1120 cm^2 (3 s.f.)

5. **a)** 693 cm^2 (3 s.f.) **b)** 137 cm^2 (3 s.f.)
c) 23.6 cm (3 s.f.)

Exercise 27.20 page 303

1. **a)** 56.5 cm^3 (3 s.f.) **b)** 264 cm^3 (3 s.f.)
c) 1.34 cm^3 (3 s.f.) **d)** 166 cm^3 (3 s.f.)

2. **a)** 6.91 cm (3 s.f.) **b)** 10.9 cm (3 s.f.)
c) 0.818 cm (3 s.f.) **d)** 51.3 cm (3 s.f.)

3. **a)** i) 7.96 cm ii) 12.7 cm iii) 843 cm^3
b) i) 15.9 cm ii) 8.41 cm iii) 2230 cm^3
c) i) 6.37 cm ii) 3.97 cm iii) 168 cm^3
d) i) 3.82 cm ii) 4.63 cm iii) 70.7 cm^3

Exercise 27.21 page 303

1. 3.88 cm (3 s.f.)

2. **a)** 33.0 cm (3 s.f.) **b)** 5.25 cm (3 s.f.)
c) 7.31 cm (3 s.f.) **d)** 211 cm^3 (3 s.f.)

3. **a)** 2304 cm^3 **b)** 603 cm^3 (3 s.f.)
c) 1700 cm^3 (3 s.f.)

4. **a)** 81.6 cm^3 (3 s.f.) **b)** 275 cm^3 (3 s.f.)
c) 8 : 27

Exercise 27.22 page 304

1. 81.8 cm^3 (3 s.f.)

2. 771 cm^3 (3 s.f.)

3. 3170 cm^3 (3 s.f.)

4. **a)** 654 cm^3 **b)** 12.5 cm **c)** 2950 cm^3

Exercise 27.23 page 305

1. **a)** 415 cm^2 (3 s.f.) **b)** 1650 cm^2 (3 s.f.)

2. 1130 cm^2 (3 s.f.)

Student assessment 1 page 306

1. 90 cm^2

2. **a)** Circumference = 34.6 cm
Area = 95.0 cm^2
b) Circumference = 50.3 mm
Area = 201 mm^2

3. 9.86 cm^2

4. **a)** 39.3 cm^2 **b)** 34 cm^2 **c)** 101.3 cm^2

5. **a)** 10.2 cm^2 **b)** 283 cm^2 **c)** 633 cm^2

6. **a)** 339 mm^3 **b)** 9.82 cm^3

Student assessment 2 page 307

1. 104 cm^2

2. **a)** Circumference = 27.0 cm
Area = 58.1 cm^2
b) Circumference = 47.1 mm
Area = 177 mm^2

3. 326 cm^2

4. **a)** 56.5 cm^2 **b)** 108 cm^2 **c)** 254.5 cm^2

5. 418 cm^3

6. **a)** 1012 cm^2 **b)** 523 cm^2

Student assessment 3 page 308

1. **a)** 11.8 cm (3 s.f.) **b)** 35.3 cm (3 s.f.)

2. **a)** 272.2° (1 d.p.) **b)** 5.7° (1 d.p.)

3. 232 cm^2 (3 s.f.)

4. **a)** 531 cm^2 **b)** 1150 cm^3

5. **a)** 1210 cm^2 (3 s.f.) **b)** 2592 cm^3

Student assessment 4 page 309

1. **a)** 178 cm (3 s.f.) **b)** 68.3 mm (3 s.f.)

2. **a)** 143.2° (1 d.p.) **b)** 2.9° (1 d.p.)

3. 95.5 cm^2 (3 s.f.)

4. **a)** 603.2 cm^2 **b)** 1072.3 cm^3

5. **a)** 22.5 cm **b)** 126 cm^3 **c)** 3270 cm^3

Student assessment 5 page 309

1. **a)** 22.9 cm (3 s.f.) **b)** 229 cm^2 (3 s.f.)
c) 985 cm^2 (3 s.f.) **d)** 1830 cm^3 (3 s.f.)

2. **a)** 905 cm^3 (3 s.f.) **b)** 12 cm
c) 13.4 cm (3 s.f.) **d)** 958.2 cm^2

3. **a)** 10 cm **b)** 82.1 cm^3 (3 s.f.)
c) 71.8 cm^3 (3 s.f.) **d)** 30.8 cm^3 (3 s.f.)
e) 41.1 cm^3 (3 s.f.)

Student assessment 6 page 310

1. **a)** 3620 cm^3 (3 s.f.) **b)** 3620 cm^3 (3 s.f.)
c) 905 cm^2 (3 s.f.) **d)** 1920 cm^2 (3 s.f.)

2. **a)** 43.3 cm^2 (3 s.f.) **b)** 173 cm^2 (3 s.f.)

3. **a)** 314 cm^2 (3 s.f.) **b)** 26.9 cm

Topic 4 Mathematical investigations and ICT

Metal trays page 311

1. **a)** length = 38 cm
width = 28 cm
height = 1 cm
b) 1064 cm^3

2. **a)** length = 36 cm
width = 26 cm
height = 2 cm
b) 1872 cm^3

3. Student's investigation and ordered table of results similar to this one.

Square length (cm)	Tray dimensions (cm)			Volume
	Length	Width	Height	
1	38	28	1	1064
2	36	26	2	1872
3	34	24	3	2448
4	32	22	4	2816
5	30	20	5	3000
6	28	18	6	3024
7	26	16	7	2912
8	24	14	8	2688
9	22	12	9	2376
10	20	10	10	2000
11	18	8	11	1584
12	16	6	12	1152
13	14	4	13	728
14	12	2	14	336
15	10	0	15	0

4. $x = 5.7$ cm

5. Maximum volume = 3032 cm^3

Tennis balls page 312

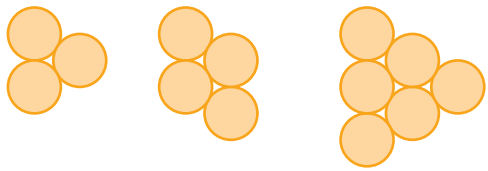
1. Cuboids of the following dimensions should be considered. Note each unit represents the diameter of one tennis ball and only different combinations are considered.
$1 \times 1 \times 12$ $1 \times 2 \times 6$ $1 \times 3 \times 4$ $2 \times 2 \times 3$

2. Total surface area of a cuboid is given by the formula: $A = 2(lw + lh + wh)$
Total surface area of the four options are as shown (to nearest whole number):

Dimensions (units)	Length (cm)	Width (cm)	Height (cm)	Surface area (cm^2)
1 × 1 × 12	6.6	6.6	79.2	2178
1 × 2 × 6	6.6	13.2	39.6	1742
1 × 3 × 4	6.6	19.8	26.4	1655
2 × 2 × 3	13.2	13.2	19.8	1394

The optimum dimensions of the box are 13.2 cm × 13.2 cm × 19.8 cm.

3. Cross-sections of possible designs are shown below:



4. Student's investigations

5. Student's conclusion based on their calculations

ICT activity page 312

1. Possible formulae are given:
In cell B2: =A2/360*2*PI()*10
C2: =B2
D2: =C2/(2*PI())
E2: =SQRT((100-D2^2))
F2: =1/3*PI()*D2^2*E2

28 Straight-line graphs

Exercise 28.1 page 319

1. **a)** 1 **b)** $\frac{3}{2}$ **c)** –2
d) $-\frac{1}{4}$ **e)** 0 **f)** infinite

2. A horizontal line has a zero gradient.

3. A vertical line has an infinite gradient.

4. Gradient of A = 2
Gradient of B = 0
Gradient of C = –3
Gradient of D = $-\frac{1}{2}$
Gradient of E = $\frac{1}{2}$
Gradient of F is infinite.

Exercise 28.2 page 321

1. **a)** $y = 7$ **b)** $y = 2$ **c)** $x = 7$
d) $x = 3$ **e)** $y = x$ **f)** $y = \frac{1}{2}x$
g) $y = -x$ **h)** $y = -2x$

Exercise 28.3 page 322

1. **a)** $y = x + 1$ **b)** $y = x + 3$ **c)** $y = x - 2$
d) $y = 2x + 2$ **e)** $y = \frac{1}{2}x + 5$ **f)** $y = \frac{1}{2}x - 1$

2. **a)** $y = -x + 4$ **b)** $y = -x - 2$
c) $y = -2x - 2$ **d)** $y = -\frac{1}{2}x + 3$
e) $y = -\frac{3}{2}x + 2$ **f)** $y = -4x + 1$

3. **a)** **1)** **a)** 1 **b)** 1 **c)** 1
d) 2 **e)** $\frac{1}{2}$ **f)** $\frac{1}{2}$
2) **a)** -1 **b)** -1 **c)** -2
d) $-\frac{1}{2}$ **e)** $-\frac{3}{2}$ **f)** -4
b) The gradient is equal to the coefficient of x.
c) The constant being added/subtracted indicates where the line intersects the y-axis.

4. Only the intercept c is different.

5. The lines are parallel.

Exercise 28.4 page 325

1. **a)** $m = 2$ $c = 1$ **b)** $m = 3$ $c = 5$
c) $m = 1$ $c = -2$ **d)** $m = \frac{1}{2}$ $c = 4$
e) $m = -3$ $c = 6$ **f)** $m = -\frac{2}{3}$ $c = 1$
g) $m = -1$ $c = 0$ **h)** $m = -1$ $c = -2$
i) $m = -2$ $c = 2$

2. **a)** $m = 3$ $c = 1$ **b)** $m = -\frac{1}{2}$ $c = 2$
c) $m = -2$ $c = -3$ **d)** $m = -2$ $c = -4$
e) $m = \frac{1}{4}$ $c = 6$ **f)** $m = 3$ $c = 2$
g) $m = 1$ $c = -2$ **h)** $m = -8$ $c = 6$
i) $m = 3$ $c = 1$

3. **a)** $m = 2$ $c = -3$ **b)** $m = \frac{1}{2}$ $c = 4$
c) $m = 2$ $c = -4$ **d)** $m = -8$ $c = 12$
e) $m = 2$ $c = 0$ **f)** $m = -3$ $c = 3$
g) $m = 2$ $c = 1$ **h)** $m = -\frac{1}{2}$ $c = 2$
i) $m = 2$ $c = -\frac{1}{2}$

4. **a)** $m = 2$ $c = -4$ **b)** $m = 1$ $c = 6$
c) $m = -3$ $c = -1$ **d)** $m = -1$ $c = 4$
e) $m = 10$ $c = -2$ **f)** $m = -3$ $c = \frac{3}{2}$
g) $m = -9$ $c = 2$ **h)** $m = 6$ $c = -14$
i) $m = 2$ $c = -\frac{3}{2}$

5. **a)** $m = 2$ $c = -2$ **b)** $m = 2$ $c = 3$
c) $m = 1$ $c = 0$ **d)** $m = \frac{3}{2}$ $c = 6$
e) $m = -1$ $c = \frac{2}{3}$ **f)** $m = -4$ $c = 2$
g) $m = 3$ $c = -12$ **h)** $m = 0$ $c = 0$
i) $m = -3$ $c = 0$

6. **a)** $m = 1$ $c = 0$ **b)** $m = -\frac{1}{2}$ $c = -2$
c) $m = -3$ $c = 0$ **d)** $m = 1$ $c = 0$
e) $m = -2$ $c = -\frac{2}{3}$ **f)** $m = \frac{2}{3}$ $c = -4$

7. **a)** $m = -\frac{2}{5}$ $c = 0$ **b)** $m = \frac{1}{3}$ $c = -\frac{5}{6}$
c) $m = 3$ $c = 0$ **d)** $m = -1$ $c = -4$

Exercise 28.5 page 327

1. Any line with a gradient of 1

2. a, b and d are parallel

3. $y = 4x$

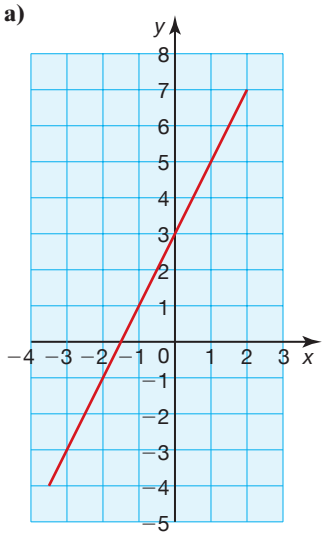
4. **a)** $y = -3x + 4$ **b)** $y = -3x - 2$

c) $y = -3x - \frac{7}{2}$

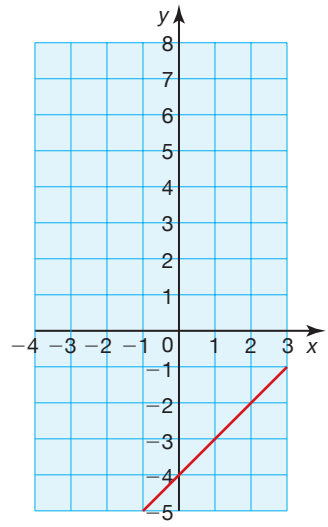
5. **a)** $y = \frac{1}{2}x + 3$ **b)** $y = \frac{1}{2}x - \frac{1}{4}$

Exercise 28.6 page 328

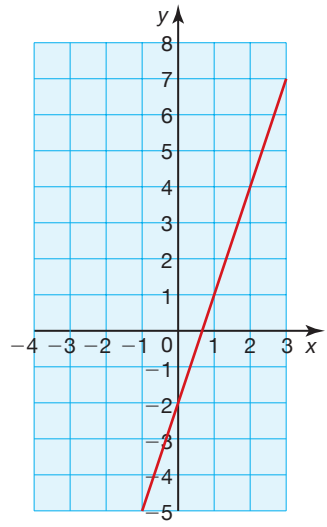
1. **a)**



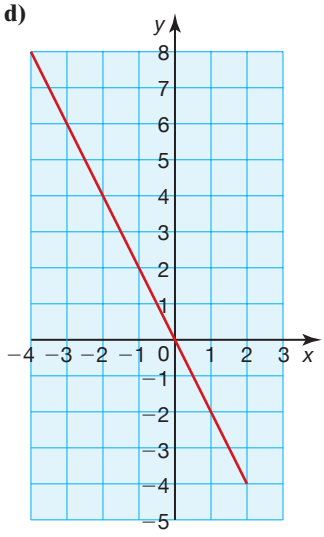
b)



c)



d)



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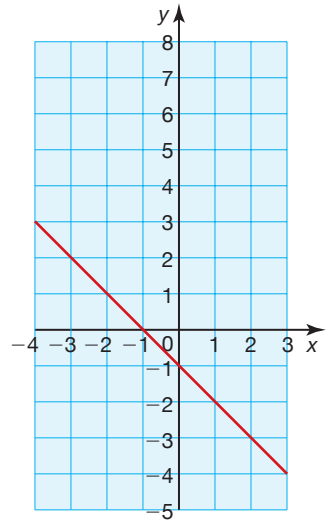

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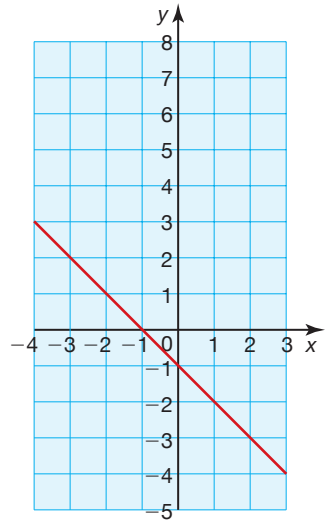
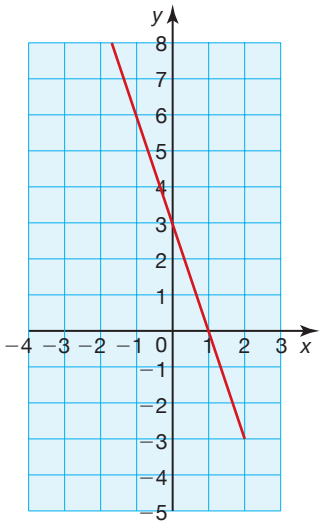
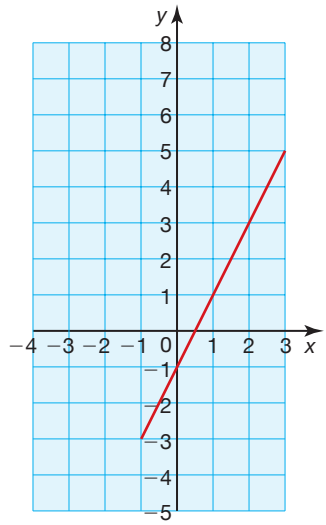


g)



h)



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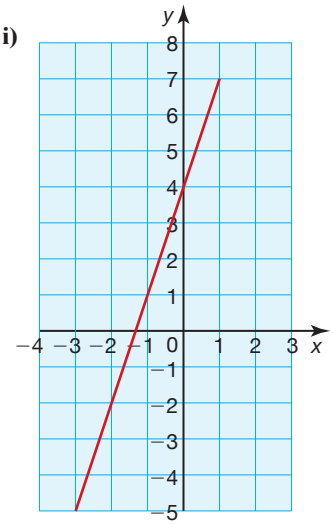

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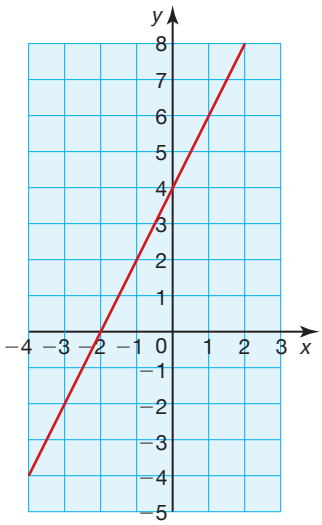
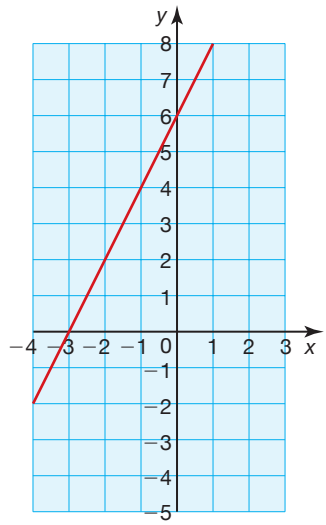
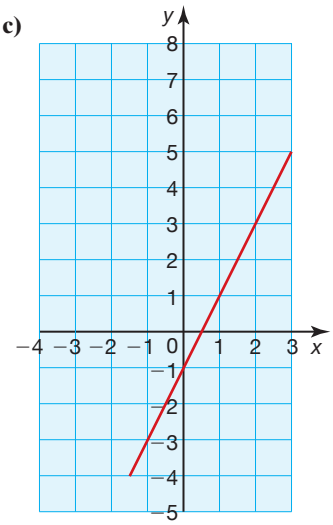


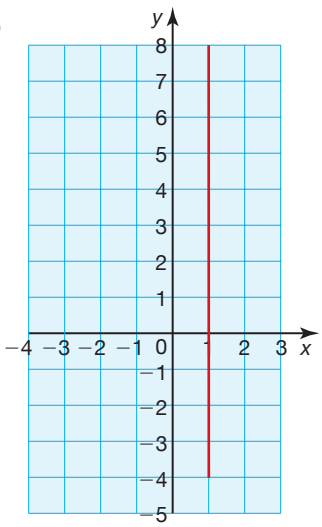
b)



c)



d)



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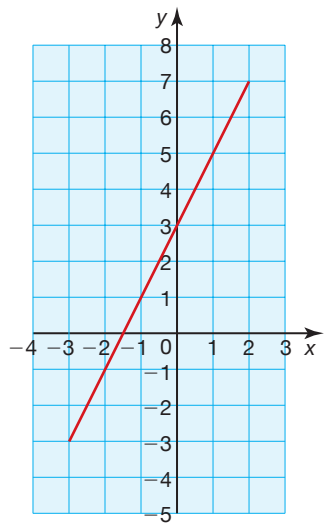

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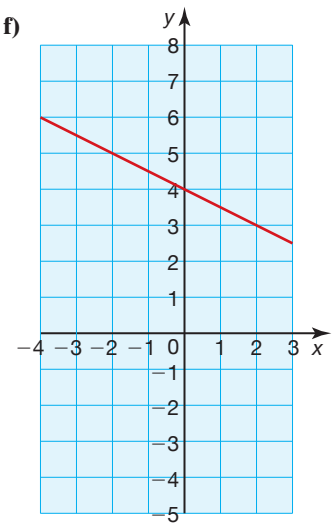
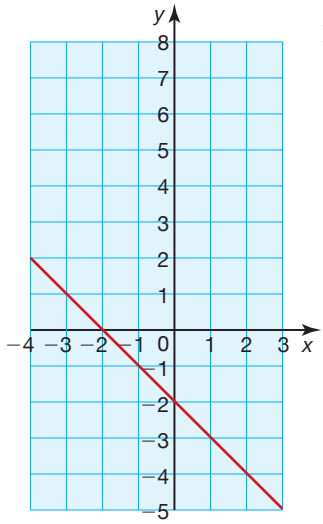


g)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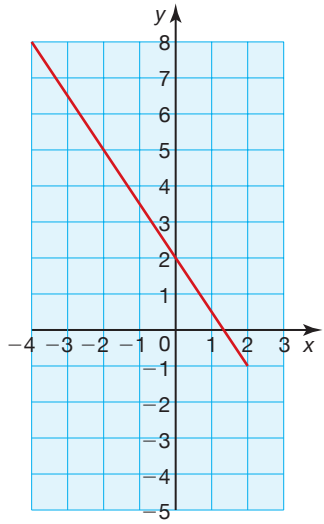
x
−4 −3 −2 −1 0 1 2 3
8 7 6 5 4 3 2 1 −1 −2 −3 −4 −5

h)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x
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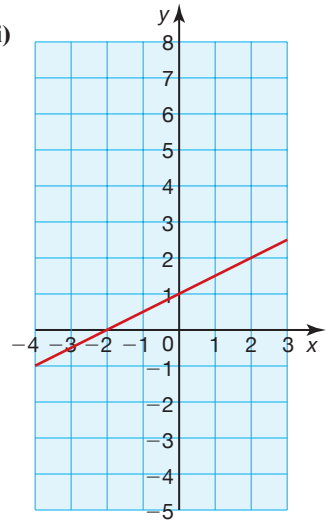
8 7 6 5 4 3 2 1 −1 −2 −3 −4 −5

i)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x
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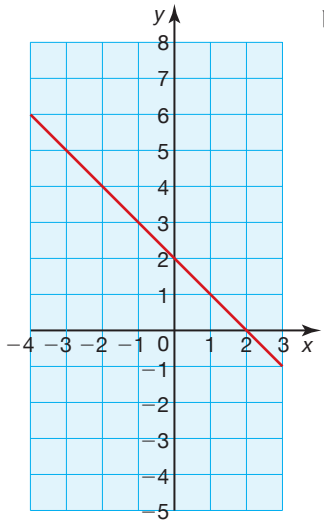
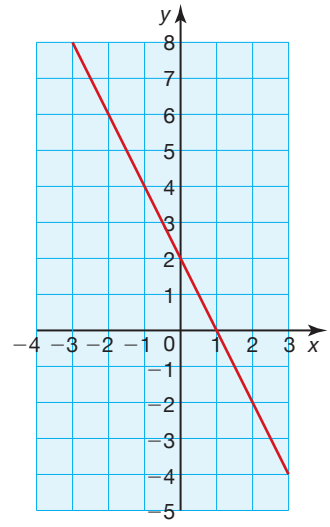
8 7 6 5 4 3 2 1 −1 −2 −3 −4 −5

3. a)


y
x
−4 −3 −2 −1 0 1 2 3
8 7 6 5 4 3 2 1 −1 −2 −3 −4 −5

b)


y
x
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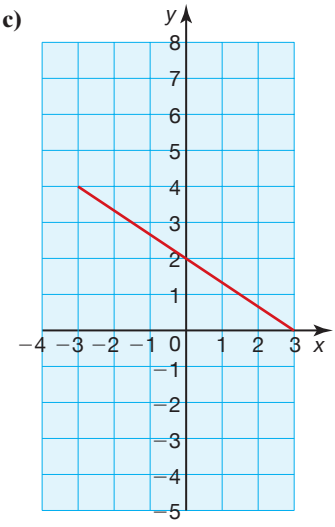
8 7 6 5 4 3 2 1 −1 −2 −3 −4 −5

c)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x
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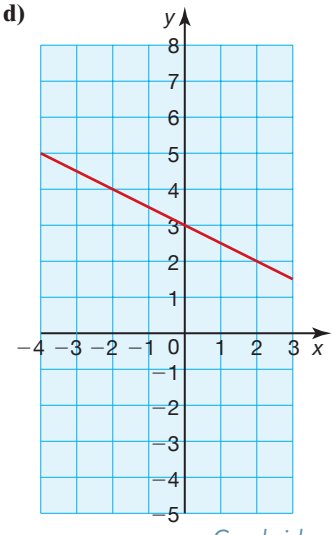
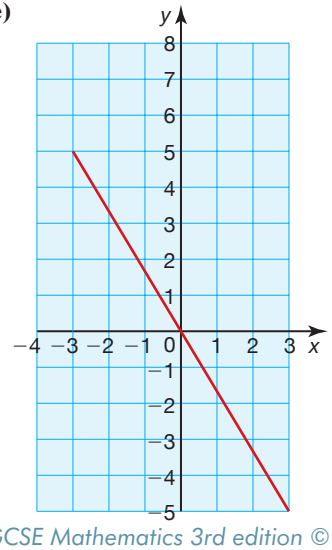
8 7 6 5 4 3 2 1 −1 −2 −3 −4 −5

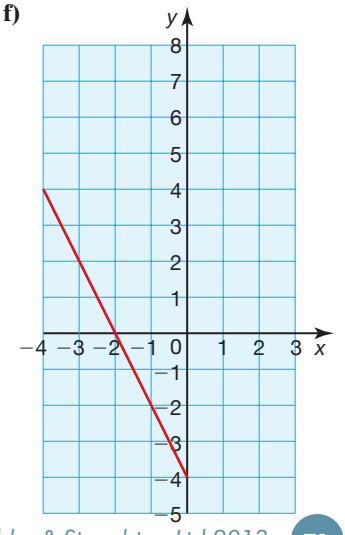
d)


y
x
−4 −3 −2 −1 0 1 2 3
8 7 6 5 4 3 2 1 −1 −2 −3 −4 −5

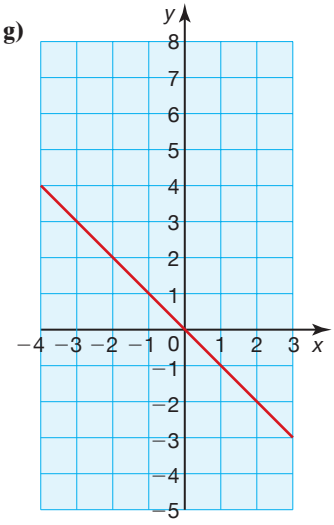
e)


y
x
−4 −3 −2 −1 0 1 2 3
8 7 6 5 4 3 2 1 −1 −2 −3 −4 −5

f)


y
x
−4 −3 −2 −1 0 1 2 3
8 7 6 5 4 3 2 1 −1 −2 −3 −4 −5

g)



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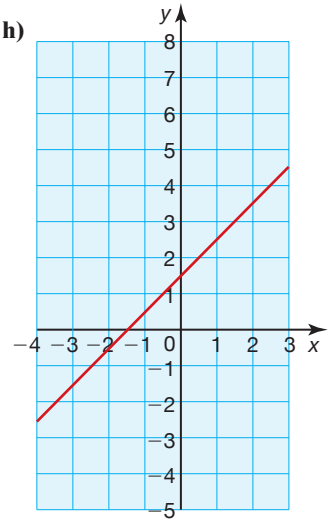
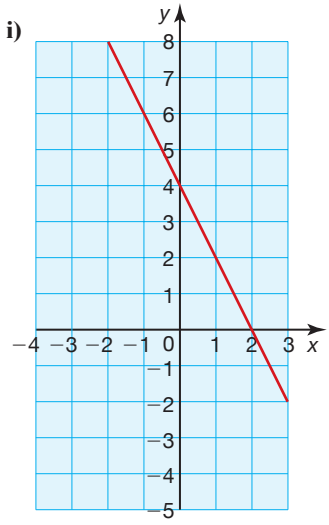


i)



Exercise 28.7 page 330

1. **a)** (3, 2) **b)** (5, 2) **c)** (2, 1)
d) (2, 1) **e)** (−4, 1) **f)** (4, −2)

2. **a)** (3, −2) **b)** (−1, −1) **c)** (−2, 3)
d) (−3, −3) **e)** Infinite solutions
f) No solution

Exercise 28.8 page 331

1. **a)** ii) 5.66 units (3 s.f.) iii) (3, 4)
b) ii) 4.24 units (3 s.f.) iii) (4.5, 2.5)
c) ii) 5.66 units (3 s.f.) iii) (3, 6)
d) ii) 8.94 units (3 s.f.) iii) (2, 4)
e) ii) 6.32 units (3 s.f.) iii) (3, 4)
f) ii) 6.71 units (3 s.f.) iii) (−1.5, 4)
g) ii) 8.25 units (3 s.f.) iii) (−2, 1)
h) ii) 8.94 units (3 s.f.) iii) (0, 0)
i) ii) 7 units iii) (0.5, 5)
j) ii) 6 units iii) (2, 3)
k) ii) 8.25 units (3 s.f.) iii) (0, 4)
l) ii) 10.8 units (3 s.f.) iii) (0, 1.5)

2. **a)** i) 4.24 units (3 s.f.) ii) (2.5, 2.5)
b) i) 5.66 units (3 s.f.) ii) (5, 4)
c) i) 8.94 units (3 s.f.) ii) (4, 2)
d) i) 8.94 units (3 s.f.) ii) (5, 0)
e) i) 4.24 units (3 s.f.) ii) (−1.5, 4.5)
f) i) 4.47 units (3 s.f.) ii) (−4, −3)
g) i) 7.21 units (3 s.f.) ii) (0, 3)
h) i) 7.21 units (3 s.f.) ii) (5, −1)
i) i) 12.4 units (3 s.f.) ii) (0, 2.5)
j) i) 8.49 units (3 s.f.) ii) (1, −1)
k) i) 11 units ii) (0.5, −3)
l) i) 8.25 units (3 s.f.) ii) (4, 2)

Exercise 28.9 page 333

1. **a)** $y = 2x - 1$ **b)** $y = 3x + 1$
c) $y = 2x + 3$ **d)** $y = x - 4$
e) $y = 4x + 2$ **f)** $y = -x + 4$
g) $y = -2x + 2$ **h)** $y = -3x - 1$
i) $y = \frac{1}{2}x$

2. **a)** $y = \frac{1}{7}x + \frac{26}{7}$ **b)** $y = \frac{6}{7}x + \frac{4}{7}$
c) $y = \frac{3}{2}x + \frac{15}{2}$ **d)** $y = 9x - 13$
e) $y = -\frac{1}{2}x + \frac{5}{2}$ **f)** $y = -\frac{3}{13}x + \frac{70}{13}$
g) $y = 2$ **h)** $y = -3x$
i) $x = 6$

Exercise 28.10 page 335

1. **a)** i) −1 ii) 1 iii) $y = x - 3$
b) i) −1 ii) 1 iii) $y = x - 5$
c) i) −2 ii) $\frac{1}{2}$ iii) $y = \frac{1}{2}x - 5$
d) i) $-\frac{1}{2}$ ii) 2 iii) $y = 2x - 20$
e) i) −1 ii) 1 iii) $y = x + 9$
f) i) −2 ii) $\frac{1}{2}$ iii) $y = \frac{1}{2}x + \frac{3}{2}$
g) i) $-\frac{3}{2}$ ii) $\frac{2}{3}$ iii) $y = \frac{2}{3}x - \frac{4}{3}$
h) i) $\frac{2}{3}$ ii) $-\frac{3}{2}$ iii) $y = -\frac{3}{2}x + 13$
i) i) $-\frac{1}{4}$ ii) 4 iii) $y = 4x + 28$
j) i) −1 ii) 1 iii) $y = x - 8$
k) i) 0 ii) Infinite iii) $x = 6$
l) i) −4 ii) $\frac{1}{4}$ iii) $y = \frac{1}{4}x - \frac{13}{4}$

2. **a)** $\frac{1}{3}$ **b)** $y = \frac{1}{3}x + \frac{5}{3}$

c) -3 **d)** $y = -3x + 15$

e) $y = -3x + 35$ **f)** $(8, 11)$

g) $(2, 9)$ **h)** $y = \frac{1}{3}x + \frac{25}{3}$

i) 6.3 units **j)** $(6, 7)$

3. **a)** $y = \frac{2}{5}x - \frac{8}{5}$

b) $y = -\frac{5}{2}x - \frac{9}{2}$

c) 15.2 units

d) Midpoint AB = (4, 0)
Midpoint AC = (−3, 3)
Midpoint BC = (2, 5)

Student assessment 1 page 337

1 **a)** 1 **b)** -3

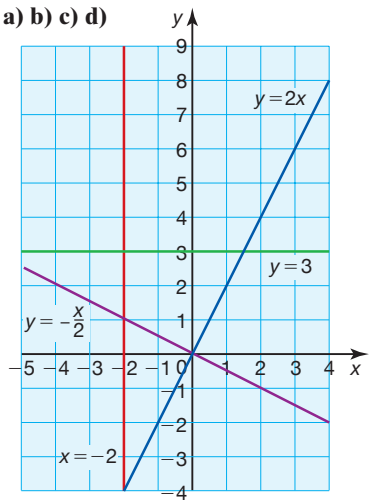
2 **a)** $y = 2x + 4$ **b)** $y = \frac{5}{2}x + 4$

3. **a)** $m = -3$ $c = 4$ **b)** $m = 3$ $c = 6$

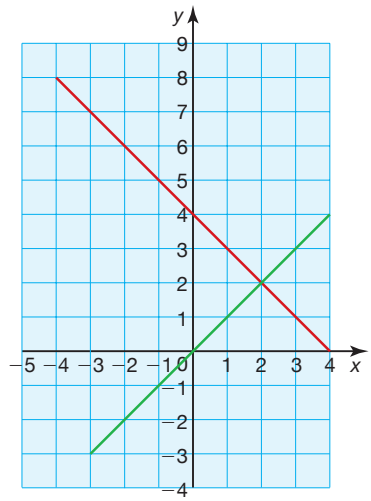
c) $m = -\frac{1}{2}$ $c = \frac{3}{2}$

4. $y = -\frac{2}{3}x + 6$

5. **a) b) c) d)**

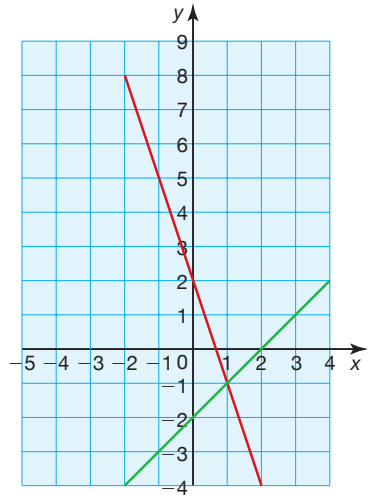


6. **a)**



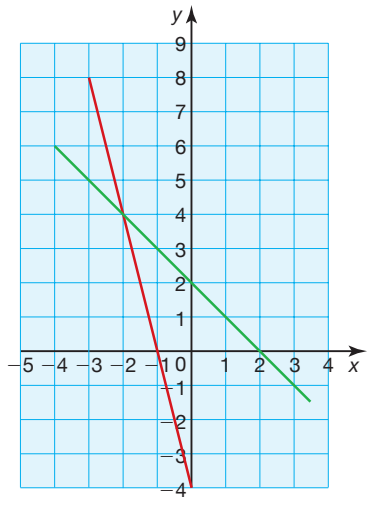
Solution is (2, 2)

b)



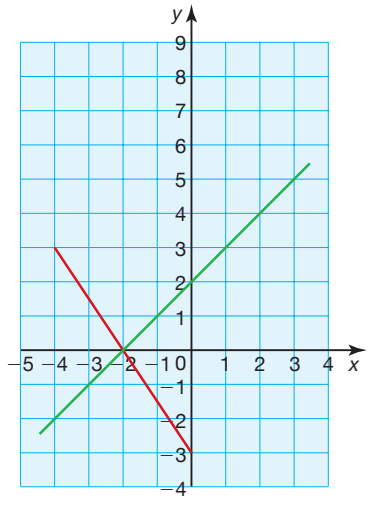
Solution is (1, −1)

c)



Solution is (–2, 4)

d)



Solution is (–2, 0)

7. **a)** i) 13 units ii) (0, 1.5)

b) i) 10 units ii) (4, 6)

8. **a)** $y = 3x - 4$ **b)** $y = -2x + 7$

9. **a)** $y = -\frac{2}{7}x + \frac{31}{7}$ **b)** $y = \frac{7}{2}x + 12$

10. **a)** i) (5, 6)

ii) $y = 3x - 9$

iii) $y = -\frac{1}{3}x + 6$

b) The diagonals are perpendicular as the product of their gradients is −1.

Student assessment 2 page 339

1 **a)** 2

b) −1

2 **a)** $y = 2x - 3$

b) $y = \frac{3}{2}x + \frac{5}{2}$

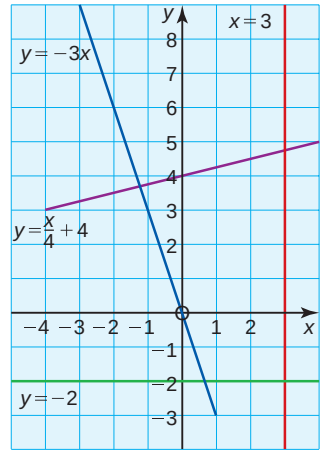
3. **a)** $m = \frac{1}{2}$ $c = 0$

b) $m = 4$ $c = 6$

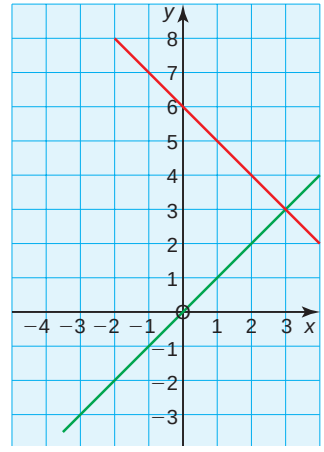
c) $m = -\frac{3}{2}$ $c = \frac{5}{2}$

4. $y = 5x$

5. **a) b) c) d)**

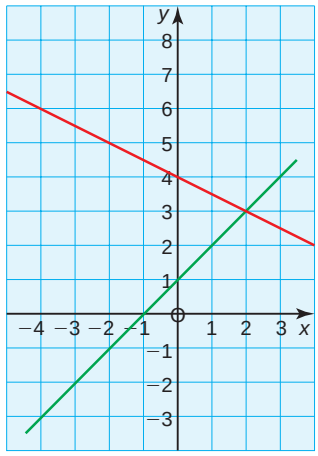


6. **a)**



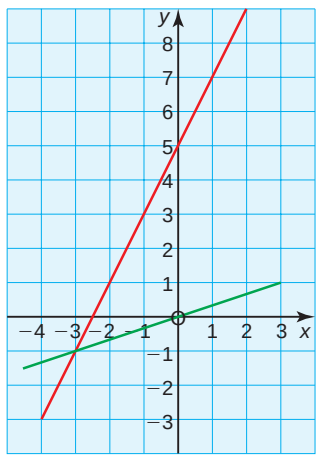
Solution is (3, 3)

b)



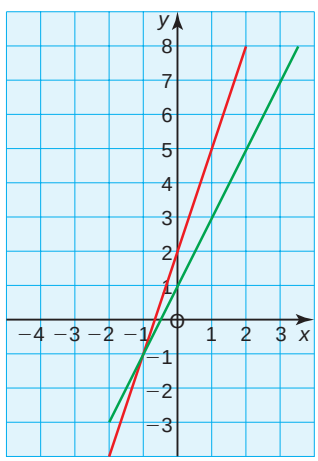
Solution is (2, 3)

c)



Solution is (–3, –1)

d)



Solution is (–1, –1)

7. **a)** i) 5 units ii) (0, 4.5)
b) i) 26 units ii) (–5, 2)

8. **a)** $y = 2x - 5$ **b)** $y = -4x + 3$

9. **a)** $y = -\frac{2}{7}x + \frac{55}{7}$ **b)** $y = -\frac{2}{7}x - \frac{19}{7}$

10. **a)** (1, 6)
b) $y = \frac{3}{4}x + \frac{21}{4}$
c) Substitute $y = 10\frac{1}{2}$ into the equation $y = \frac{3}{4}x + \frac{21}{4}$ and rearrange to find $x = 7$.
d) 7.5 units

Topic 5 Mathematical investigations and ICT

Plane trails page 341

1. Student's investigation

2. Student's ordered table similar to the one shown

Number of planes (p)	Maximum number of crossing points (n)
1	0
2	1
3	3
4	6
5	10
…	…

3. The sequence of the number of crossing points is the sequence of triangular numbers.

$n = \frac{1}{2}p(p-1)$

Hidden treasure page 342

1–2. The results for up to 20 contestants are given in the table below:

Number of contestants (n)	Winning chest (x)
1	1
2	2
3	2
4	4
5	2
6	4
7	6
8	8
9	2
10	4
11	6
12	8
13	10
14	12
15	14
16	16
17	2
18	4
19	6
20	8

3. Student's observed pattern: key pattern is that $x = n$ when n is a power of 2.

4. 31 contestants, winning chest is 30.
32 contestants, winning chest is 32.
33 contestants, winning chest is 2.

5. $x = 2(n - T)$, where x = the winning chest, n = number of contestants and T = the nearest power of 2 below n.

ICT activity page 343

Student's own investigation

29 Bearings

Exercise 29.1 page 347

1. Student's own diagrams

2. Student's own diagrams leading to:

a) 343° **b)** 034°

3. Student's own diagrams leading to:

a) 120° **b)** 102°

Student assessment 1 page 348

1. Student's own diagram

2. Student's own diagram

3. Student's own diagram

30 Trigonometry

Exercise 30.1 page 350

1. **a)** 1.82 cm **b)** 4.04 cm **c)** 19.2 cm
d) 4.87 cm **e)** 37.3 cm **f)** 13.9 cm

2. **a)** 14.3 cm **b)** 8.96 cm **c)** 9.33 cm
d) 4.10 cm **e)** 13.9 cm **f)** 6.21 cm

3. **a)** 49.4° **b)** 51.1° **c)** 51.3°
d) 63.4° **e)** 50.4° **f)** 71.6°

Exercise 30.2 page 352

1. **a)** 2.44 cm **b)** 18.5 cm **c)** 6.19 cm
d) 2.44 cm **e)** 43.8 cm **f)** 31.8 cm

2. **a)** 38.7° **b)** 48.6° **c)** 38.1°
d) 49.8° **e)** 32.6° **f)** 14.5°

Exercise 30.3 page 353

1. **a)** 36.0 cm **b)** 15.1 cm **c)** 48.2°
d) 81.1° **e)** 6.7 cm **f)** 16.8 cm
g) 70.5° **h)** 2.1 cm

Exercise 30.4 page 354

1. **a)** 5 cm **b)** 11.4 mm (3 s.f.)
c) 12 cm **d)** 13.2 cm (3 s.f.)

2. **a)** 11.0 cm (3 s.f.) **b)** 14.8 cm (3 s.f.)
c) 7.86 cm (3 s.f.) **d)** 7.35 cm (3 s.f.)
e) 3 cm **f)** 13.9 cm (3 s.f.)

3. 71.6 km

4. 66.9 km

5. **a)** 225° − 135° = 90° **b)** 73.8 km

6. 57 009 m

7. **a)** 8.5 km **b)** 15.5 km (3 s.f.)

8. **a)** 13.3 m (3 s.f.) **b)** 15.0 m (3 s.f.)

Exercise 30.5 page 356

1. a) 43.6° b) 19.5 cm
c) 16.7 cm d) 42.5°

2. a) 20.8 km (3 s.f.) b) 215.2° (1 d.p.)

3. **a)** 228 km (3 s.f.) **b)** 102 km (3 s.f.)
c) 103 km (3 s.f.) **d)** 147 km (3 s.f.)
e) 415 km (3 s.f.) **f)** 217° (3 s.f.)

4. **a)** 6.71 m (3 s.f.) **b)** 19.6 m (3 s.f.)
c) 15.3 m (3 s.f.)

5. **a)** 48.2° (3 s.f.) **b)** 41.8° (3 s.f.)
c) 8 cm **d)** 8.94 cm (3 s.f.)
e) 76.0 cm^2 (3 s.f.)

Exercise 30.6 page 357

1. **a)** 12.2 km (3 s.f.) **b)** 9.5° (1 d.p.)

2. **a)** 10.1 km (3 s.f.) **b)** 1.23 km (3 s.f.)

3. **a)** 22.6° (1 d.p.) **b)** 130 m

4. **a)** 0.342 km (3 s.f.) **b)** 0.940 km (3 s.f.)

5. **a)** 64.0 m (3 s.f.) **b)** 30.2 m (3 s.f.)

6. 6.93 km (3 s.f.)

7. **a)** 7.46 km (3 s.f.) **b)** 3.18 km (3 s.f.)

8. **a)** 2.9 km (1 d.p.) **b)** 6.9 km (1 d.p.)
c) 11.4° (1 d.p.) **d)** 20.4 km (1 d.p.)

9. **a)** 2.68 km (3 s.f.) **b)** 1.02 km (3 s.f.)
b) 3.5° (1 d.p.) **d)** 16.83 km (2 d.p.)

10. **a)** 225 m **b)** 48.4° (1 d.p.)

Exercise 30.7 page 360

1. **a)** sin 120° **b)** sin 100° **c)** sin 65°
d) sin 40° **e)** sin 52° **f)** sin 13°

2. **a)** sin 145° **b)** sin 130° **c)** sin 150°
d) sin 132° **e)** sin 76° **f)** sin 53°

3. **a)** 19°, 161° **b)** 82°, 98° **c)** 5°, 175°
d) 72°, 108° **e)** 13°, 167° **f)** 28°, 152°

4. **a)** 70°, 110° **b)** 9°, 171° **c)** 53°, 127°
d) 34°, 146° **e)** 16°, 164° **f)** 19°, 161°

Exercise 30.8 page 361

1. **a)** –cos 160° **b)** –cos 95° **c)** –cos 148°
d) –cos 85° **e)** –cos 33° **f)** –cos 74°

2. **a)** –cos 82° **b)** –cos 36° **c)** –cos 20°
d) –cos 37° **e)** –cos 9° **f)** –cos 57°

3. **a)** cos 80° **b)** –cos 90° **c)** cos 70°
d) cos 135° **e)** cos 58° **f)** cos 155°

4. **a)** −cos 55° **b)** −cos 73° **c)** cos 60°
d) cos 82° **e)** cos 88° **f)** cos 70°

Student assessment 1 page 362

1. **a)** 4 cm **b)** 43.9 cm **c)** 20.8 cm
d) 3.91 cm

2. **a)** 36.9° **b)** 56.3° **c)** 31.0°
d) 33.8°

3. **a)** 5 cm **b)** 6.63 cm **c)** 9.29 cm
d) 28.5 cm

Student assessment 2 page 363

1. **a)** 160.8 km **b)** 177.5 km

2. **a)** $\tan\theta = \frac{5}{x}$ **b)** $\tan\theta = \frac{7.5}{(x+16)}$

c) $\frac{5}{x} = \frac{7.5}{(x+16)}$ **d)** 32 m

e) 8.9° (1 d.p.)

3. **a)** 285 m (3 s.f.) **b)** 117° (3 s.f.)
c) 297° (3 s.f.)

4. **a)** 1.96 km (3 s.f.) **b)** 3.42 km (3 s.f.)
c) 3.57 km (3 s.f.)

Student assessment 3 page 364

1. **a)** 4003 m **b)** 2.35° (3 s.f.)

2. Student's graph

3. **a)** sin 130° **b)** sin 30° **c)** –cos 135°
d) –cos 60°

4. **a)** 38° **b)** 106°

5. **a)** 24.6° (1 d.p.) **b)** 32.6° (1 d.p.)
c) 5.94 km (3 s.f.) **d)** 1641

Student assessment 4 page 364

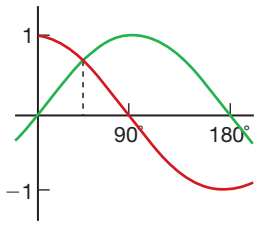
1. Student's graph

2. **a)** $-\cos 52°$ **b)** $\cos 100°$

3. **a)** 678 m (3 s.f.) **b)** 11.6° (1 d.p.)
c) 718 m (3 s.f.)

4. **a)** 21.8° (1 d.p.) **b)** 8.5° (1 d.p.)
c) 2.2 km **d)** 10.5° (1 d.p.)
e) 15.3° (1 d.p.) **f)** 1.76 km (3 s.f.)

5. **a)**



b) $\theta = 45°$

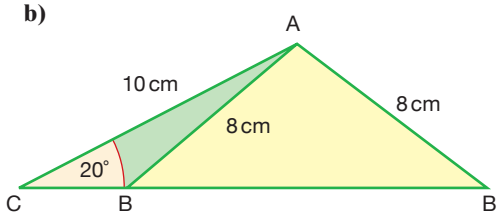
31 Further trigonometry

Exercise 31.1 page 367

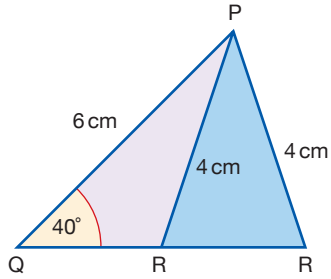
1. **a)** 8.91 cm **b)** 8.93 cm **c)** 5.96 mm
d) 8.64 cm

2. **a)** 33.2° **b)** 52.7° **c)** 77.0°
d) 44.0°

3. **a)** 25°, 155° (nearest degree)
b)



4. **a)** 75°, 105° (nearest degree)
b)



Exercise 31.2 page 368

1. **a)** 4.71 m **b)** 12.1 cm **c)** 9.15 cm
d) 3.06 cm **e)** 10.7 cm

2. **a)** 125.1° **b)** 108.2° **c)** 33.6°
d) 37.0° **e)** 122.9°

Exercise 31.3 page 369

1. **a)** 42.9 m (3 s.f.) **b)** 116.9° (1 d.p.)
c) 24.6° (1 d.p.) **d)** 33.4° (1 d.p.)
e) 35.0 m (3 s.f.)

2. 370 m

3. 73.9 m (3 s.f.)

Exercise 31.4 page 370

1. **a)** 70.0 cm^2 **b)** 70.9 mm^2 **c)** 122 cm^2
d) 17.0 cm^2

2. **a)** 24.6° **b)** 13.0 cm **c)** 23.1 cm
d) 63.2°

3. 16 800 m^2

4. **a)** 3.90 m^2 (3 s.f.) **b)** 222 m^3 (3 s.f.)

Exercise 31.5 page 372

1. **a)** 5.66 cm (3 s.f.) **b)** 6.93 cm (3 s.f.)
c) 54.7° (1 d.p.)

2. **a)** 5.83 cm (3 s.f.) **b)** 6.16 cm (3 s.f.)
c) 18.9° (1 d.p.)

3. **a)** 6.40 cm (3 s.f.) **b)** 13.6 cm (3 s.f.)
c) 61.9° (1 d.p.)

4. **a)** 75.3° (1 d.p.) **b)** 56.3° (1 d.p.)

5. **a)** i) 7.21 cm (3 s.f.) ii) 21.1° (1 d.p.)
b) i) 33.7° (1 d.p.) ii) 68.9° (1 d.p.)

6. **a)** i) 8.54 cm (3 s.f.) ii) 28.3° (1 d.p.)
b) i) 20.6° (1 d.p.) ii) 61.7° (1 d.p.)

7. **a)** 6.5 cm **b)** 11.3 cm (3 s.f.)
c) 70.7 cm (1 d.p.)

8. **a)** 11.7 cm **b)** 7.55 cm (3 s.f.)

9. **a)** TU = TQ = 10 cm
QU = 8.49 (3 s.f.)
b) 90°, 36.9°, 53.1° **c)** 24 cm^2

Exercise 31.6 page 375

1. **a)** RW **b)** TQ **c)** SQ
d) WU **e)** QV **f)** SV

2. **a)** JM **b)** KN **c)** HM
d) HO **e)** JO **f)** MO

3. **a)** ∠TPS **b)** ∠UPQ **c)** ∠VSW
d) ∠RTV **e)** ∠SUR **f)** ∠VPW

4. **a)** 5.83 cm (3 s.f.) **b)** 31.0° (1 d.p.)

5. **a)** 10.2 cm (3 s.f.) **b)** 29.2° (1 d.p.)
c) 51.3° (1 d.p.)

6. **a)** 6.71 cm (3 s.f.) **b)** 61.4° (1 d.p.)

7. **a)** 7.81 cm (3 s.f.) **b)** 11.3 cm (3 s.f.)
c) 12.4° (1 d.p.)

8. **a)** 14.1 cm (3 s.f.) **b)** 8.49 cm (3 s.f.)
c) 7.48 cm (3 s.f.) **d)** 69.3° (1 d.p.)

9. **a)** 17.0 cm (3 s.f.) **b)** 5.66 cm (3 s.f.)
c) 7 cm **d)** 51.1° (1 d.p.)

Student assessment 1 page 377

1. 134° (3 s.f.)

2. **a)** 11.7 cm (3 s.f.)
b) 12.3 cm (3 s.f.)
c) 29.1° (1 d.p.)

3. **a)** 18.0 m (3 s.f.) **b)** 26.5° (1 d.p.)
c) 28.8 m (3 s.f.) **d)** 278 m^2 (3 s.f.)

4. **a)** 12.7 cm (3 s.f.) **b)** 66.6° (1 d.p.)
c) 93.5 cm^2 (3 s.f.) **d)** 14.7 cm (1 d.p.)

Student assessment 2 page 377

1. **a)** 10.8 cm (3 s.f.) **b)** 11.9 cm (3 s.f.)
c) 30.2° (1 d.p.) **d)** 41.0° (1 d.p.)

2. **a)** 9.81 cm (3 s.f.) **b)** 30°
c) 19.6 cm (3 s.f.)

3. **a)** 5.83 cm (3 s.f.) **b)** 6.71 cm (3 s.f.)
c) 7.81 cm (3 s.f.) **d)** 46.6° (1 d.p.)
e) 19.0 cm^2 (3 s.f.) **f)** 36.7° (1 d.p.)

Topic 6 Mathematical investigations and ICT

Numbered balls page 378

1. If a ball is odd (n), the next ball is $n + 1$.
If the ball is even (n), the next ball is $\frac{n}{2}$.

2. 65, 66, 33, 34, 17, 18, 9, 10, 5, 6, 3, 4, 2, 1

3. The first ball must be odd. Start at 1 and work backwards.

4. 513, 514, 257, 258, 129, 130, 65, 66, 33, 34, 17, 18, 9, 10, 5, 6, 3, 4, 2, 1

Towers of Hanoi page 378

1. 3

2. 15

3. Student's investigation

4. The results up to 8 discs are given below:

Number of discs	Smallest number of moves
1	1
2	3
3	7
4	15
5	31
6	63
7	127
8	255

5. The number of moves are 1 less than the powers of 2.

6. 1023

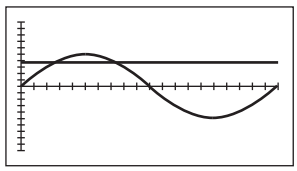
7. Number of moves = $2^n - 1$, where n = number of discs.

8. Time taken to move 64 discs is $2^{64} - 1$ seconds
This equates to 5.85×10^{11} years, i.e.
585 billion years.
Therefore according to the legend we needn't be too worried!

ICT activity page 379

1. **b)** i) 0.940 (3 d.p.)
ii) 0.819 (3 d.p.)
iii) –0.866 (3 d.p.) or $-\frac{\sqrt{3}}{2}$

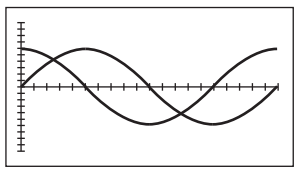
c)



The graph of $y = \sin x$ intersects the line $y = 0.7$ in two places as shown.

d) 30° and 150°

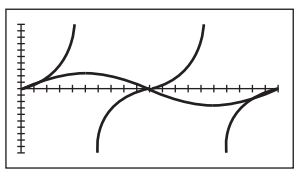
2. **a)**



b) Two solutions

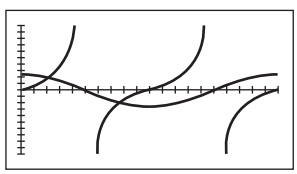
c) 225°

3. **a)**



Solutions are 0°, 180° and 360°

b)



Solutions are 38.2° and 141.8° (1 d.p.)

32 Vectors

Exercise 32.1 page 383

1. **a)** $\begin{pmatrix}2\\4\end{pmatrix}$ **b)** $\begin{pmatrix}3\\1\end{pmatrix}$ **c)** $\begin{pmatrix}-2\\-4\end{pmatrix}$

d) $\begin{pmatrix}3\\-2\end{pmatrix}$ **e)** $\begin{pmatrix}-6\\1\end{pmatrix}$ **f)** $\begin{pmatrix}6\\-1\end{pmatrix}$

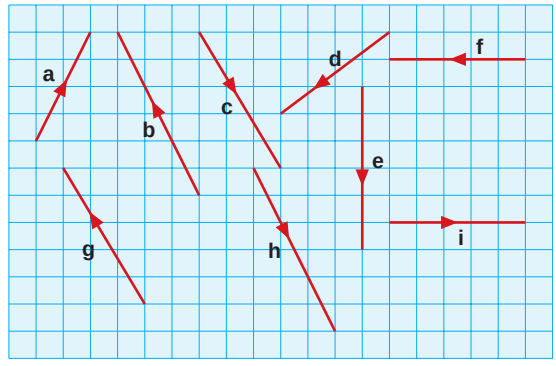
g) $\begin{pmatrix}-3\\-1\end{pmatrix}$ **h)** $\begin{pmatrix}-5\\-5\end{pmatrix}$ **i)** $\begin{pmatrix}-1\\3\end{pmatrix}$

2. **a)** $\begin{pmatrix}4\\4\end{pmatrix}$ **b)** $\begin{pmatrix}0\\4\end{pmatrix}$ **c)** $\begin{pmatrix}-3\\0\end{pmatrix}$

d) $\begin{pmatrix}-2\\6\end{pmatrix}$ **e)** $\begin{pmatrix}-4\\-2\end{pmatrix}$ **f)** $\begin{pmatrix}0\\-4\end{pmatrix}$

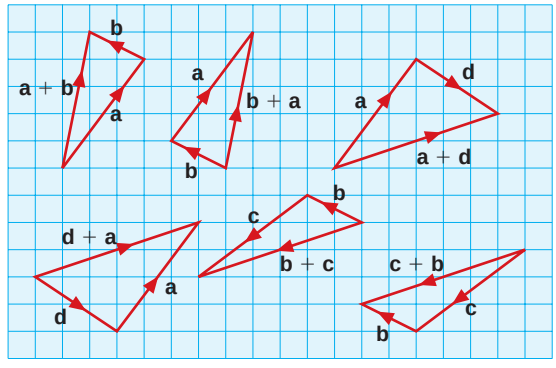
g) $\begin{pmatrix}3\\0\end{pmatrix}$ **h)** $\begin{pmatrix}2\\-6\end{pmatrix}$ **i)** $\begin{pmatrix}-4\\-4\end{pmatrix}$

3.



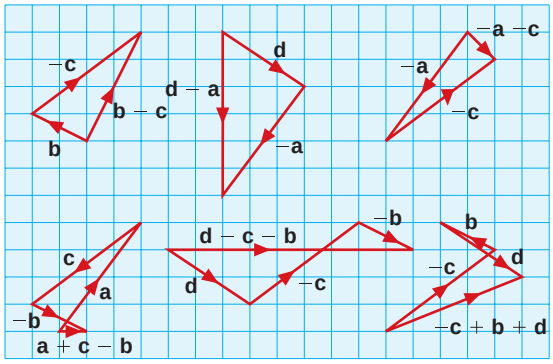
Exercise 32.2 page 384

1. **a) b) c) d) e) f)**



2. $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$, $\mathbf{a} + \mathbf{d} = \mathbf{d} + \mathbf{a}$, $\mathbf{b} + \mathbf{c} = \mathbf{c} + \mathbf{b}$

3. a) b) c) d) e) f)



4. a) $\begin{pmatrix}2\\4\end{pmatrix}$ b) $\begin{pmatrix}0\\-6\end{pmatrix}$ c) $\begin{pmatrix}1\\-1\end{pmatrix}$
d) $\begin{pmatrix}1\\0\end{pmatrix}$ e) $\begin{pmatrix}9\\0\end{pmatrix}$ f) $\begin{pmatrix}5\\2\end{pmatrix}$

Exercise 32.3 page 385

1. $\mathbf{d} = -\mathbf{c}$ $\mathbf{e} = -\mathbf{a}$ $\mathbf{f} = 2\mathbf{a}$ $\mathbf{g} = \frac{1}{2}\mathbf{c}$ $\mathbf{h} = \frac{1}{2}\mathbf{b}$
$\mathbf{i} = -\frac{1}{2}\mathbf{b}$ $\mathbf{j} = \frac{3}{2}\mathbf{b}$ $\mathbf{k} = -\frac{3}{2}\mathbf{a}$

2. a) $\begin{pmatrix}4\\6\end{pmatrix}$ b) $\begin{pmatrix}-12\\-3\end{pmatrix}$ c) $\begin{pmatrix}2\\-4\end{pmatrix}$
d) $\begin{pmatrix}-2\\2\end{pmatrix}$ e) $\begin{pmatrix}-2\\-5\end{pmatrix}$ f) $\begin{pmatrix}-8\\9\end{pmatrix}$
g) $\begin{pmatrix}-10\\-5\end{pmatrix}$ h) $\begin{pmatrix}3\\2\end{pmatrix}$ i) $\begin{pmatrix}10\\-6\end{pmatrix}$

3. a) $2\mathbf{a}$ b) $-\mathbf{b}$ c) $\mathbf{b} + \mathbf{c}$
d) $\mathbf{a} - \mathbf{b}$ e) $2\mathbf{c}$ f) $2\mathbf{c} - \mathbf{a}$

Exercise 32.4 page 386

1. $|\mathbf{a}| = 5.0$ units $|\mathbf{b}| = 4.1$ units $|\mathbf{c}| = 4.5$ units
$|\mathbf{d}| = 7.0$ units $|\mathbf{e}| = 7.3$ units $|\mathbf{f}| = 6.4$ units

2. a) $|AB| = 4.0$ units b) $|BC| = 5.4$ units
c) $|CD| = 7.2$ units d) $|DE| = 13.0$ units
e) $|2AB| = 8.0$ units f) $|-CD| = 7.2$ units

3. a) 4.1 units b) 18.4 units c) 15.5 units
d) 17.7 units e) 31.8 units f) 19.6 units

Exercise 32.5 page 387

1. $A = \begin{pmatrix}3\\3\end{pmatrix}$ $B = \begin{pmatrix}1\\2\end{pmatrix}$ $C = \begin{pmatrix}-3\\2\end{pmatrix}$
$D = \begin{pmatrix}-3\\0\end{pmatrix}$ $E = \begin{pmatrix}4\\-2\end{pmatrix}$ $F = \begin{pmatrix}2\\-4\end{pmatrix}$
$G = \begin{pmatrix}-1\\-1\end{pmatrix}$ $H = \begin{pmatrix}0\\-3\end{pmatrix}$

Exercise 32.6 page 388

1. a) $\mathbf{a}$ b) $\mathbf{a}$ c) $\mathbf{b}$
d) $-\mathbf{b}$ e) $2\mathbf{b}$ f) $-2\mathbf{a}$
g) $\mathbf{a} + \mathbf{b}$ h) $\mathbf{b} - \mathbf{a}$ i) $\mathbf{b} - 2\mathbf{a}$

2. a) $2\mathbf{a}$ b) $\mathbf{b}$ c) $\mathbf{b} - \mathbf{a}$
d) $\mathbf{b} - \mathbf{a}$ e) $-\mathbf{a}$ f) $\mathbf{a} - 2\mathbf{b}$

3. a) $-2\mathbf{a}$ b) $-\mathbf{a}$ c) $\mathbf{b}$
d) $\mathbf{b} - \mathbf{a}$ e) $2(\mathbf{b} - \mathbf{a})$ f) $2\mathbf{b} - \mathbf{a}$
g) $-2\mathbf{b}$ h) $\mathbf{b} - 2\mathbf{a}$ i) $-\mathbf{a} - \mathbf{b}$

4. a) $5\mathbf{a}$ b) $\frac{8}{3}\mathbf{b}$
c) $\frac{1}{3}(8\mathbf{b} - 15\mathbf{a})$ d) $\frac{1}{15}(8\mathbf{b} - 15\mathbf{a})$
e) $\mathbf{b} - 2\mathbf{a}$ f) $5\mathbf{a} - 2\mathbf{b}$
g) $\frac{8}{5}\mathbf{b}$ h) $\frac{1}{5}(10\mathbf{a} + 8\mathbf{b})$
i) $\frac{1}{5}(8\mathbf{b} - 5\mathbf{a})$

Exercise 32.7 page 389

1. a) i) $2\mathbf{a}$ ii) $2\mathbf{a} - \mathbf{b}$ iii) $\frac{1}{4}(2\mathbf{a} + 3\mathbf{b})$
b) Proof

2. a)b) Proofs

3. a) i) $4\mathbf{a}$ ii) $2\mathbf{a}$ iii) $2(\mathbf{a} + \mathbf{b})$ iv) $\frac{3}{2}\mathbf{a}$
b) Proof

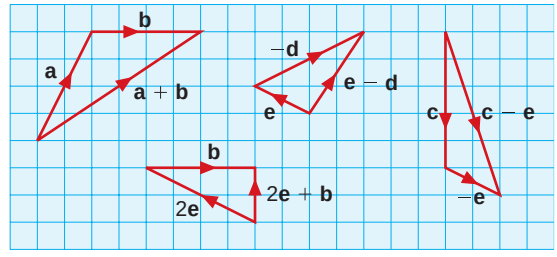
4. a) i) $\frac{1}{2}(2\mathbf{q} - \mathbf{p})$ ii) $\frac{2}{7}(\mathbf{p} - \mathbf{q})$ b) Proof

Student assessment 1 page 390

1. a) $\begin{pmatrix}-2\\3\end{pmatrix}$ b) $\begin{pmatrix}7\\2\end{pmatrix}$ c) $\begin{pmatrix}6\\-2\end{pmatrix}$

2. $\mathbf{a} = \begin{pmatrix}2\\4\end{pmatrix}$ $\mathbf{b} = \begin{pmatrix}4\\0\end{pmatrix}$ $\mathbf{c} = \begin{pmatrix}0\\-5\end{pmatrix}$
$\mathbf{d} = \begin{pmatrix}-4\\-2\end{pmatrix}$ $\mathbf{e} = \begin{pmatrix}-2\\1\end{pmatrix}$

3.



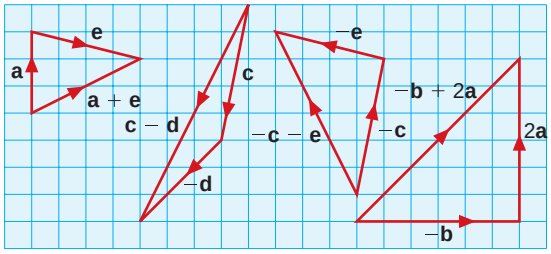
4. a) $\begin{pmatrix}-1\\5\end{pmatrix}$ b) $\begin{pmatrix}1\\5\end{pmatrix}$ c) $\begin{pmatrix}1\\11\end{pmatrix}$ d) $\begin{pmatrix}0\\14\end{pmatrix}$

Student assessment 2 page 390

1. **a)** $\begin{pmatrix}-2\\7\end{pmatrix}$ **b)** $\begin{pmatrix}-5\\-4\end{pmatrix}$ **c)** $\begin{pmatrix}-1\\-3\end{pmatrix}$

2. **a)** $\begin{pmatrix}0\\3\end{pmatrix}$ **b)** $\begin{pmatrix}-6\\0\end{pmatrix}$ **c)** $\begin{pmatrix}-1\\-5\end{pmatrix}$
d) $\begin{pmatrix}3\\3\end{pmatrix}$ **e)** $\begin{pmatrix}4\\-1\end{pmatrix}$

3.



4. **a)** $\begin{pmatrix}7\\-11\end{pmatrix}$ **b)** $\begin{pmatrix}-3\\9\end{pmatrix}$ **c)** $\begin{pmatrix}6\\-6\end{pmatrix}$ **d)** $\begin{pmatrix}-18\\28\end{pmatrix}$

Student assessment 3 page 391

1. **a)** $|\overrightarrow{AB}| = 7.21$ units
b) $|\mathbf{a}| = 9.22$ units $|\mathbf{b}| = 8.06$ units
$|\mathbf{c}| = 13$ units

2. **a)** 17.5 units **b)** 2.69 units

3. $A = \begin{pmatrix}2\\4\end{pmatrix}$ $B = \begin{pmatrix}-1\\2\end{pmatrix}$ $C = \begin{pmatrix}-3\\-1\end{pmatrix}$
$D = \begin{pmatrix}0\\-3\end{pmatrix}$ $E = \begin{pmatrix}1\\-4\end{pmatrix}$

4. **a)** Student's own vector
b) $\overrightarrow{DF} = \frac{1}{2}\overrightarrow{BC}$
c) $\overrightarrow{CF} = -\overrightarrow{DE}$

Student assessment 4 page 392

1. **a)** $|\overrightarrow{FG}| = 5$ units
b) $|\mathbf{a}| = 6.1$ units $|\mathbf{b}| = 12.4$ units
$|\mathbf{c}| = 14.1$ units

2. **a)** 29.7 units **b)** 11.4 units

3. **a)** $2\mathbf{a}$ **b)** $-\mathbf{b}$ **c)** $\mathbf{b} - \mathbf{a}$

Student assessment 5 page 392

1. **a)** i) $\frac{1}{4}\mathbf{b}$ ii) $\frac{1}{4}(4\mathbf{a} - \mathbf{b})$ iii) $\frac{2}{5}(\mathbf{b} - \mathbf{a})$
b) Proof

2. **a)** Proof **b)** Proof

3. **a)** i) $\mathbf{b} - \mathbf{a}$ ii) $\mathbf{a}$ iii) $\mathbf{a} + \mathbf{b}$
b) i) 4 : 25 ii) 20 : 25

Student assessment 6 page 393

1. **a)** $\mathbf{a}$ **b)** $-\mathbf{b}$ **c)** $(1 + \sqrt{2})\mathbf{b}$

2. **a)** $5\mathbf{b}$ **b)** $5\mathbf{b} - \mathbf{a}$ **c)** $\frac{1}{2}(\mathbf{a} + 3\mathbf{b})$

3. **a)** i) $\frac{1}{2}\mathbf{a}$ ii) $\mathbf{b} - \mathbf{a}$ **b)** 2 : 3

33 Matrices

Exercise 33.1 page 394

1. **a)** $\mathbf{P} = 2 \times 3$ **b)** $\mathbf{Q} = 2 \times 4$ **c)** $\mathbf{R} = 4 \times 2$
d) $\mathbf{S} = 4 \times 5$ **e)** $\mathbf{T} = 5 \times 1$ **f)** $\mathbf{F} = 1 \times 5$

2. Student's own matrices

3. $\begin{pmatrix}6500 & 900\\7200 & 1100\\7300 & 1040\end{pmatrix}$

4. $\begin{pmatrix}3 & 4 & 2 & 1\\0 & 6 & 2 & 0\\1 & 3 & 0 & 2\end{pmatrix}$

5. $\begin{pmatrix}8 & 6 & 9 & 3\end{pmatrix}$

6. $\begin{pmatrix}37 & 49\\74 & 58\\76 & 62\\89 & 56\end{pmatrix}$

7. $\begin{pmatrix}20 & 35 & 15\\45 & 25 & 40\\30 & 30 & 10\\0 & 0 & 25\\5 & 10 & 10\end{pmatrix}$

8. $\begin{pmatrix}8000 & 3000 & 5000\\8000 & 6000 & 10\,000\\5000 & 11\,000 & 9000\\9000 & 13\,000 & 6000\end{pmatrix}$

9. $\begin{pmatrix}6 & 12 & 43 & 18 & 6 & 9 & 6\\9 & 15 & 28 & 18 & 12 & 12 & 6\\12 & 19 & 30 & 12 & 9 & 9 & 9\end{pmatrix}$

10. Student's own matrix

Exercise 33.2 page 398

1. **a)** $\begin{pmatrix}14 & 8\\ 7 & 16\end{pmatrix}$ **b)** $\begin{pmatrix}9 & 3 & 24\\ 19 & 35 & 5\end{pmatrix}$

c) $\begin{pmatrix}24\\ 3\\ 18\end{pmatrix}$ **d)** $\begin{pmatrix}-10 & 13 & 17\\ 2 & -13 & 18\\ 5 & 10 & 11\end{pmatrix}$

e) $\begin{pmatrix}-3 & 1 & -1\end{pmatrix}$ **f)** $\begin{pmatrix}14 & -9\\ 15 & -5\\ -5 & 10\end{pmatrix}$

2. **a)** $\begin{pmatrix}5 & 4\\ 1 & 0\end{pmatrix}$ **b)** $\begin{pmatrix}2 & 2 & 2\\ 2 & 2 & 2\end{pmatrix}$

c) $\begin{pmatrix}-5\\ 0\\ 4\end{pmatrix}$ **d)** $\begin{pmatrix}4 & -14 & -1\\ 2 & 0 & 4\\ -9 & -1 & -5\end{pmatrix}$

e) $\begin{pmatrix}-3 & 18\\ -1 & -12\end{pmatrix}$ **f)** $\begin{pmatrix}-3 & -4\\ -21 & 13\\ -3 & 2\end{pmatrix}$

3. **a)** $\begin{pmatrix}9 & 11\\ 15 & 5\\ 9 & 8\end{pmatrix}$ **b)** $\begin{pmatrix}9 & 11\\ 15 & 5\\ 9 & 8\end{pmatrix}$ **c)** $\begin{pmatrix}-1 & 5\\ 6 & 6\\ 7 & -5\end{pmatrix}$

d) $\begin{pmatrix}10 & 3\\ -3 & 14\\ 3 & 4\end{pmatrix}$ **e)** $\begin{pmatrix}10 & 6\\ 9 & -1\\ 2 & 13\end{pmatrix}$ **f)** $\begin{pmatrix}11 & -2\\ -9 & 8\\ -4 & 9\end{pmatrix}$

g) $\begin{pmatrix}5 & 9\\ 6 & 0\\ -5 & 13\end{pmatrix}$ **h)** $\begin{pmatrix}3 & -1\\ -12 & 12\\ -5 & 0\end{pmatrix}$ **i)** $\begin{pmatrix}13 & -7\\ -6 & 10\\ 9 & 0\end{pmatrix}$

4. **a)** $\begin{pmatrix}2 & 3 & 3\\ 3 & 1 & 4\\ 1 & 4 & 3\end{pmatrix}$ $\begin{pmatrix}5 & 1 & 1\\ 1 & 4 & 2\\ 1 & 3 & 3\end{pmatrix}$ **b)** 15 races

c) $\begin{pmatrix}7 & 4 & 4\\ 4 & 5 & 6\\ 2 & 7 & 6\end{pmatrix}$

5. **a)** 444

b) $\begin{pmatrix}265 & 312\\ 140 & 132\end{pmatrix} - \begin{pmatrix}189 & 204\\ 121 & 68\end{pmatrix} = \begin{pmatrix}76 & 108\\ 19 & 64\end{pmatrix}$

c) 267

Exercise 33.3 page 400

1. **a)** $\begin{pmatrix}8 & 12\\ 14 & 6\end{pmatrix}$ **b)** $\begin{pmatrix}21 & 6\\ 3 & 0\end{pmatrix}$ **c)** $\begin{pmatrix}9 & 3\\ 0 & 6\end{pmatrix}$

d) $\begin{pmatrix}10 & 5\\ 0 & 20\end{pmatrix}$ **e)** $\begin{pmatrix}28 & 20\\ 16 & 32\end{pmatrix}$ **f)** $\begin{pmatrix}21 & 42\\ 14 & 7\end{pmatrix}$

2. **a)** $\begin{pmatrix}4 & 1\\ 0 & 2\end{pmatrix}$ **b)** $\begin{pmatrix}4 & 3\\ 2 & 1\end{pmatrix}$ **c)** $\begin{pmatrix}3 & 6\\ 0.75 & 1.5\end{pmatrix}$

d) $\begin{pmatrix}6 & 4\\ 0.8 & 2\end{pmatrix}$ **e)** $\begin{pmatrix}2.5 & 7.5\\ 7.5 & 10\end{pmatrix}$ **f)** $\begin{pmatrix}12 & 3\\ 3 & 6\end{pmatrix}$

Exercise 33.4 page 402

1. **a)** $\begin{pmatrix}18 & 78\\ 14 & 54\end{pmatrix}$ **b)** $\begin{pmatrix}12 & 72\\ 0 & 24\end{pmatrix}$

2. **a)** $\begin{pmatrix}52 & 12 & 18\\ 44 & 22 & 12\end{pmatrix}$ **b)** $\begin{pmatrix}18 & 18\\ 8 & 8\\ 6 & 6\end{pmatrix}$

3. **a)** $\begin{pmatrix}28 & 4 & -8\\ -31 & 19 & 40\end{pmatrix}$ **b)** $\begin{pmatrix}27 & -17\end{pmatrix}$

4. **a)** $\begin{pmatrix}6 & 4 & -2\\ -3 & -2 & 1\\ -12 & -8 & 4\\ 18 & 12 & -6\end{pmatrix}$ **b)** $\begin{pmatrix}30 & 22 & 38\\ 20 & 13 & 2\\ 9 & 7 & 17\\ 24 & 19 & 50\end{pmatrix}$

Exercise 33.5 page 402

1. $\mathbf{VW} = \begin{pmatrix}0 & 7\\ 24 & 8\end{pmatrix}$ $\mathbf{WV} = \begin{pmatrix}-12 & 4\\ -18 & 20\end{pmatrix}$

2. $\mathbf{VW} = \begin{pmatrix}-10 & -9\\ -20 & -18\end{pmatrix}$

$\mathbf{WV} = \begin{pmatrix}11 & 14 & -1\\ -25 & -30 & 5\\ 39 & 46 & -9\end{pmatrix}$

3. $\mathbf{VW} = (-11)$

$\mathbf{WV} = \begin{pmatrix}4 & -10 & 18 & 4\\ 0 & 0 & 0 & 0\\ -6 & 15 & -27 & -6\\ 12 & -30 & 54 & 12\end{pmatrix}$

4. $\mathbf{VW} = \begin{pmatrix}7 & 7 & 9\\ -6 & -4 & -14\end{pmatrix}$ **WV** is not possible

5. $\mathbf{VW} = \begin{pmatrix}-33 & 41 & -10\\ 17 & -11 & -8\end{pmatrix}$ **WV** is not possible

Exercise 33.6 page 403

1. $\mathbf{AI} = \begin{pmatrix}2 & 1\\ 3 & 2\end{pmatrix}$ $\mathbf{IA} = \begin{pmatrix}2 & 1\\ 3 & 2\end{pmatrix}$

2. $\mathbf{AI} = \begin{pmatrix}-2 & -4\\ 3 & 6\end{pmatrix}$ $\mathbf{IA} = \begin{pmatrix}-2 & -4\\ 3 & 6\end{pmatrix}$

3. $\mathbf{AI} = \begin{pmatrix}4 & 8\\ -2 & 4\end{pmatrix}$ $\mathbf{IA} = \begin{pmatrix}4 & 8\\ -2 & 4\end{pmatrix}$

4. $\mathbf{AI} = \begin{pmatrix}3 & 2\\ 1 & 6\\ -2 & 5\end{pmatrix}$ **IA** is not possible

5. $\mathbf{AI} = (-5 \quad -6)$ **IA** is not possible

6. $\begin{pmatrix} 4 & -3 \\ 5 & -6 \\ 3 & 2 \\ 1 & 4 \end{pmatrix}$ **IA** is not possible

7. When **AI** exists, it is equal to A.

8. For a 2×2 matrix **AI** = **IA** = **A.**

Exercise 33.7 page 404

1. **a)** 3 **b)** 3 **c)** 4 **d)** 2

2. **a)** –4 **b)** –10 **c)** –10 **d)** –1

3. **a)** 18 **b)** 14 **c)** –54 **d)** 4

4. Student's own matrices

5. Student's own matrices

6. Student's own matrices

7. **a)** 16 **b)** 131 **c)** 18 **d)** 936
e) –104 **f)** –576 **g)** 254 **h)** –576
i) 147

Exercise 33.8 page 406

1. **a)** $\begin{pmatrix} 4 & -5 \\ -7 & 9 \end{pmatrix}$ **b)** $\begin{pmatrix} 5 & -7 \\ -7 & 10 \end{pmatrix}$ **c)** $\begin{pmatrix} 1 & -1 \\ -0.8 & 1 \end{pmatrix}$

d) $\begin{pmatrix} -1\frac{1}{3} & 3 \\ -1 & 2 \end{pmatrix}$ **e)** $\begin{pmatrix} -1.8 & -0.8 \\ 2 & 1 \end{pmatrix}$

f) Not possible

2. See answers for Q.1.

3. It has no inverse as the determinant = 0.

4. A, B, D have no inverse.

5. **a)** $\begin{pmatrix} -9 & 4 \\ 7 & -3 \end{pmatrix}$ **b)** $\begin{pmatrix} -\frac{5}{4} & -\frac{3}{4} \\ 2 & 1 \end{pmatrix}$ **c)** $\begin{pmatrix} 6 & -\frac{11}{4} \\ -11 & 5 \end{pmatrix}$

d) $\begin{pmatrix} -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{11} \end{pmatrix}$ **e)** $\begin{pmatrix} \frac{9}{2} & \frac{5}{2} \\ -\frac{31}{12} & -\frac{17}{12} \end{pmatrix}$ **f)** $\begin{pmatrix} -\frac{3}{4} & \frac{1}{2} \\ 2 & -1 \end{pmatrix}$

Student assessment 1 page 406

1. **a)** $\begin{pmatrix} 4 & -1 \\ -7 & 8 \end{pmatrix}$ **b)** $\begin{pmatrix} 6 & 11 \\ -1 & -12 \\ 10 & -5 \end{pmatrix}$

c) $\begin{pmatrix} -9 & -6 \\ -1 & 1 \end{pmatrix}$ **d)** $\begin{pmatrix} 5 & -4 \\ 1 & 5 \\ 1 & 5 \end{pmatrix}$

e) $\begin{pmatrix} 6 & 16 & -8 \\ 2 & -12 & 14 \end{pmatrix}$ **f)** $\begin{pmatrix} 1 & -2 \\ 0 & -1.5 \end{pmatrix}$

2. **a)** $\begin{pmatrix} 18 & -17 \\ 45 & 0 \\ 30 & 15 \end{pmatrix}$ **b)** $\begin{pmatrix} -83 & -14 \end{pmatrix}$

3. **a)** 1 **b)** 6 **c)** –6 **d)** –4

4. **a)** $\begin{pmatrix} 1 & -2 \\ -\frac{3}{2} & \frac{7}{2} \end{pmatrix}$ **b)** $\begin{pmatrix} 8 & -7 \\ -9 & 8 \end{pmatrix}$

c) $\begin{pmatrix} \frac{5}{9} & -\frac{11}{18} \\ -\frac{2}{3} & \frac{5}{6} \end{pmatrix}$ **d)** $\begin{pmatrix} -\frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & -\frac{1}{12} \end{pmatrix}$

Student assessment 2 page 407

1. **a)** $\begin{pmatrix} 3 & -1 \\ -8 & 2 \end{pmatrix}$ **b)** $\begin{pmatrix} 11 & 7 \\ 6 & -12 \\ 8 & -5 \end{pmatrix}$

c) $\begin{pmatrix} -10 & -7 \\ -4 & 1 \end{pmatrix}$ **d)** $\begin{pmatrix} 3 & 1 \\ -4 & 1 \\ 6 & -3 \end{pmatrix}$

e) $\begin{pmatrix} 3 & 12 & -3 \\ 6 & -12 & 15 \end{pmatrix}$ **f)** $\begin{pmatrix} 1 & -2 \\ 0 & -4 \end{pmatrix}$

2. **a)** $\begin{pmatrix} 3 & -15 \\ 42 & 10 \\ -36 & 26 \end{pmatrix}$ **b)** $\begin{pmatrix} -19 & -32 \end{pmatrix}$

3. **a)** 1 **b)** 66 **c)** 66 **d)** 18

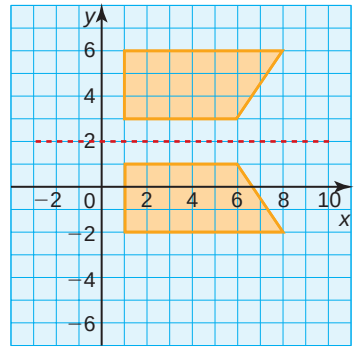
4. **a)** $\begin{pmatrix} 9 & -8 \\ -10 & 9 \end{pmatrix}$ **b)** $\begin{pmatrix} \frac{7}{2} & -3 \\ -\frac{9}{2} & 4 \end{pmatrix}$

c) Not possible **d)** $\begin{pmatrix} 67.5 & -59 \\ -75.5 & 66 \end{pmatrix}$

34 Transformations

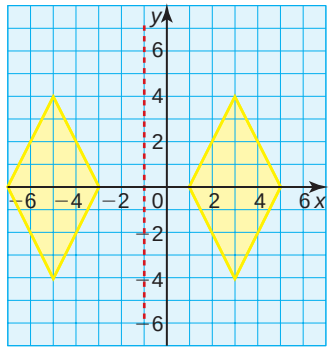
Exercise 34.1 page 409

1. **a)**



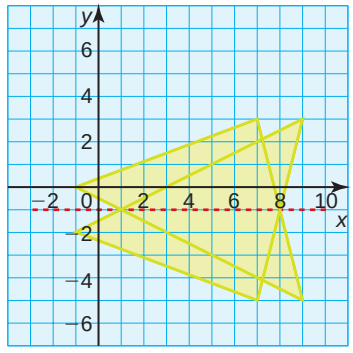
b) $y = 2$

2. **a)**



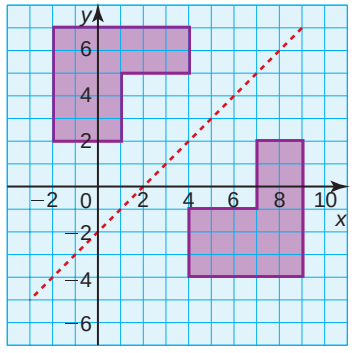
b) $x = -1$

3. **a)**



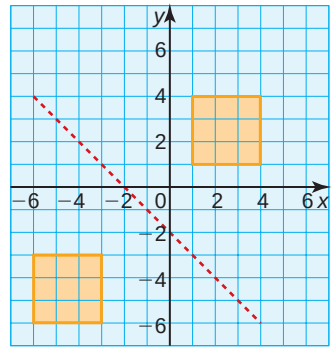
b) $y = -1$

4. **a)**



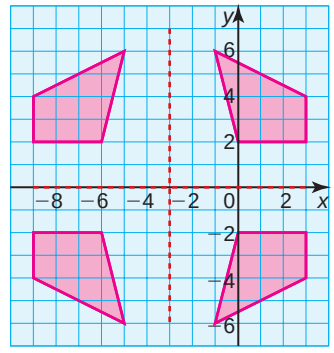
b) $y = x - 2$

5. **a)**



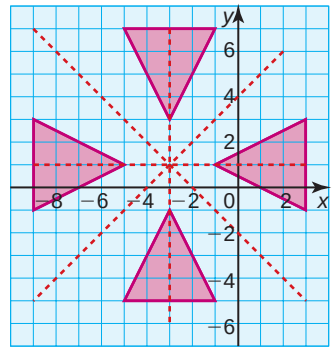
b) $y = -x - 2$

6. **a)**



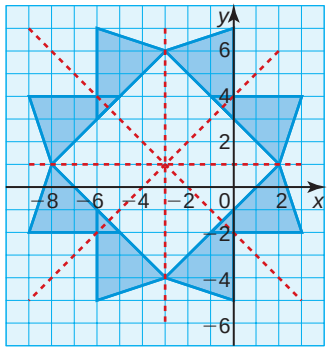
b) $y = 0, x = -3$

7. **a)**



b) $y = 1, x = -3, y = x + 4, y = -x - 2$

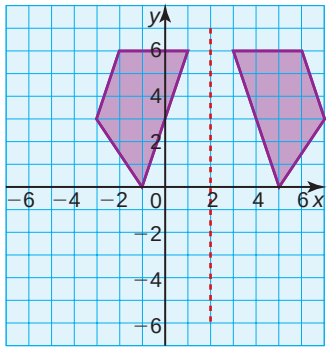
8. a)



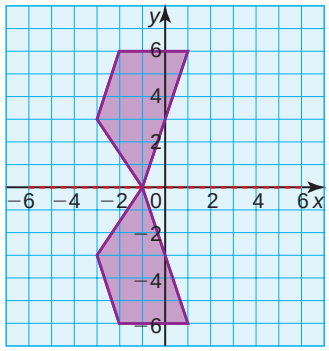
b) $y = 1$, $x = -3$, $y = x + 4$, $y = -x - 2$

Exercise 34.2 page 410

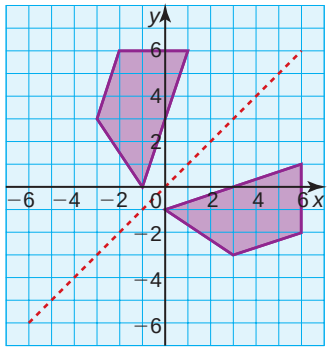
1. a)



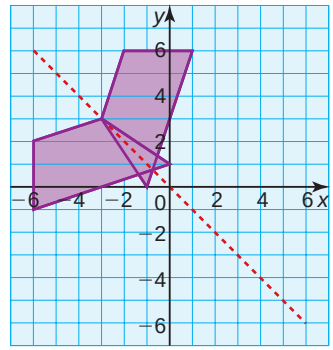
b)



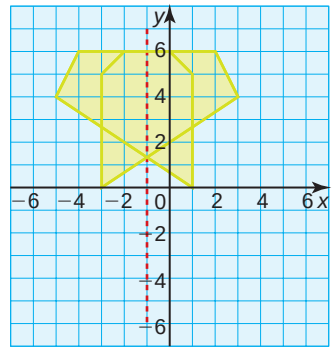
c)



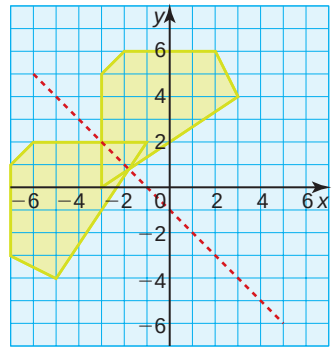
d)



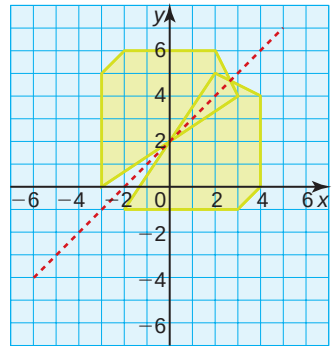
2. a)



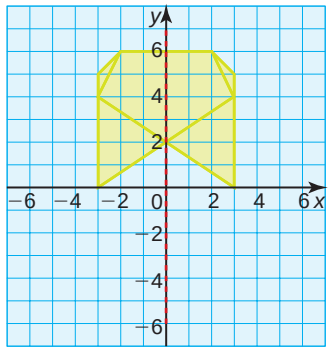
b)



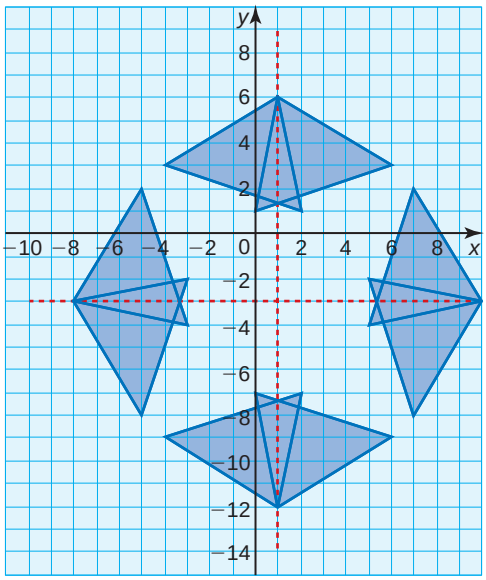
c)



d)

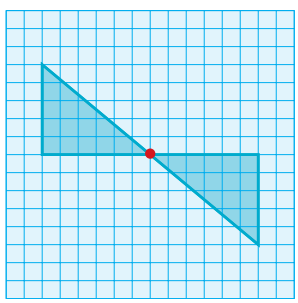


3.

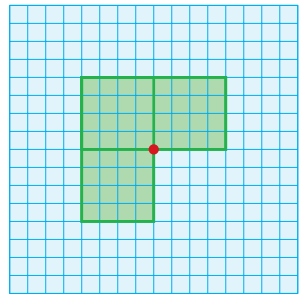


Exercise 34.3 page 411

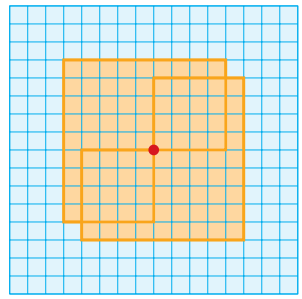
1.



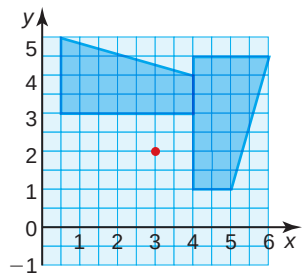
2.



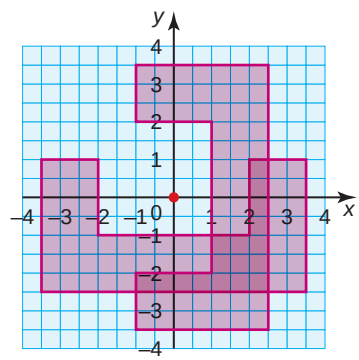
3.



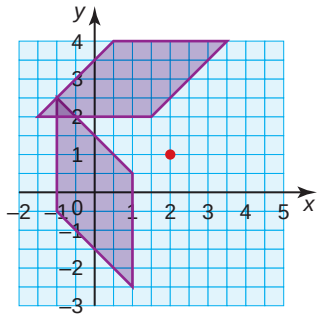
4.



5.

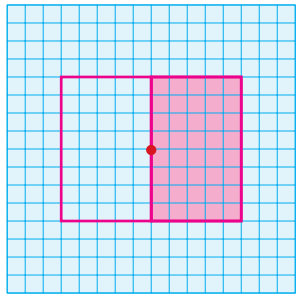


6.



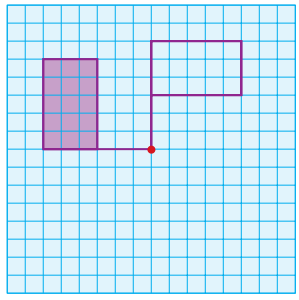
Exercise 34.4 page 411

1. a)



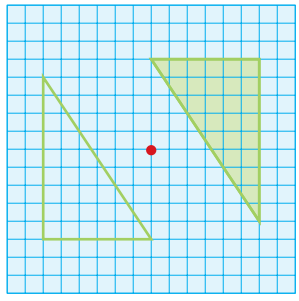
b) 180° clockwise/anti-clockwise

2. a)



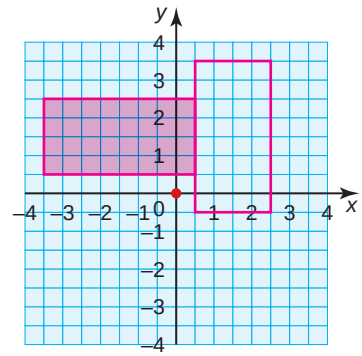
b) 90° anti-clockwise

3. a)



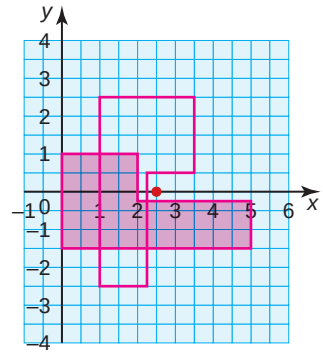
b) 180° clockwise/anti-clockwise

4. a)



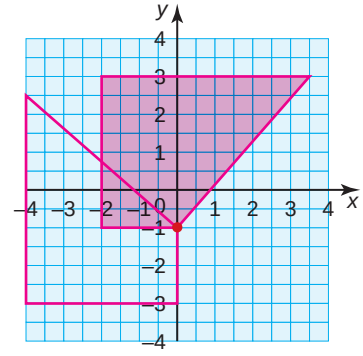
b) 90° anti-clockwise about (0, 0)

5. a)



b) 90° anti-clockwise about (2.5, 0)

6. a)



b) 90° clockwise about (0, −1)

Exercise 34.5 page 413

Student's own diagrams.

Exercise 34.6 page 414

1. $A \to B = \begin{pmatrix} -6 \\ 0 \end{pmatrix}$ $A \to C = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$

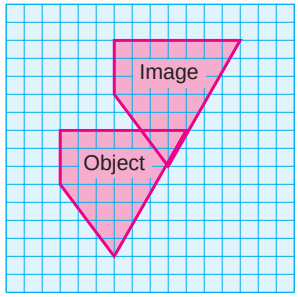
2. $A \to B = \begin{pmatrix} 0 \\ -7 \end{pmatrix}$ $A \to C = \begin{pmatrix} -6 \\ 1 \end{pmatrix}$

3. $A \to B = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$ $A \to C = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$

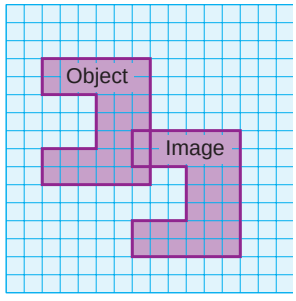
4. $A \to B = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ $A \to C = \begin{pmatrix} -3 \\ -6 \end{pmatrix}$

Exercise 34.7 page 415

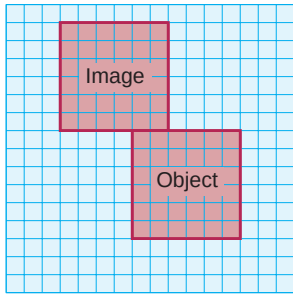
1.



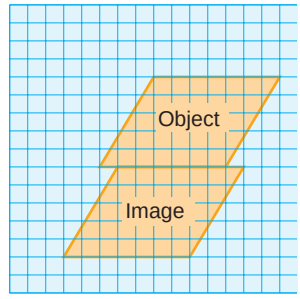
2.



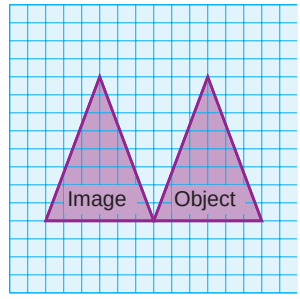
3.



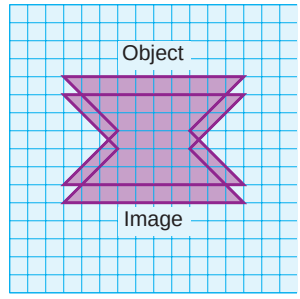
4.



5.

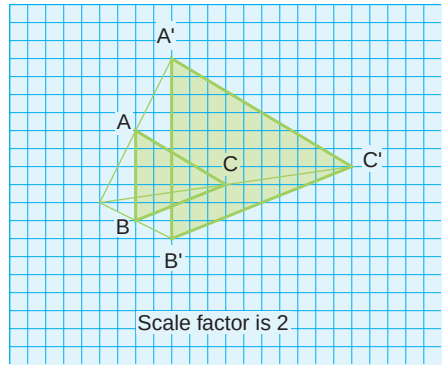


6.

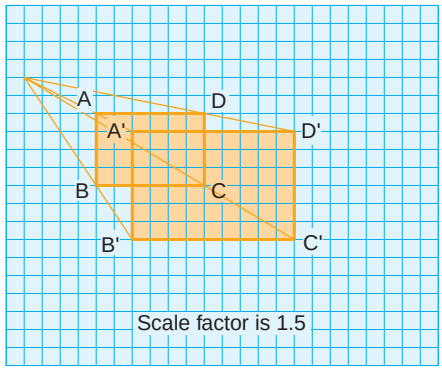


Exercise 34.8 page 418

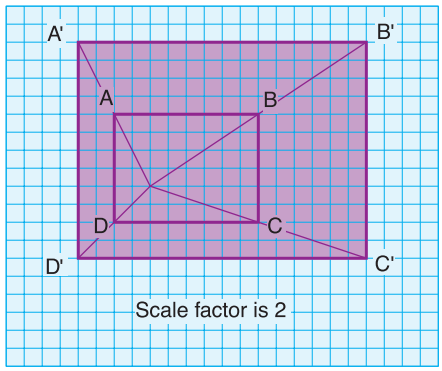
1.



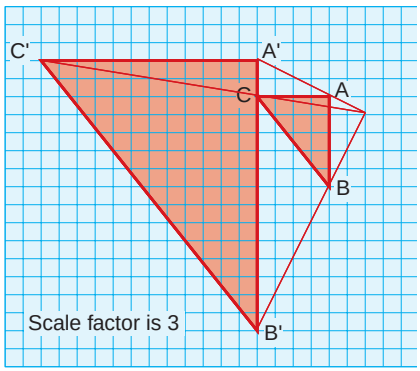
2.



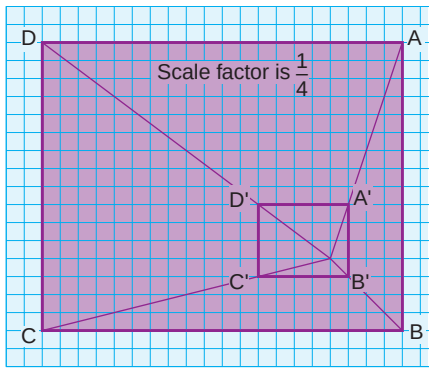
3.



4.

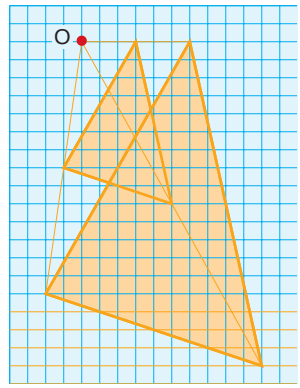


5.

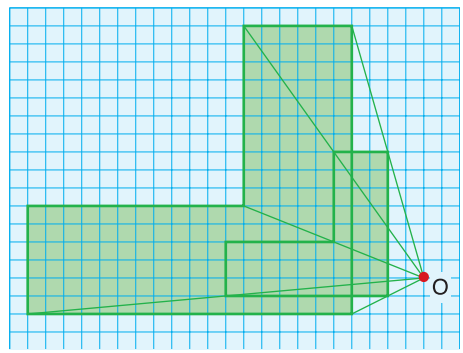


Exercise 34.9 page 419

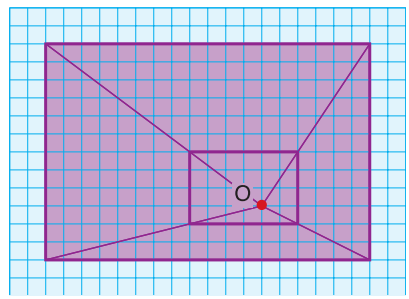
1.



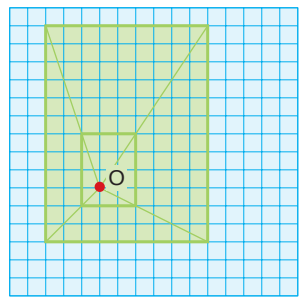
2.



3.

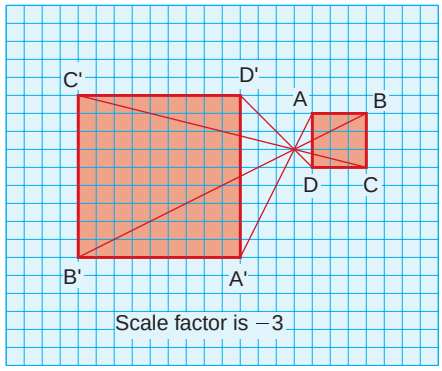


4.

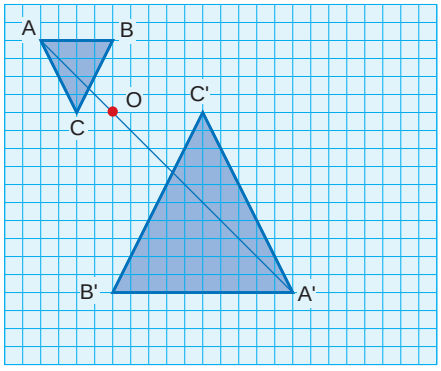


Exercise 34.10 page 420

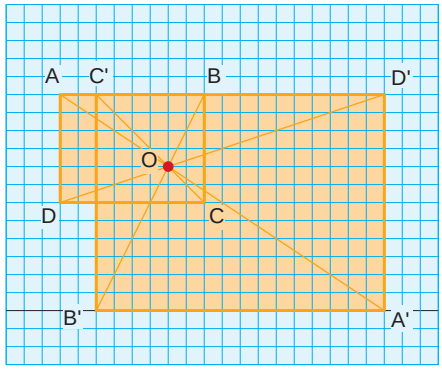
1.



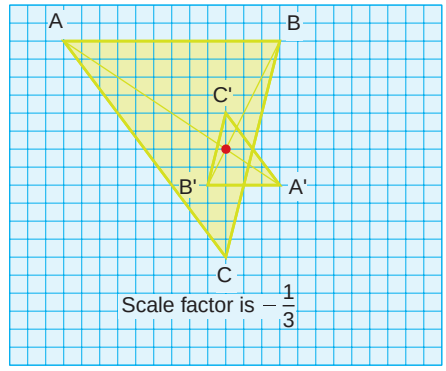
2.



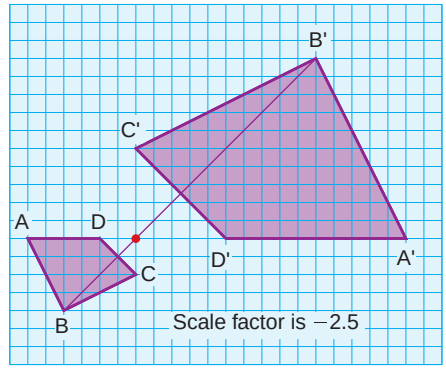
3.



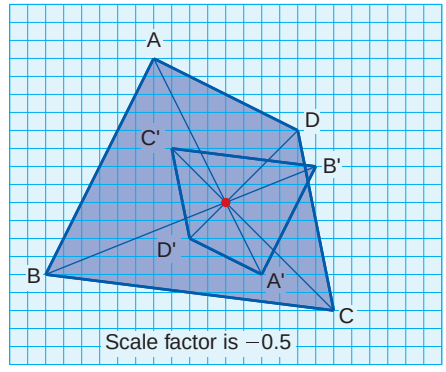
4.



5.

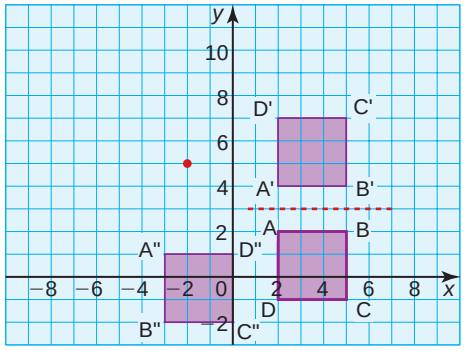


6.

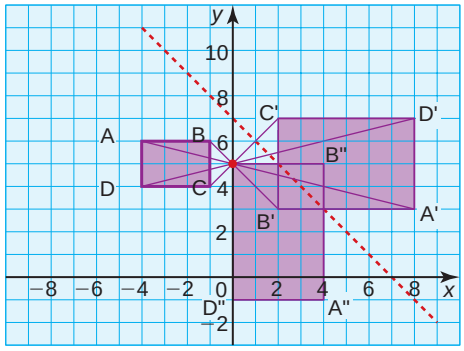


Exercise 34.11 page 422

1.

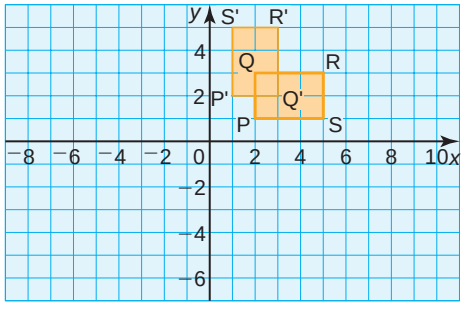


2.

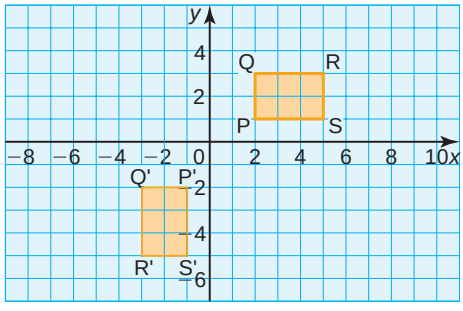


Exercise 34.12 page 424

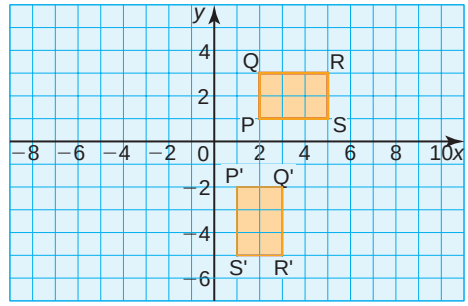
1.



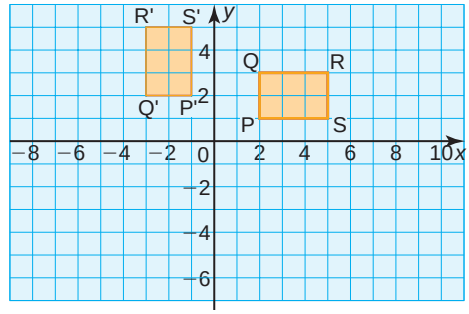
2.



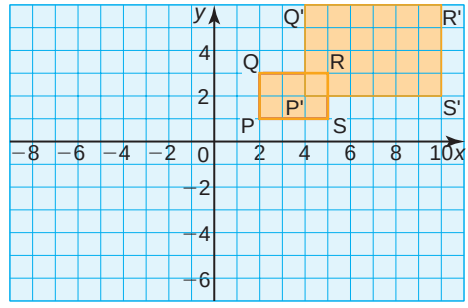
3.



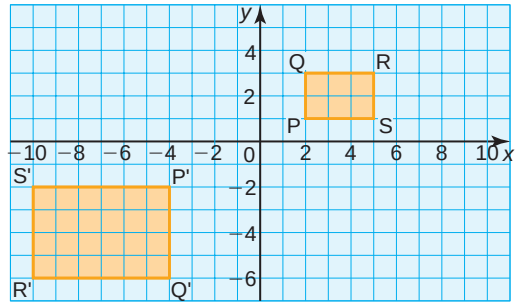
4.



5.

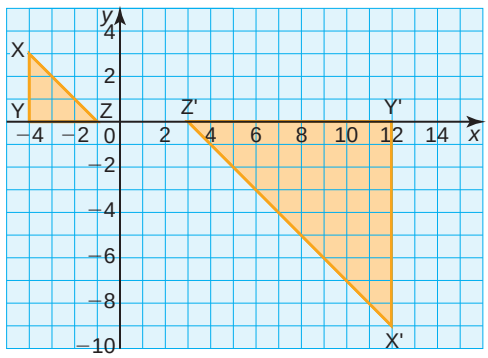


6.



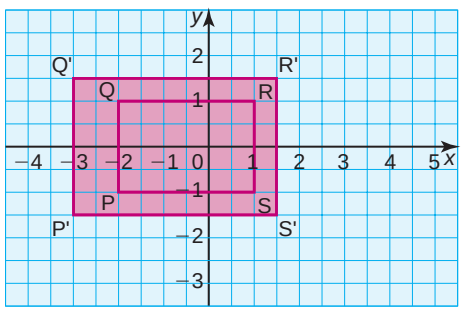
7. Q.1 represents a reflection in the line $y = x$.
Q.2 represents a reflection in the line $y = -x$.
Q.3 represents a clockwise rotation of 90° about the origin.
Q.4 represents an anti-clockwise rotation of 90° about the origin.
Q.5 represents an enlargement of scale factor 2 with its centre at the origin.
Q.6 represents an enlargement of scale factor –2 with its centre at the origin.

8. a)



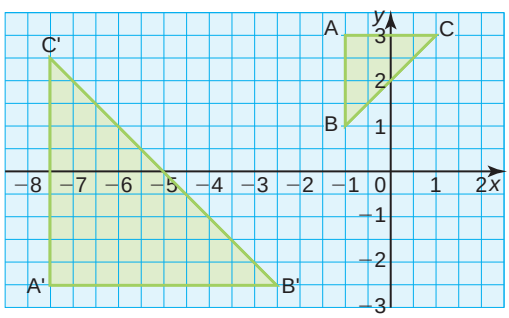
b) 4.5 units^2 **c)** 40.5 units^2 **d)** 9 **e)** 9

9. a)



b) 6 units^2 **c)** 13.5 units^2 **d)** 2.25 **e)** 2.25

10. a)



b) 2 units^2 **c)** 12.5 units^2 **d)** 6.25 **e)** 6.25

Exercise 34.13 page 426

1. a) Student's own diagram
b) Student's diagram should show a reflection in $y = -x$ followed by enlargement from the origin ×2
c) $\begin{pmatrix} 0 & -0.5 \\ -0.5 & 0 \end{pmatrix}$

2. a) Student's own diagram
b) Student's diagram should show a rotation 180° about the origin
c) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

3. a) Student's own diagram
b) Student's diagram should show a rotation 90° clockwise about the origin followed by enlargement from the origin ×1.5
c) $\begin{pmatrix} 0 & -\frac{2}{3} \\ \frac{2}{3} & 0 \end{pmatrix}$

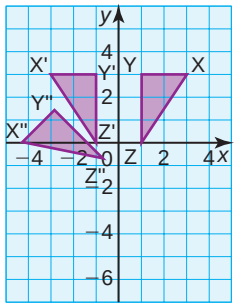
4. a) Student's own diagram
b) Student's diagram should show an enlargment from the origin ×0.5 followed by reflection in the y-axis
c) $\begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$

5. a) Student's own diagram
b) Student's diagram should show an enlargment from the origin ×2.5 followed by reflection in the x-axis
c) $\begin{pmatrix} \frac{2}{5} & 0 \\ 0 & -\frac{2}{5} \end{pmatrix}$

6. a) Student's own diagram
b) Student's diagram should show a rotation about the origin 60° anti-clockwise followed by an enlargement from origin ×(−2)
c) $\begin{pmatrix} -\frac{1}{4} & -\frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & -\frac{1}{4} \end{pmatrix}$

Exercise 34.14 page 429

1. a) c)

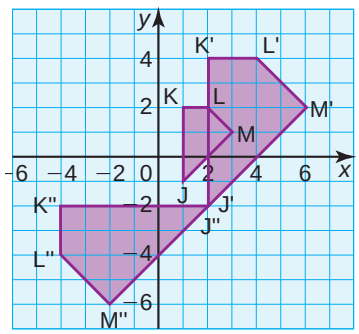


b) Reflection in the y-axis

d) Rotation 45° anti-clockwise about the origin

e) $\begin{pmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$ **f)** $\begin{pmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$

2. a) c)



b) Enlargement scale factor 2, with centre at the origin

d) Reflection in the line $y = -x$

e) $\begin{pmatrix} 0 & -2 \\ -2 & 0 \end{pmatrix}$ **f)** $\begin{pmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix}$

3. a) $\begin{pmatrix} 0 & -\frac{3}{2} \\ \frac{3}{2} & 0 \end{pmatrix}$ **b)** $\begin{pmatrix} 0 & \frac{2}{3} \\ -\frac{2}{3} & 0 \end{pmatrix}$

4. a) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ **b)** $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

c) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

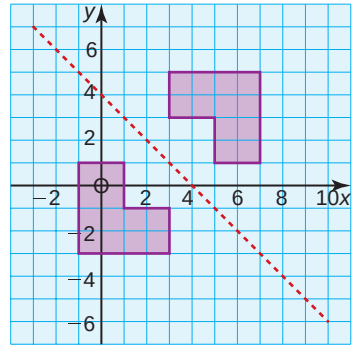
d) Rotation 180° about the origin

e) Reflection in the line $y = -x$

f) Reflection in the line $y = x$

Student assessment 1 page 431

1. a)



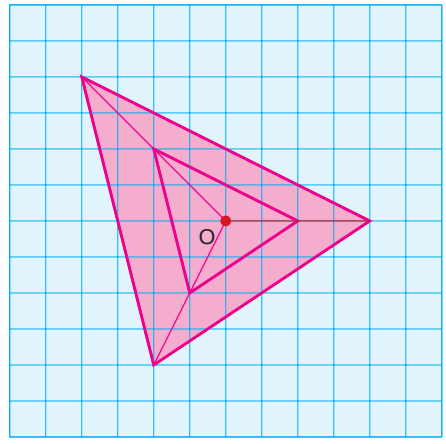
$y = x - 2$ is also a mirror line

b) $y = -x + 4$

2. a) (3, 2) **b)** 90° clockwise

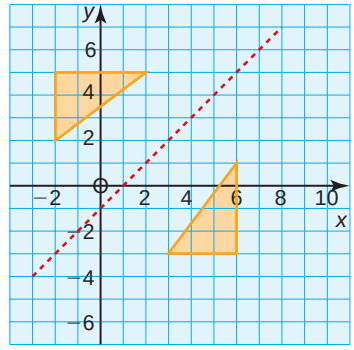
3. a) $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$ **b)** $\begin{pmatrix} -3 \\ -5 \end{pmatrix}$

4.



Student assessment 2 page 432

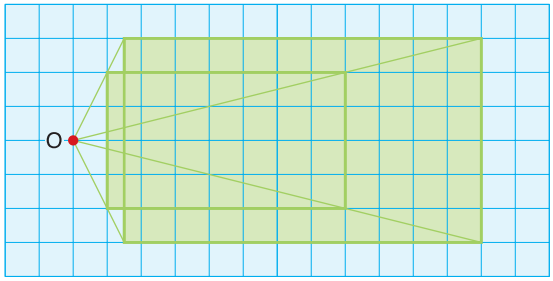
1.



2. **a)** Student's own construction
b) 180° clockwise/anti-clockwise

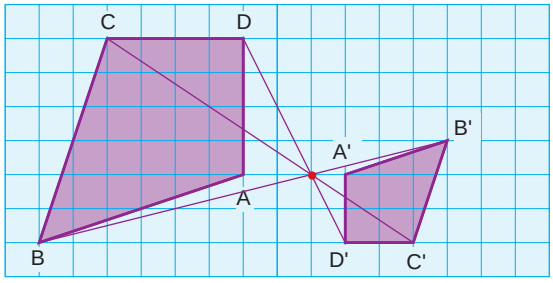
3. **a)** $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ **b)** $\begin{pmatrix} -8 \\ 2 \end{pmatrix}$

4.



Student assessment 3 page 433

1. **a)**

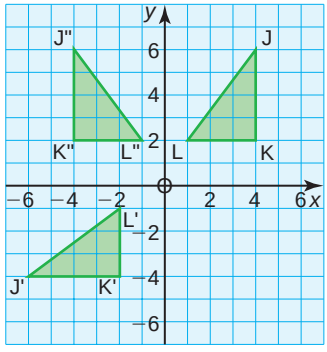


b) The scale factor of enlargement is –0.5.

2. **a)** An enlargement of scale factor 2. Centre of enlargement (3, 3)
b) A reflection about the line $y = -x - 1$

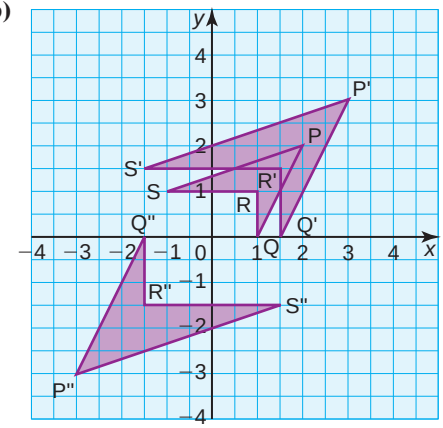
3. **a)** A reflection in the line $x = 0$
b) An enlargement by scale factor –0.5. Centre of enlargement (0, –1)

4. **a) b)**



c) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

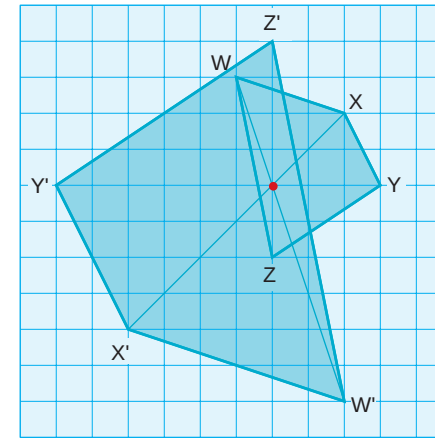
5. **a) b)**



c) $\begin{pmatrix} 1.5 & 0 \\ 0 & 1.5 \end{pmatrix}$ **d)** $\begin{pmatrix} -\frac{2}{3} & 0 \\ 0 & -\frac{2}{3} \end{pmatrix}$

Student assessment 4 page 435

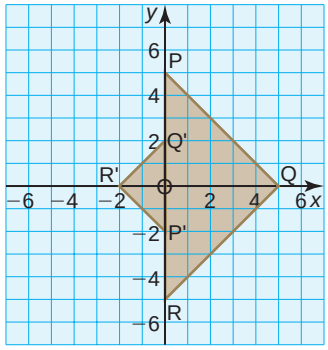
1. **a)**



b) The scale factor of enlargement is –2.

2. **a)** A translation of vector $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$
b) An enlargement of scale factor 2. Centre of enlargement (6, 8)

3. **a)**



b) $\frac{4}{25} = 0.16$ **c)** $\frac{4}{25}$ or 0.16

Topic 7 Mathematical investigations and ICT

A painted cube page 436

1. **a)** 8 **b)** 12 **c)** 6 **d)** 1

2. A: 8, B: 24, C: 24, D: 8

3. A: 8, B: 96, C: 384, D: 512

4. When $(n-2) \geq 0$, A: 8, B: $12(n-2)$, C: $6(n-2)^2$, D: $(n-2)^3$

5. When $(l-2) \geq 0$, $(w-2) \geq 0$ and $(h-2) \geq 0$,

A: 8,
B: $4(l-2) + 4(w-2) + 4(h-2)$,
C: $2(l-2)(w-2) + 2(l-2)(h-2) + 2(w-2)(h-2)$,
D: $(l-2)(w-2)(h-2)$

Triangle count page 437

1. 9

2. Student's investigation and ordered table

Number of horizontal lines	Number of triangles
0	3
1	6
2	9
3	12
…	…

3. $n = 3(h+1)$

4. 12

5. Student's investigation and ordered table

Number of horizontal lines	Number of triangles
0	6
1	12
2	18
3	24
…	…

6. $n = 6(h+1)$

ICT activity 1 page 438

Student's help sheet

ICT activity 2 page 439

3. Reflection in $y = x$

4. **a)** Reflection in $y = -x$
b) Clockwise rotation about the origin of 90°
c) Anti-clockwise rotation about the origin of 90°
d) Enlargement of scale factor 2, centred at the origin.
e) Englargement of scale factor – 2, centred at the origin

35 Probability

Exercise 35.1 page 443

1. Student's own drawing

2. Student's own answers

Exercise 35.2 page 444

1. **a)** $\frac{1}{6}$ **b)** $\frac{5}{6}$ **c)** $\frac{1}{2}$ **d)** $\frac{5}{6}$ **e)** 0 **f)** 1

2. **a)** i) $\frac{1}{7}$ ii) $\frac{6}{7}$ **b)** Total = 1

3. **a)** $\frac{1}{250}$ **b)** $\frac{1}{2}$ **c)** $\frac{151}{250}$ **d)** 1

4. **a)** $\frac{5}{8}$ **b)** $\frac{3}{8}$

5. **a)** $\frac{1}{13}$ **b)** $\frac{5}{26}$ **c)** $\frac{21}{26}$ **d)** $\frac{3}{26}$

6. $\frac{1}{6}$

7. **a)** i) $\frac{1}{10}$ ii) $\frac{1}{4}$ **b)** i) $\frac{1}{19}$ ii) $\frac{3}{19}$

8. **a)** $\frac{1}{37}$ **b)** $\frac{36}{37}$ **c)** $\frac{18}{37}$ **d)** $\frac{1}{37}$
e) $\frac{21}{37}$ **f)** $\frac{12}{37}$ **g)** $\frac{17}{37}$ **h)** $\frac{11}{37}$

9. **a)** RCA RAC CRA CAR ARC ACR
b) $\frac{1}{6}$ **c)** $\frac{1}{3}$ **d)** $\frac{1}{2}$ **e)** $\frac{1}{24}$

10. **a)** $\frac{1}{4}$ **b)** $\frac{3}{4}$ **c)** $\frac{1}{13}$ **d)** $\frac{1}{26}$
e) $\frac{3}{13}$ **f)** $\frac{1}{52}$ **g)** $\frac{5}{13}$ **h)** $\frac{4}{13}$

Exercise 35.3 page 445

1. **a)** 140
b) i) $\frac{1}{5}$ ii) $\frac{3}{70}$ iii) $\frac{6}{7}$

2. **a)** 32
b) i) $\frac{9}{16}$ ii) $\frac{5}{32}$ iii) $\frac{3}{8}$ iv) $\frac{5}{8}$

3. **a)** 70
b) i) $\frac{11}{35}$ ii) $\frac{2}{7}$ iii) $\frac{5}{7}$ iv) $\frac{33}{35}$

Exercise 35.4 page 448

1. 2

2. 25

3. **a)** $\frac{1}{16}$ **b)** $\frac{7}{16}$ **c)** $\frac{1}{2}$ **d)** $\frac{15}{16}$ **e)** $\frac{11}{16}$

4. **a)** $\frac{14}{45}$ **b)** $\frac{7}{45}$ **c)** $\frac{13}{30}$ **d)** $\frac{1}{15}$ **e)** $\frac{14}{15}$

5. 35 blue, 28 red, 21 yellow, 49 green, 7 white

6. 300

7. 14

8. 200

9. 2

Student assessment 1 page 449

1. **a)** $\frac{1}{6}$ **b)** $\frac{5}{6}$ **c)** $\frac{2}{3}$ **d)** 0

2. **a)** $\frac{1}{4}$ **b)** $\frac{1}{13}$ **c)** $\frac{1}{2}$ **d)** $\frac{3}{13}$
e) $\frac{4}{13}$

3. **a)** $\frac{1}{250}$ **b)** $\frac{1}{50}$ **c)** $\frac{2}{25}$ **d)** $\frac{3}{10}$
e) 1

4. **a)** $\frac{11}{40}$ **b)** $\frac{1}{8}$ **c)** $\frac{1}{4}$ **d)** $\frac{4}{5}$

5. 160 red, 96 blue, 64 green

6. **a)** $\frac{41}{150}$ **b)** $\frac{29}{150}$ **c)** $\frac{1}{25}$ **d)** $\frac{4}{15}$
e) $\frac{11}{15}$ **f)** $\frac{137}{150}$ **g)** $\frac{1}{30}$

7. **a)** The girl's results are likely to be more reliable as she repeated the experiment more times.
b) It is likely that the dice is biased towards the number 3.

Student assessment 2 page 450

1. **a)** $\frac{1}{8}$ **b)** $\frac{7}{8}$ **c)** $\frac{5}{8}$ **d)** 0

2. **a)** 32 cards in pack
b) i) $\frac{1}{8}$ ii) $\frac{3}{8}$ iii) $\frac{1}{4}$ iv) $\frac{1}{8}$ v) $\frac{11}{32}$

3. **a)** $\frac{1}{180}$ **b)** $\frac{1}{20}$ **c)** $\frac{1}{12}$ **d)** $\frac{2}{9}$ **e)** 1

4. **a)** $\frac{9}{32}$ **b)** $\frac{3}{8}$ **c)** $\frac{21}{32}$

5. 128 red, 80 blue, 112 green

6. **a)** $\frac{3}{20}$ **b)** $\frac{19}{20}$ **c)** $\frac{7}{10}$

7. **a)** 50 000
b) You have to assume that all entrants have an equal chance of winning.

36 Further probability

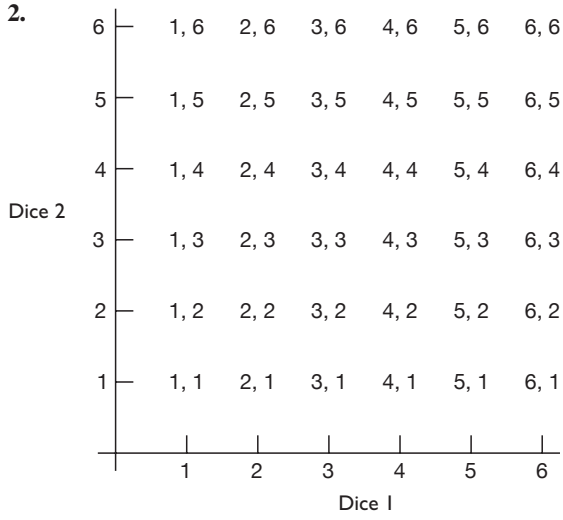
Exercise 36.1 page 452

1. **a)**

		Dice 1			
		1	2	3	4
Dice 2	1	1, 1	2, 1	3, 1	4, 1
	2	1, 2	2, 2	3, 2	4, 2
	3	1, 3	2, 3	3, 3	4, 3
	4	1, 4	2, 4	3, 4	4, 4

b) $\frac{1}{4}$ **c)** $\frac{1}{4}$ **d)** $\frac{9}{16}$

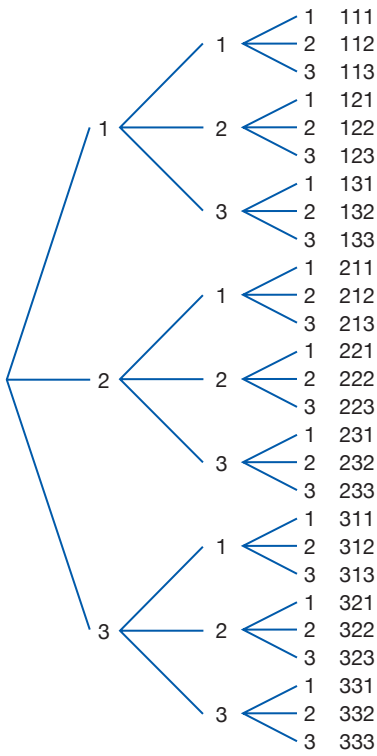
2.



a) $\frac{1}{36}$ **b)** $\frac{1}{6}$ **c)** $\frac{1}{18}$ **d)** $\frac{1}{6}$ **e)** $\frac{1}{4}$ **f)** $\frac{3}{4}$
g) $\frac{5}{18}$ **h)** $\frac{1}{6}$ **i)** $\frac{11}{18}$

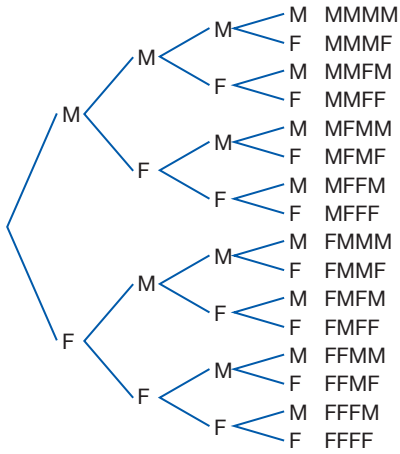
Exercise 36.2 page 454

1. a)



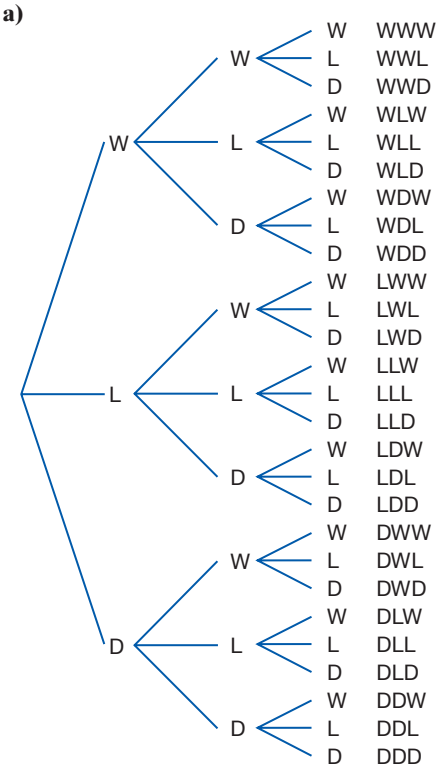
b) i) $\frac{1}{27}$ ii) $\frac{1}{3}$ iii) $\frac{1}{9}$ iv) $\frac{1}{3}$ v) $\frac{5}{9}$

2. a)



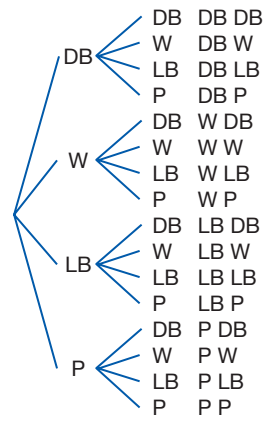
b) i) $\frac{1}{16}$ ii) $\frac{3}{8}$ iii) $\frac{15}{16}$ iv) $\frac{5}{16}$

3. a)



b) i) $\frac{1}{27}$ ii) $\frac{10}{27}$ iii) $\frac{19}{27}$ iv) $\frac{8}{27}$

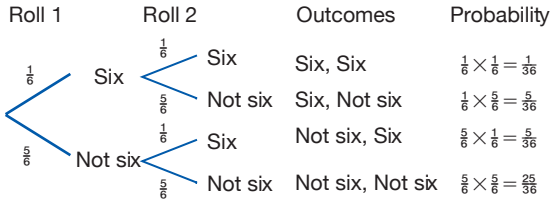
4. a)



b) i) $\frac{1}{16}$ ii) $\frac{1}{4}$ iii) $\frac{1}{8}$

Exercise 36.3 page 456

1. a)

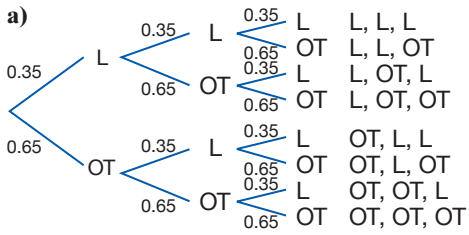


b) i) $\frac{1}{6}$ ii) $\frac{11}{36}$ iii) $\frac{5}{36}$ iv) $\frac{125}{216}$ v) $\frac{91}{216}$

c) Add up to 1

2. a) $\frac{4}{25}$ **b)** $\frac{54}{125}$ **c)** $\frac{98}{125}$

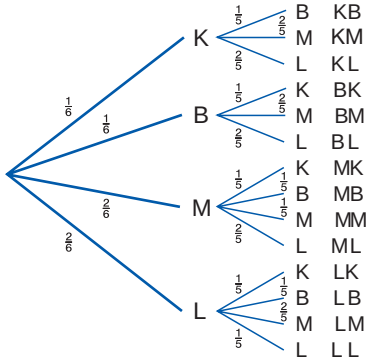
3. a)



b) i) 0.275 (3 s.f.) ii) 0.123 (3 s.f.)
iii) 0.444 (3 s.f.) iv) 0.718 (3 s.f.)

4. a) 0.0588 (3 s.f.) **b)** 0.0129 (3 s.f.)
c) 0.414 (3 s.f.) **d)** 0.586 (3 s.f.)

5. a)



b) i) $\frac{1}{15}$ ii) $\frac{1}{15}$ iii) $\frac{3}{5}$

6. 0.027

7. a) 0.75^2 (0.563)
b) 0.75^3 (0.422)
c) 0.75^{10} (0.056)

Student assessment 1 page 457

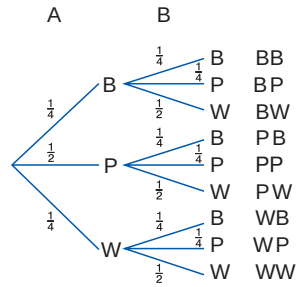
1. a) i) $\frac{3}{5}$ ii) $\frac{1}{19}$
b) Infinite

2. a)

	1	2	3	4	5	6
H	1H	2H	3H	4H	5H	6H
T	1T	2T	3T	4T	5T	6T

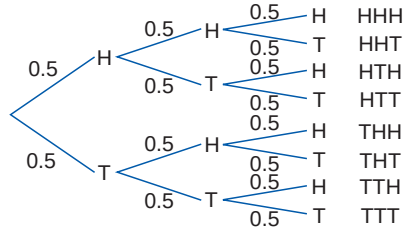
b) i) $\frac{1}{12}$ ii) $\frac{1}{4}$ iii) $\frac{1}{4}$

3. a)



b) i) $\frac{1}{16}$ ii) $\frac{1}{8}$ iii) $\frac{1}{4}$

4. a)



b) i) $\frac{1}{8}$ ii) $\frac{3}{8}$ iii) $\frac{1}{8}$ iv) $\frac{7}{8}$

5. a) $\frac{4}{9}$ **b)** $\frac{26}{27}$ **c)** $\frac{2}{9}$

6. a) $\frac{9}{16}$ **b)** $\frac{1}{16}$ **c)** $\frac{1}{4}$

7. 0.009 95 (3 s.f.)

Student assessment 2 page 458

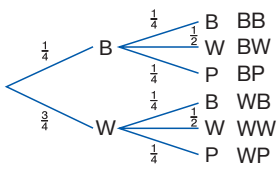
1. **a)**

Dice 2 \ Dice 1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b) i) $\frac{1}{36}$ ii) $\frac{1}{6}$ iii) $\frac{1}{12}$ iv) $\frac{7}{12}$
c) 25

2. **a)** i) $\frac{1}{4}$ ii) $\frac{3}{8}$
b) i) $\frac{3}{64}$ ii) $\frac{3}{32}$ iii) $\frac{7}{16}$ iv) $\frac{39}{64}$

3. **a)**

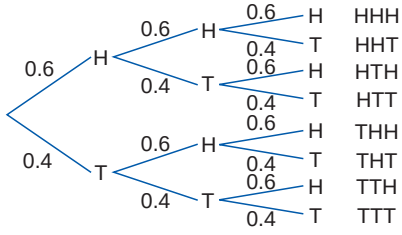


b) i) $\frac{1}{16}$ ii) $\frac{3}{8}$ iii) $\frac{3}{16}$ iv) $\frac{7}{8}$

4. **a)** $\frac{1}{12}$ **b)** $\frac{1}{6}$ **c)** $\frac{1}{6}$

5. **a)** 0.72 **b)** 0.729

6. **a)**



b) i) 0.216 ii) 0.064 iii) 0.648

7. **a)** i) 0.8 ii) 0.7 iii) 0.3
b) i) 0.06 ii) 0.56 iii) 0.38

Topic 8 Mathematical investigations and ICT

Probability drop page 460

1. Tray 1: LLL
Tray 2: LLR LRL RLL
Tray 3: LRR RLR RRL
Tray 4: RRR

2. Tray 1: LLLL
Tray 2: LLLR LLRL LRLL RLLL
Tray 3: LLRR LRLR LRRL RLLR
RLRL RRLL
Tray 4: RRRL RRLR RLRR LRRR
Tray 5: RRRR

3. $\frac{1}{16}$ because there are 16 possible routes and only one results in the marble landing in Tray 1.

4. Tray 2: $\frac{4}{16} = \frac{1}{4}$
Tray 3: $\frac{6}{16} = \frac{3}{8}$
Tray 4: $\frac{4}{16} = \frac{1}{4}$
Tray 5: $\frac{1}{16}$

5. Student's investigation

6. $\frac{210}{1024} = \frac{105}{512}$

7. Each number in each row of Pascal's triangle corresponds to the number of routes to landing in each tray of the game.

8. The binomial expansion generates the numbers in Pascal's triangle and therefore the number of routes to landing in each tray of the game.

Dice sum page 461

1.

Dice 2 \ Dice 1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

2. 36

3. 7

4. $\frac{3}{36} = \frac{1}{12}$

5. $\frac{3}{36} = \frac{1}{6}$

6. You are four times more likely to get a 5 than a 2.

7.

		Dice 1			
		1	2	3	4
Dice 2	1	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

8. 16

9. 5

10. $\frac{4}{16} = \frac{1}{4}$

11. Student's investigation

12. **a)** m^2 **b)** $m + 1$ **c)** $\frac{m}{m^2} = \frac{1}{m}$

13. **a)** $m \times n$

b) The total can take any integer value in the range $n + 1 \rightarrow m + 1$.

c) $\frac{n}{nm} = \frac{1}{m}$

ICT activity: Buffon's needle experiment page 462

1.–9. Student's experiment and results entered in a spreadsheet

10. The value of $\frac{2}{p}$ should tend to π.

37 Mean, median, mode and range

Exercise 37.1 page 467

1. Mean = 1.67 (3 s.f.) Median = 1
Mode = 1 Range = 5

2. Mean = 6.2 Median = 6.5
Mode = 7 Range = 9

3. Mean = 14 yrs 3 mths
Median = 14 yrs 3 mths
Mode = 14 yrs 3 mths
Range = 8 mths

4. Mean = 26.4 Median = 27
Mode = 28 Range = 5

5. Mean = 13.9 s (3 s.f.)
Median = 13.9 s
Mode = 13.8 s
Range = 0.6 s

6. 91.1 kg

7. 103 points

Exercise 37.2 page 468

1. Mean = 3.35 Median = 3
Mode = 1 and 4 Range = 5

2. Mean = 7.03

3. **a)** Mean = 6.33 (3 s.f.)
Median = 7
Mode = 8
Range = 5

b) The mode, as it gives the highest number of flowers per bush

Exercise 37.3 page 469

1. **a)**

Height (m)	Frequency	Mid-interval value	Frequency × mid-interval value
1.8–	2	**1.85**	**3.7**
1.9–	5	**1.95**	**9.75**
2.0–	10	**2.05**	**20.5**
2.1–	22	**2.15**	**47.3**
2.2–	7	**2.25**	**15.75**
2.3–2.4	4	**2.35**	**9.4**

b) Mean = 2.13 m (3 s.f.)
c) Modal class = 2.1–2.2 m

2. **a)** Mean = 33 h (2 s.f.)
b) Modal class = 30–39 h

3. **a)** Mean = 6.2 cm
b) Modal class = 6.0–6.5 cm

Student assessment 1 page 470

1. **a)** Mean = 16
b) Median = 16.5
c) Mode = 18
d) Range = 39

2. **a)** 28
b) i) Mean 7.75
ii) Median = 8
iii) Mode = 8
iv) Range = 5

3. **a)** Mean = 19
b) Modal class is $19 \leqslant M < 20$

Student assessment 2 page 471

1. Mean = 86.8 m Median = 90.5 m
Mode = 93 m Range = 18 m

2. **a)** 26
b) i) Mean = 7.73 (3 s.f.)
ii) Median = 7.5
iii) Mode = 10
iv) Range = 6

3. **a)** Mean $\approx$ 10.0 kg (3 s.f.)
b) Modal class is $10.0 \leqslant M < 10.1$ kg

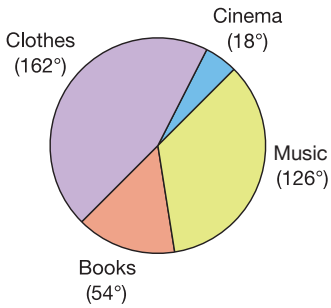
38 Collecting and displaying data

Exercise 38.1 page 476

1.

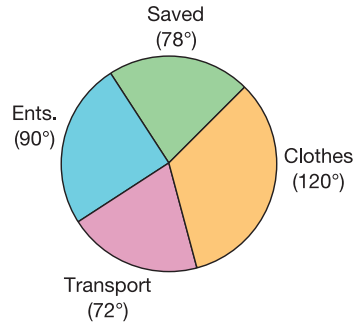
	Sleep	Meals	Sport	TV	School
Ayse	8 h 20	2 h	5 h	2 h	6 h 40
Ahmet	8 h 40	2 h	5 h 20	2 h	6 h

2.

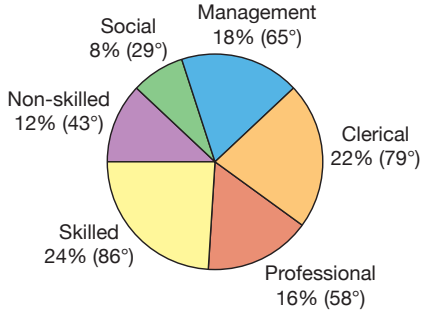


3.

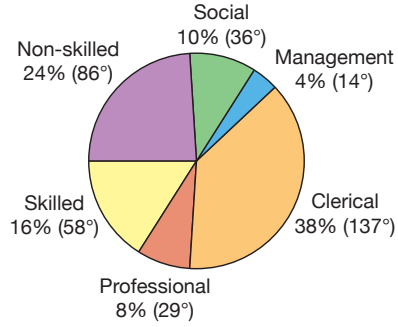
	Fraction	$	Degrees
Clothes	$\frac{1}{3}$	800	120
Transport	$\frac{1}{5}$	480	72
Ents.	$\frac{1}{4}$	600	90
Saved	$\frac{13}{60}$	520	78



4. Male



Female



a) Student's own two statements
b) Professional = 8 million × 8% = 640 000
Management = 8 million × 4% = 320 000

Exercise 38.2 page 477

1. Student's own survey results and pie charts

2. Student's report

Exercise 38.3 page 481

1. Student's answers may differ from those given below.

a) Possible positive correlation (strength depending on topics tested)

b) No correlation

c) Positive correlation (likely to be quite strong)

d) Negative correlation (likely to be strong). Assume that motorcycles are not rare/vintage.

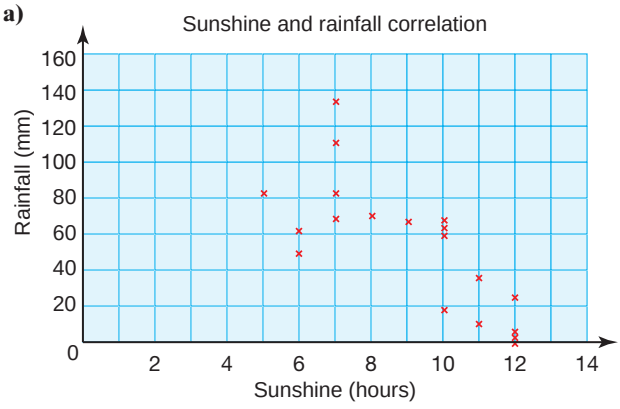
e) Factors such as social class, religion and income are likely to affect results. Therefore little correlation is likely.

f) Negative correlation (likely to be strong)

g) 0–16 years likely to be a positive correlation

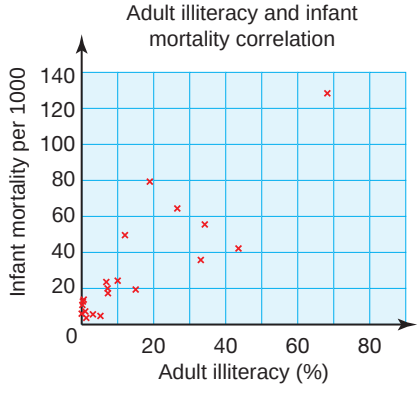
h) Strong positive correlation

2. **a)**



b) Graph shows a very weak negative correlation.

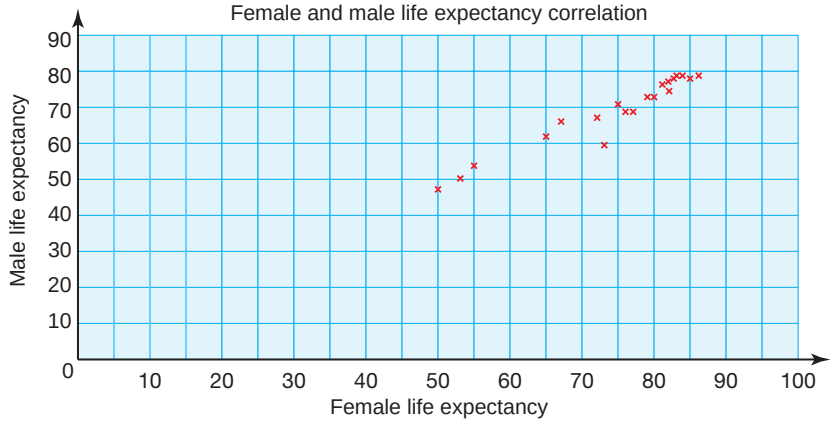
3. **a)**



b) Positive correlation

c) Student's answer. However, although there is a correlation, it doesn't imply that one variable affects the other

d)

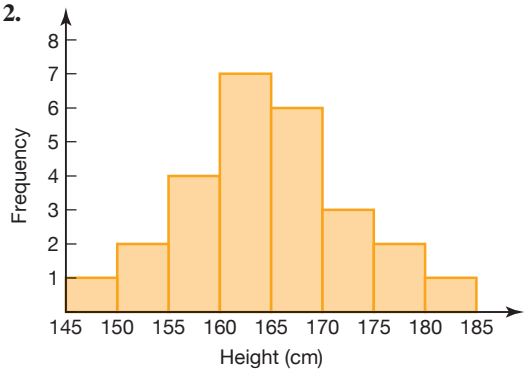


4. **a)** Moderate/strong positive correlation
b) Approx. 31 tomatoes
c) Approx. 60 cm

Exercise 38.4 page 485

1.

2.

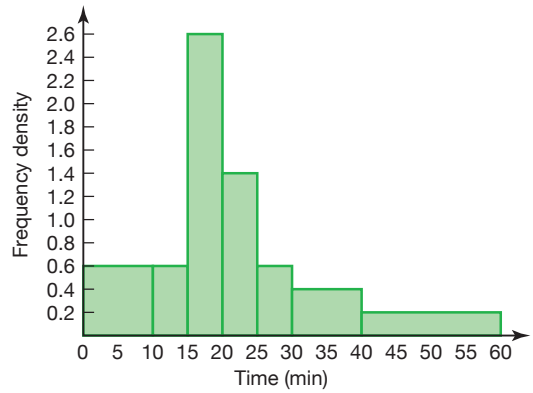


Exercise 38.5 page 487

1. **a)**

Time (min)	0–	10–	15–	20–	25–	30–	40–60
Freq.	6	3	13	7	3	4	4
Freq. density	**0.6**	**0.6**	**2.6**	**1.4**	**0.6**	**0.4**	**0.2**

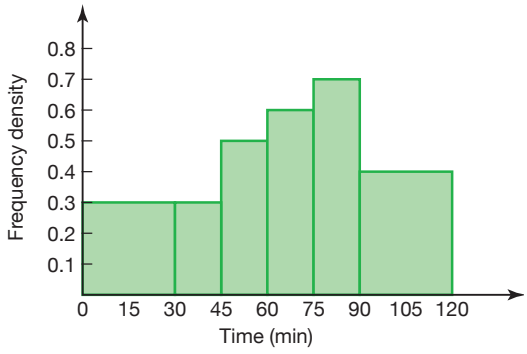
b)



2. **a)**

Time (min)	Frequency	Frequency density
$0 \leqslant t < 30$	8	**0.3**
$30 \leqslant t < 45$	5	**0.3**
$45 \leqslant t < 60$	8	**0.5**
$60 \leqslant t < 75$	9	**0.6**
$75 \leqslant t < 90$	10	**0.7**
$90 \leqslant t < 120$	12	**0.4**

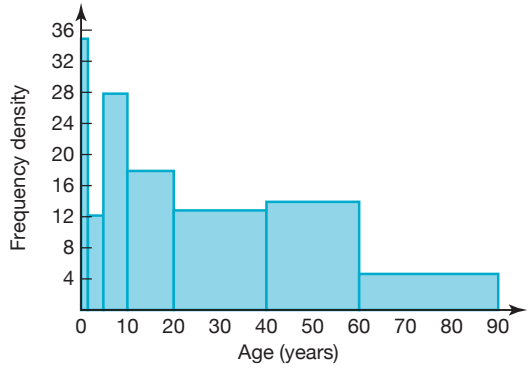
b)



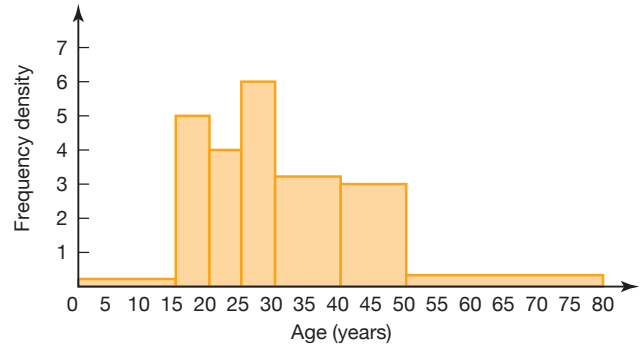
3. **a)**

Age (years)	0–	1–	5–	10–	20–	40–	60–90
Freq.	35	**48**	**140**	180	260	**280**	150
Freq. density	**35**	12	28	**18**	**13**	14	**5**

b)



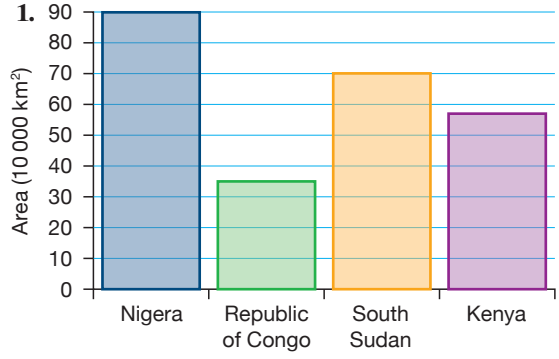
4. **a)**



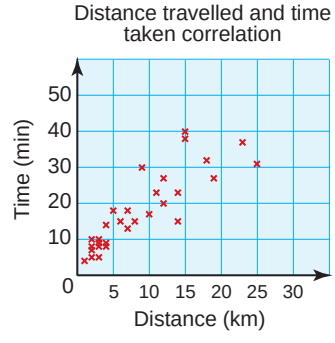
b) Student's own answers

Student assessment 1 page 489

1.



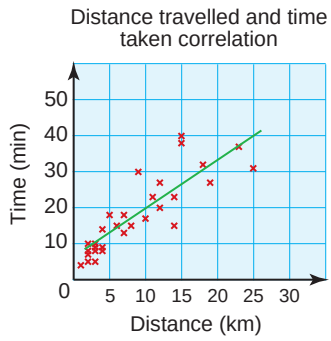
2. **a)**



b) (Strong) positive correlation

c) It depends on their mode of transport.

d)

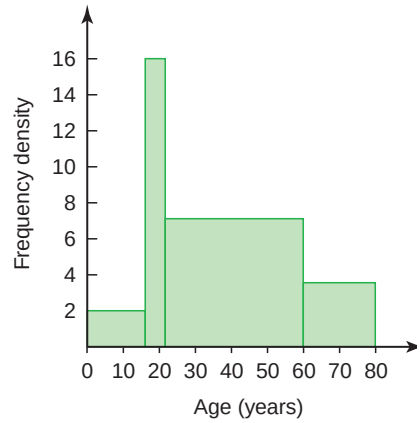


e) Approx. 9.5 km

3. a)

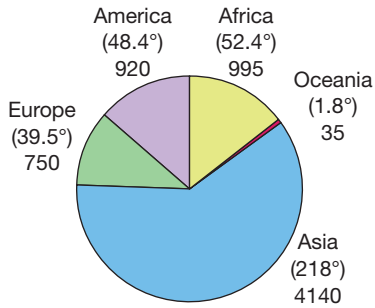
	Age	Number	Frequency density
Juniors	0–	32	**2**
Intermediates	16–	80	**16**
Full members	21–	273	**7**
Seniors	60–80	70	**3.5**

b)

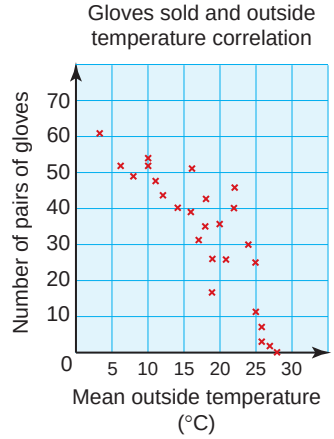


Student assessment 2 page 490

1.



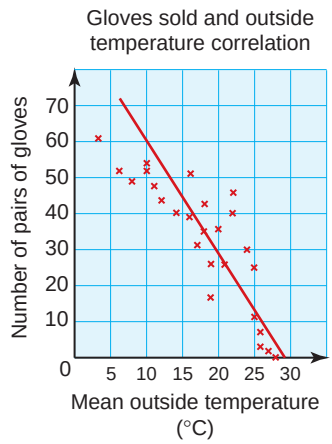
2. a)



b) Negative correlation

c) Student's own answer

d)

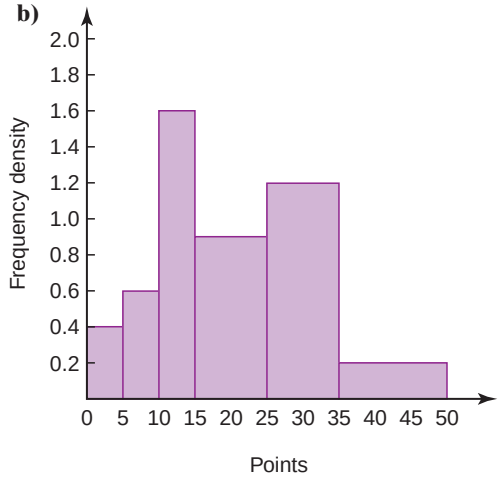


Approx. 30 pairs

3. a)

Points	0–	5–	10–	15–	25–	35–50
Number of games	2	3	8	9	12	3
Freq. density	**0.4**	**0.6**	**1.6**	**0.9**	**1.2**	**0.2**

b)



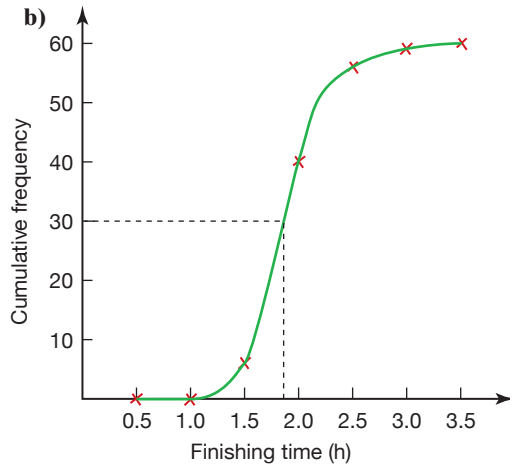
39 Cumulative frequency

Exercise 39.1 page 492

1. a)

Finishing time (h)	0–	0.5–	1.0–	1.5–	2.0–	2.5–	3.0–3.5
Freq.	0	0	6	34	16	3	1
Cum. freq.	**0**	**0**	**6**	**40**	**56**	**59**	**60**

b)



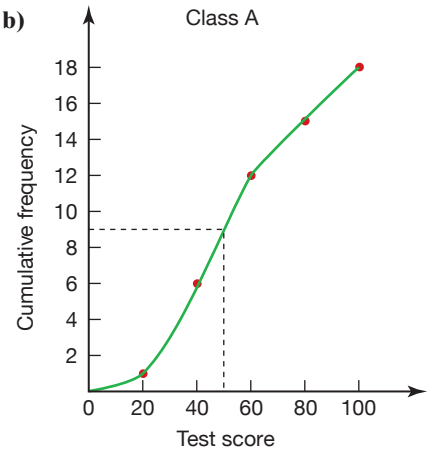
c) Median ≈ 1.8 h

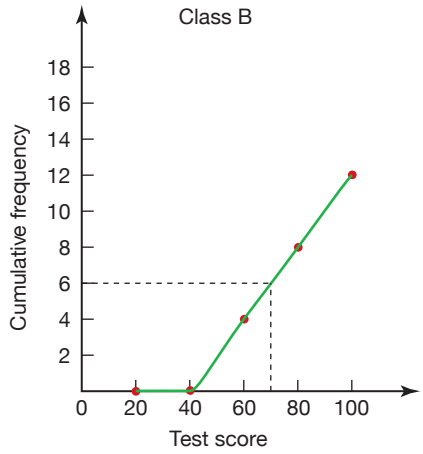
d) As many runners finished before as after the median.

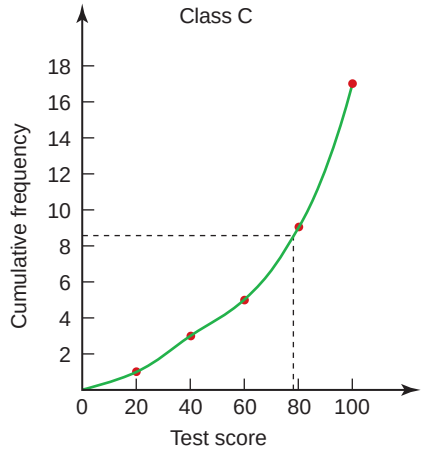
2. a)

Score	Class A		Class B		Class C	
	Freq.	Cum. freq.	Freq.	Cum. freq.	Freq.	Cum. freq.
$0 \leqslant x < 20$	1	1	0	0	1	1
$20 \leqslant x < 40$	5	6	0	0	2	3
$40 \leqslant x < 60$	6	12	4	4	2	5
$60 \leqslant x < 80$	3	15	4	8	4	9
$80 \leqslant x < 100$	3	18	4	12	8	17

b)







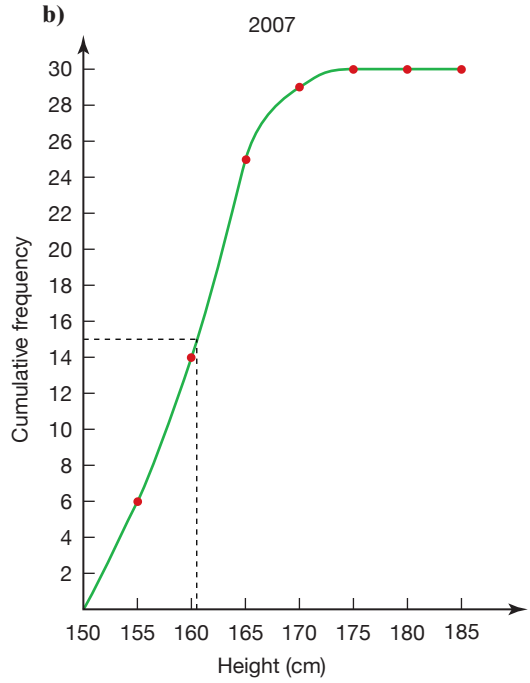
c) Class A median ≈ 50
Class B median ≈ 70
Class C median ≈ 78

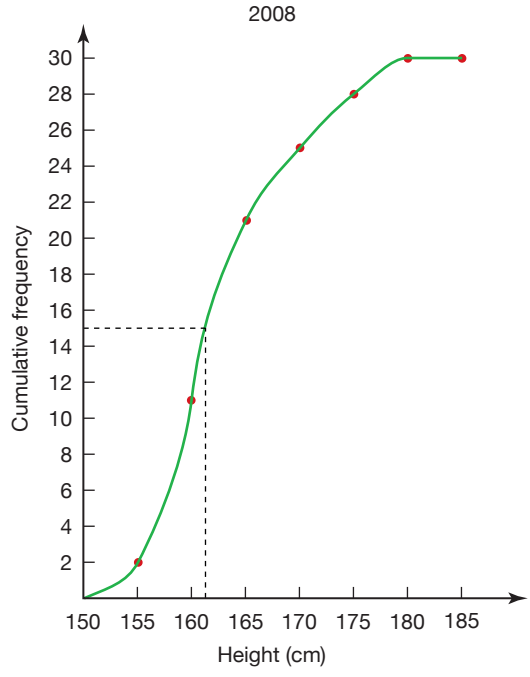
d) As many students were above as below the median.

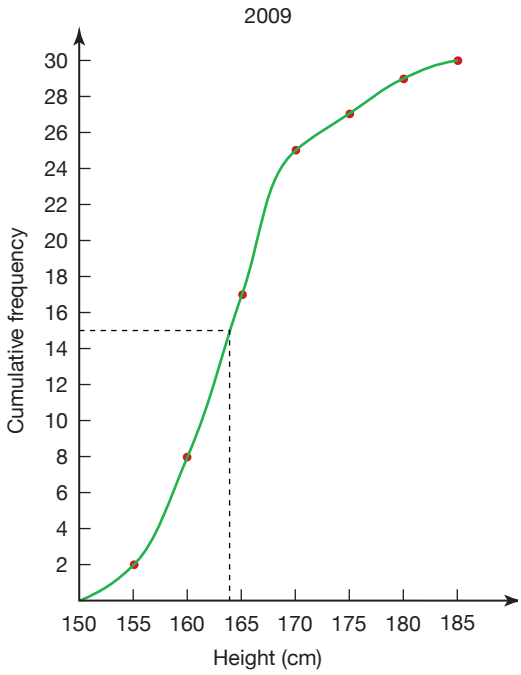
3. a)

Height (cm)	2007		2008		2009	
	Freq.	Cum. freq.	Freq.	Cum. freq.	Freq.	Cum. freq.
150–	6	6	2	2	2	2
155–	8	14	9	11	6	8
160–	11	25	10	21	9	17
165–	4	29	4	25	8	25
170–	1	30	3	28	2	27
175–	0	30	2	30	2	29
180–185	0	30	0	30	1	30

b)







c) Median (2007) ≈ 161 cm
Median (2008) ≈ 162 cm
Median (2009) ≈ 164 cm

d) As many students are taller than the median as shorter than the median.

Exercise 39.2 page 495

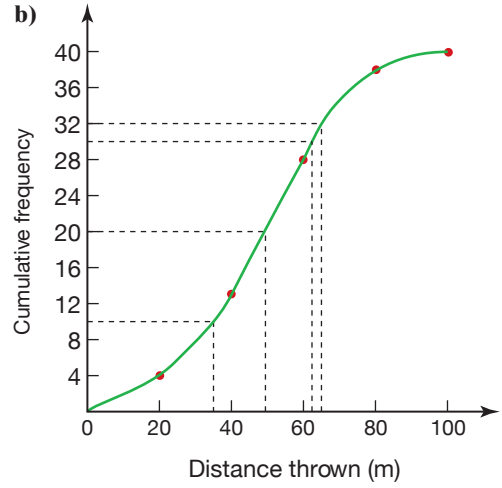
1. **a)** Class A ≈ 30 Class B ≈ 30 Class C ≈ 40
b) Student's own responses

2. **a)** 2007 ≈ 7 cm 2008 ≈ 8 cm 2009 ≈ 8 cm
b) Student's own responses

3. **a)**

Distance thrown (m)	0–	20–	40–	60–	80–100
Freq.	4	9	15	10	2
Cum. freq.	4	13	28	38	40

b)



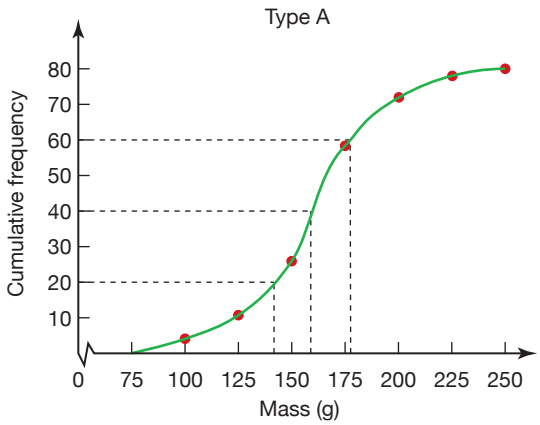
c) Qualifying distance ≈ 66 m
d) Inter-quartile range ≈ 28 m
e) Median ≈ 50 m

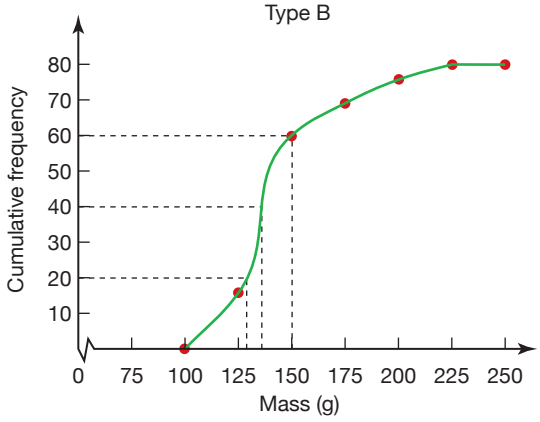
4. **a)**

Type A		
Mass (g)	**Frequency**	**Cum. freq.**
75–	4	4
100–	7	11
125–	15	26
150–	32	58
175–	14	72
200–	6	78
225–250	2	80

Type B		
Mass (g)	**Frequency**	**Cum. freq.**
75–	0	0
100–	16	16
125–	43	59
150–	10	69
175–	7	76
200–	4	80
225–250	0	80

b)





c) Median type A ≈ 157 g
Median type B ≈ 137 g

d) i) Lower quartile type A ≈ 140 g
Lower quartile type B ≈ 127 g
ii) Upper quartile type A ≈ 178 g
Upper quartile type B ≈ 150 g
iii) Inter-quartile type range type A ≈ 38 g
Inter-quartile type range type B ≈ 23 g

e) Student's own report

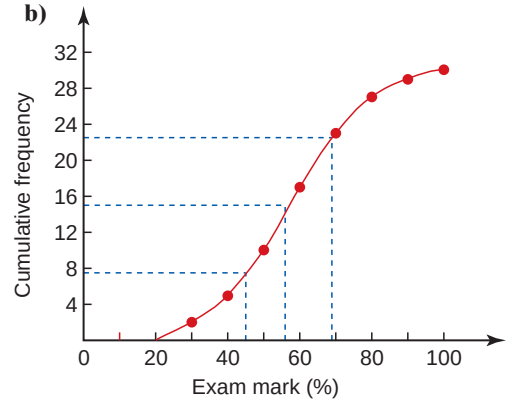
5. a) Student's own explanation
b) Student's own explanation

Student assessment 1 page 497

1. a)

Mark	20–	30–	40–	50–	60–	70–	80–	90–100
Freq.	2	3	5	7	6	4	2	1
Cum. freq.	**2**	**5**	**10**	**17**	**23**	**27**	**29**	**30**

b)

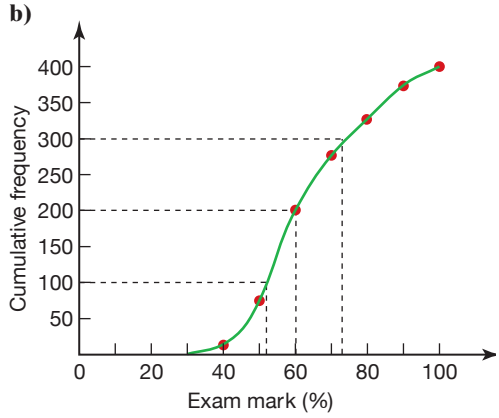


c) i) Median ≈ 57%
ii) Lower quartile ≈ 45%
Upper quartile ≈ 69%
iii) Inter-quartile range ≈ 24%

2. a)

Mark (%)	Frequency	Cumulative frequency
31–40	21	21
41–50	55	76
51–60	125	201
61–70	74	275
71–80	52	327
81–90	45	372
91–100	28	400

b)

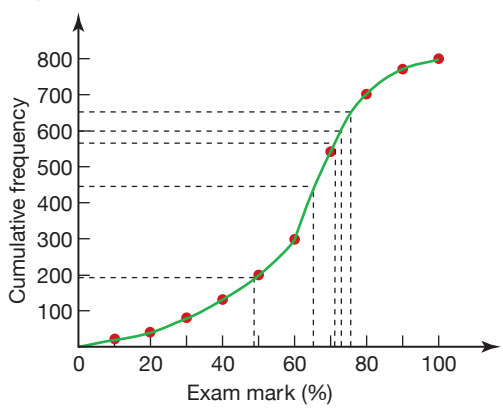


c) i) Median ≈ 60%
ii) Lower quartile ≈ 52%
Upper quartile ≈ 73%
iii) Inter-quartile range ≈ 21%

3. a)

Mark (%)	Frequency	Cumulative frequency
1–10	10	10
11–20	30	40
21–30	40	80
31–40	50	130
41–50	70	200
51–60	100	300
61–70	240	540
71–80	160	700
81–90	70	770
91–100	30	800

b)



c) 'A' grade ≈ 75%
d) Lower boundary ≈ 66%
Upper boundary ≈ 72%
e) Inter-quartile range ≈ 25%

Topic 9 Mathematical investigations and ICT

Heights and percentiles page 499

1. Approx. 167 cm

2. Approx. 168 cm (Note: This corresponds to the 25th percentile not the 75th.)

3. Approx. 151 cm

4. Student's calculations could include mean, median, inter-quartile range and comparisons with printed charts.

5. It is likely that different cultures have different charts as some races are taller on average than others.

Reading ages page 501

1. Possible answers include length of sentences, number of words with 3 or more syllables, size of type, etc.

2. Student's choices

3. Student's calculations

4. Students should choose articles on a similar topic. Ignore proper nouns. Choose more than one article from each paper.

ICT activity page 501

1.–6. Student's data, graph and comparisons