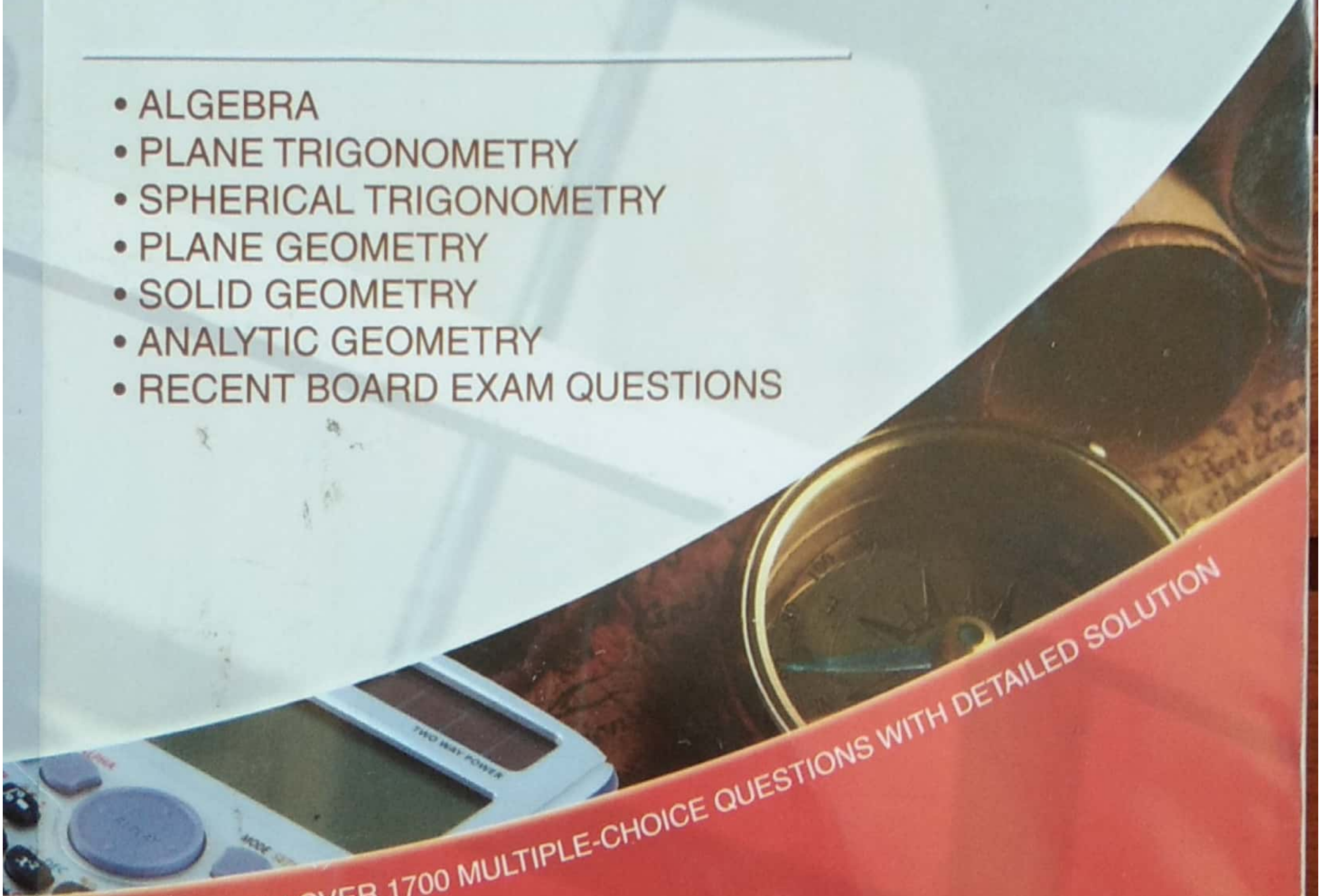
Engineering Mathematics

VOLUME 1

Third Edition

- ALGEBRA
- PLANE TRIGONOMETRY
- SPHERICAL TRIGONOMETRY
- PLANE GEOMETRY
- SOLID GEOMETRY
- ANALYTIC GEOMETRY
- RECENT BOARD EXAM QUESTIONS



NOW WITH OVER 1700 MULTIPLE-CHOICE QUESTIONS WITH DETAILED SOLUTION

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CERTC review

Part 1
ALGEBRA & ADVANCED MATH

BASIC LAW OF NATURAL NUMBERS

Let a, b, and c be any number.

1. Law of closure for addition:
   $a + b$
2. Commutative law for addition:.
   $a + b = b + a$
3. Associative law for addition:
   $a + (b + c) = (a + b) + c$
4. Law of closure for multiplication:
   $a \times b$
5. Commutative law for multiplication
   $a \times b = b \times a$
6. Associative law for multiplication
   $a(bc) = (ab)c$
7. Distributive Law
   $a(b + c) = ab + ac$

BASIC LAWS OF EQUALITY

1. Reflexive property
   $a = a$
2. Symmetric property
   If $a = b$, then $b = a$
3. Transitive property
   If $a = b$ and $b = c$, then $a = c$. That is, things equal to the same thing are equal to each other.
4. If $a = b$ and $c = d$, then $a + c = b + d$. That is, if equals are added to equals, the results are equal.
5. If $a = b$ and $c = d$, then $ac = bd$. That is, if equals are multiplied to equals, the results are equal.

INEQUALITY

A statement that one quantity is greater than or less than another quantity

Symbols used in inequality

$a > b$	a is greater than b
$a < b$	a is less than b
$a \leq b$	a is less than or equal to b
$a \geq b$	a is greater than or equal to b

Theorems on Inequalities

1. $a > b$ if and only if $-a < -b$
2. If $a > 0$, then $-a < 0$
3. If $-a < 0$, then $a < 0$
4. If $a > b$, $c < 0$, then $ac < bc$
5. If $a > b$, $c > d$, then $(a + c) > (b + d)$
6. If $a > b$, $c > d$, and $a, b, c, d > 0$, then $ac > bd$
7. If $a > 0$, $b > 0$, $a > b$, then $\frac{1}{a} < \frac{1}{b}$

OTHER IMPORTANT PROPERTIES IN ALGEBRA

1. $a \times 0 = 0$
2. If $a \times b = 0$, then either $a =$ or $b = 0$ or both a and b are zero.
3. $\frac{0}{a} = 0$ if $a \neq 0$
4. $\frac{a}{0}$ = undefined
5. $\frac{a}{\infty} = 0$

LAWS OF EXPONENTS (INDEX LAW)

1. $a^n = a \times a \times a \ldots$ (n factors)
2. $a^m \times a^n = a^{m+n}$
3. $\frac{a^m}{a^n} = a^{m-n}$
4. $(a^m)^n = a^{mn}$
5. $(abc)^n = a^n b^n c^n$
6. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
7. $a^{\frac{m}{n}} = \sqrt[n]{a^m}$
8. $a^{-m} = \frac{1}{a^m}$ and $\frac{1}{a^{-m}} = a^m$
9. $a^0 = 1$
10. If $a^m = a^n$, then $m = n$ (provided $a \neq 0$)

PROPERTIES OF RADICALS

1. $a^{\frac{1}{n}} = \sqrt[n]{a}$
2. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$
3. $\left(\sqrt[n]{a}\right)^n = a$
4. $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$
5. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$ provided that $b \neq 0$

PROPERTIES OF LOGARITHM

1. $\log_a MN = \log_a M + \log_a N$
2. $\log_a \dfrac{M}{N} = \log_a M - \log_a N$
3. $\log_a M^n = n \log_a M$
4. $\log_a a = 1$
5. $\log_a a^x = x \log_a a = x$
6. $\log_a 1 = 0$
7. If $\log_a M = N$, then $a^N = M$
8. If $\log_a M = \log_a N$, then $M = N$
9. $\log_e M = \ln M$
   $e = 2.71828\ldots$ (Naperian logarithm)
10. $\log_{10} M = \log M$ (Common logarithm)
11. $\log_n M = \log M / \log n = \ln M / \ln n$
12. If $\log_b x = a$ then $x = \text{antilog}_b\, a$
13. $a^x = \text{antilog}_a\, x$
14. $\log_{10} 4250 = \log_{10} (1000 \times 4.25)$
    $= \log 1000 + \log 4.25$
    $\log_{10} 4250 = 3 + 0.6284 =$ **3.6284**

3, the *integral part*, is called the characteristic
0.6284, a *non-negative decimal* fraction part, is called the mantissa

POLYNOMIALS

Expanding Brackets

By multiplying two brackets together, each term in one bracket is multiplied by each term of the other bracket.

$$(a + b + c)(d + e) = ad + ae + bd + be + cd + ce$$

Factorization

Factorization is the opposite process of expanding brackets. The usual process includes changing a long expression without any brackets to a shorter expression that includes brackets.

$$2x^2 - 6x + 4 = 2(x^2 - 3x + 2) = 2(x - 2)(x - 1)$$

Special Products and Factoring

1. $(x + y)(x - y) = x^2 - y^2$
2. $(x + y)^2 = x^2 + 2xy + y^2$
3. $(x - y)^2 = x^2 - 2xy + y^2$
4. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2xz + 2yz$
5. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
6. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
7. $x^6 - y^6 = (x^2)^3 - (y^2)^3 = (x^2 - y^2)[(x^2)^2 + (x^2)(y^2) + (y^2)^2]$
   $= (x + y)(x - y)(x^4 + x^2 y^2 + y^4)$

Division of Polynomials

Carrying out the division of polynomials is no different, in principle, to numerical division. Consider the following example.

Example

Divide $x^4 - 10x^2 - 9x - 20$ by $x - 4$.

Solution A

By long division

$$\begin{array}{rll}
 & \mathbf{x^3 + 4x^2 + 6x + 15 \text{ remainder } 40} & \\
x - 4 \,) & x^4 - 10x^2 - 9x - 20 & 1.\ x^4 \div x = x^3 \\
- & x^4 - 4x^3 & \\ \hline
 & 4x^3 - 10x^2 & 2.\ 4x^3 \div x = 4x^2 \\
- & 4x^3 - 16x^2 & \\ \hline
 & 6x^2 - 9x & 3.\ 6x^2 \div x = 6x \\
- & 6x^2 - 24x & \\ \hline
 & 15x - 20 & 4.\ 15x \div x = 15 \\
- & 15x - 60 & \\ \hline
 & \text{remainder} \rightarrow 40 &
\end{array}$$

Solution B

BY SYNTHETIC DIVISION

Write the coefficients of the terms, supplying zero as the coefficient of the missing power of x.

1	0	-10	-9	-20	4
	4	16	24	60	
1	4	6	15	40	

The quotient is $x^3 + 4x^2 + 6x + 15$ remainder 40

Factor Theorem

Consider a function $f(x)$. If $f(1) = 0$ then $(x - 1)$ is a factor of $f(x)$. If $f(-3) = 0$ then $(x + 3)$ is a factor of $f(x)$. Use of factor theorem can produce the factors of an expression in a *trial and error* manner.

Example

Factorize $2x^3 + 5x^2 - x - 6$

Solution

$f(x) = 2x^3 + 5x^2 - x - 6$

$f(1) = 2(1)^3 + 5(1)^2 - (1) - 6 = 0$, hence $(x - 1)$ is a factor

$f(-1) = 2(-1)^3 + 5(-1)^2 - (-1) - 6 = -2$, hence $(x + 1)$ is not a factor

$f(2) = 2(2)^3 + 5(2)^2 - (2) - 6 = 28$, hence $(x - 2)$ is not a factor

$f(-2) = 2(-2)^3 + 5(-2)^2 - (-2) - 6 = 0$, hence $(x + 2)$ is a factor

$f(-3/2) = 2(-3/2)^3 + 5(-3/2)^2 - (-3/2) - 6 = 0$, hence $2x + 3$ is a factor.

Thus, $2x^3 + 5x^2 - x - 6 = \mathbf{(x - 1)(x + 2)(2x + 3)}$

Remainder Theorem ✓4

If a polynomial $f(x)$ is divided by $(x - r)$ until a remainder which is free of x is obtained, the remainder is $f(r)$.
If $f(r) = 0$ then $(x - r)$ is a factor of $f(x)$.

Example

Find the remainder when $x^4 - 10x^2 - 9x - 20$ is divided by $x - 4$.

Solution

$f(x) = x^4 - 10x^2 - 9x - 20$

$x - r = x - 4$

$r = 4$

Remainder $= f(4) = 4^4 - 10(4)^2 - 9(4) - 20$

Remainder = **40**

Example

Find k such that $x - 3$ is a factor of $kx^3 - 6x^2 + 2kx - 12$.

Solution

Remainder $= f(3) = k(3)^3 - 6(3)^2 + 2k(3) - 12 = 0$

$\mathbf{k = 2}$

BINOMIAL THEOREM ok

Expansion of $(a+b)^n$

Properties

1. The number of terms in the expansion $n + 1$,
2. The first term is a^n & the last term is b^n,
3. The exponent of a descends linearly from n to 0,

4. The exponent of b ascends linearly from 0 to n,
5. The sum of the exponents of a and b in any of the terms is equal to n,
6. The coefficient of the second term and the second from the last term is n,

Pascal's Triangle

Used to determine the coefficients of the terms in a binomial expansion.

$(a+b)^0$	1
$(a+b)^1$	1 1
$(a+b)^2$	1 2 1
$(a+b)^3$	1 3 3 1
$(a+b)^4$	1 4 6 4 1
$(a+b)^5$	1 5 10 10 5 1

r^{th} term of $(a+b)^n$

$$r^{th}\text{ term} = \frac{n!}{(n-r+1)!(r-1)!}\, a^{n-r+1}\, b^{r-1}$$

To get the middle term (for even value of n),

$$\text{set } r = \frac{n}{2} + 1$$

Example

Find the 3rd term in the expansion of $(x^2 + y)^5$.

Solution A

Using the properties and Pascal's triangle:

$$(x^2+y)^5 = (x^2)^5 + 5(x^2)^4 y + 10(x^2)^3 y^2$$
$$= x^{10} + 5x^8 y + \mathbf{10x^6 y^2}$$

Solution B

Using the formula:

$$r^{th}\text{ term} = \frac{n!}{(n-r+1)!(r-1)!}\, a^{n-r+1}\, b^{r-1}$$

$r = 3$ $n = 5$
$a = x^2$ $b = y$

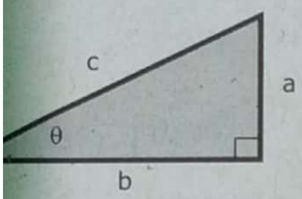
$$3^{rd}\text{ term} = \frac{5!}{(5-3+1)!(3-1)!}(x^2)^{5-3+1}(y)^{3-1}$$
$$= \mathbf{10\, x^6 y^2}$$

$nCR = \frac{n!}{(n-R)!.(R)!}$

Part 2
PLANE & SPHERICAL TRIGONOMETRY

Plane Trigonometry

UNCTIONS F A RIGHT RIANGLE



From the right triangle shown: (*soh-cah-toa*)

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c} \qquad (soh)$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c} \qquad (cah)$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b} \qquad (toa)$$

$$\cot\theta = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{b}{a} \qquad (tao)$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{c}{b} \qquad (cha)$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{c}{a} \qquad (sho)$$

thagorean eorem

"In any right triangle, the square of the longest side (hypotenuse) equals the sum of the squares of the other two sides".

From the right triangle shown above:

$$c^2 = a^2 + b^2$$

TRIGONOMETRIC IDENTITIES

Identity is a type of equation which is satisfied with any value of the variable or variables. Equations that are satisfied by some value or values of the variable are called *conditional equation.*

Consider the following equations:

$x^2 - 4 = 0$....................... *conditional equation* true only for $x = \pm 2$

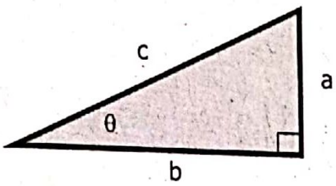
$(x + 2)^2 = x^2 + 4x + 4$.... *identity*

$\sin\theta = 0.5$..................... *conditional equation* true only if $\theta = 30°, 150°$

$\sin^2\theta + \cos^2\theta = 1$........ *identity*

Basic Identities

From the right triangle shown:



$$\tan\theta = \frac{a}{b} = \frac{a/c}{b/c} = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{b}{a} = \frac{b/c}{a/c} = \frac{\cos\theta}{\sin\theta}$$

$$\sec\theta = \frac{c}{b} = \frac{c/c}{b/c} = \frac{1}{\cos\theta}$$

$$\csc\theta = \frac{c}{a} = \frac{c/c}{a/c} = \frac{1}{\sin\theta}$$

Pythagorean Relations

From the Pythagorean theorem:

$$a^2 + b^2 = c^2$$

dividing both side by c^2:

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{c^2}{c^2} \quad \text{or} \quad \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$$

then;

$$\sin^2\theta + \cos^2\theta = 1$$

Dividing $a^2 + b^2 = c^2$ by b^2 we get,

$$\tan^2\theta + 1 = \sec^2\theta$$

Dividing $a^2 + b^2 = c^2$ by a^2 we get,

$$1 + \cot^2\theta = \csc^2\theta$$

Sum and Difference of Two Angles

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$
$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$
$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$
$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$
$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Double Angle Formulas

Double angle formulas can be derived using the sum of angle formulas.

Consider the following example:

$$\sin 2x = \sin(x + x) = \sin x \cos x + \cos x \sin x$$

Thus;

$$\sin 2x = 2 \sin x \cos x$$

We ca apply similar procedure to the rest of the formulas.

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Half-Angle Formulas

The half-angle formulas may be derived from the following relations from double angle formula:

$$\cos 2x = 1 - 2\sin^2 x$$

$$\sin x = \sqrt{\frac{1 - \cos 2x}{2}}$$

Let $2x = \theta$, then $x = \frac{\theta}{2}$

then $\sin\frac{\theta}{2} = \sqrt{\frac{1 - \cos\theta}{2}}$

Applying similar procedure, the following formulas can be derived:

$$\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Powers of Functions

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\tan^2 x = \frac{1-\cos 2x}{1+\cos 2x}$$

Product of Functions

$$\sin x \cos y = \tfrac{1}{2}[\sin(x+y) + \sin(x-y)]$$

$$\sin x \sin y = \tfrac{1}{2}[\cos(x-y) - \cos(x+y)]$$

$$\cos x \cos y = \tfrac{1}{2}[\cos(x+y) + \cos(x-y)]$$

Sum and Difference of Functions (Factoring Formulas)

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\tan x + \tan y = \frac{\sin(x+y)}{\cos x \cos y}$$

$$\tan x - \tan y = \frac{\sin(x-y)}{\cos x \cos y}$$