*Module in*

# MATHEMATICS IN THE MODERN WORLD


## ISCOF

ILOILO STATE COLLEGE OF FISHERIES

## Vision

A premier academic institution in Southeast

## Mission

To upgrade quality education, generate jobs, and alleviate poverty through excellent academic programs

## Goals

Quality Instruction, Relevance and Responsiveness, Access and Equity, Human Resource , Development, Linkages and Infrastructure Development

# OBJECTIVES OF ISCOF

***The objectives are to:***

1. produce quality graduates who will contribute to the advancement of their chosen field;
2. conduct viable researches, develop and discriminate technologies, and provide technical assistance to the community for an increase production;
3. offers courses that are responsive to the needs of the community and industries for the national development;
4. Promote self employment and entrepreneurship;
5. Strengthen opportunities for student scholarship and access to quality education;
6. Promote faculty and staff development;
7. Strengthen and develop linkages with other agencies or institutions to achieve both in the national and international levels; and
8. Improve facilities and structures that will efficiently carry out quality instruction, research and development, extension and production.

## Goals of the College of Education

The Iloilo State College of Fisheries, School of Education provides quality education and training in academic, professional, and technological competencies through instruction, extension and research development.

## Program Objectives

1. Produce competent and effective teachers for Mathematics, English, Filipino, Science and other major fields.
2. Develop competence in research and extension through collaboration of faculty and students.
3. Produce quality instructional materials through the application of various technologies in teaching.

# Program Learning Outcomes

1. Exhibit competence in mathematical concepts and procedures.
2. Exhibit proficiency in relating mathematics to other curricular areas.
3. Manifest meaningful and comprehensive pedagogical content knowledge (PCK) of mathematics
4. Demonstrate competence in designing, constructing and utilizing different forms of assessment in mathematics.
5. Demonstrate proficiency in problem-solving by solving and creating routine and non-routine problems with different levels of complexity.
6. Use effectively appropriate approaches, methods and techniques in teaching mathematics including technological tools.
7. Appreciate mathematics as an opportunity for creative work, moments of enlightenment, discovery and gaining insights of the world.

# Letter to My Students


***Good Day!***

***This module on Mathematics in the Modern World is uniquely prepared for you to understand that Mathematics can help you do many essential things in your everyday lives. It provides you with aspects of the nature of mathematics and how it is used to understand our world. It is to remind you that mathematics does not only take place within the four walls in the classroom but more so in real-life situations.***

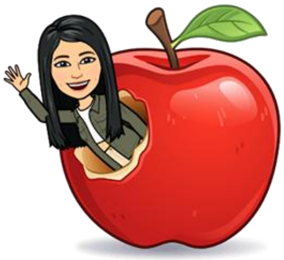
***In this module, you are expected to see the relevance of mathematics in your lives. It develop a deeper appreciation for mathematics after the topic in regular patterns in nature and the environment.***

***This module also contains examples with detailed solutions, designed to help you solve your own problems after studying and practicing a variety of problem-solving techniques and strategies that will enhance your critical and creative thinking skills. By learning basic statistics, you will be able to properly apply the statistical method in the management, analysis, and interpretation of data. It will also enhance your research skills.***

***It also includes codes and linear programming for you to capture text data, errors, and error detection and correction to determine the right strategy to correct them. Mathematics on finance, apportionment, and voting and logic is also included in order for you to apply the concept of mathematics in other fields like politics, economics, and other social sciences.***

***The theories on graphs and number systems are made part of the highlights of this module to elucidate the foundation of mathematics through Euler's formula, modulo, and group concept. All these contributions will make you grasp and internalize mathematics is vital in your academic journey and in your realization of your dream to become a lifelong learner professional.***

***I hope you enjoy the journey!***



# Course Outline

**I. COURSE NUMBER** : GE 3
**II. DESCRIPTIVE TITLE** : Mathematics in the Modern World
**III. CREDIT** : 3 Units – 3 hours lecture per week
**PRE-REQUISITES** : NONE
**IV. COURSE DESCRIPTION** :

This course deals with nature of mathematics, appreciation of its practical, intellectual, and aesthetic dimensions, and application of mathematical tools in daily life.

The course begins with an introduction to the nature of mathematics as an exploration of patterns (in nature and the environment) and as an application of inductive and deductive reasoning. By exploring these topics, students are encouraged to go beyond the typical understanding of mathematics as merely a set of formulas but as a source of aesthetics in patterns of nature, for example, and a rich language in itself (and of science) governed by logic and reasoning.



The course then proceeds to survey ways in which mathematics provides a tool for understanding and dealing with various aspects of present-day living, such as managing personal finances, making social choices, appreciating geometric designs, understanding codes used in data transmission and security, and dividing limited resources fairly. These aspects will provide opportunities for actually doing mathematics in a broad range of exercises that bring out the various dimensions of mathematics as a way of knowing, and test the students' understanding and capacity. (CMO No. 20, series of 2013)

**V. COURSE MATRIX:**

| Desired Learning Outcomes | Course Topics |
|---|---|
| **MIDTERM** | |
| **At the end of the unit, students must have:**<br>1. Identified, described, explained and internalized the vision, mission, core values of the university and the institutional college/campus, degree and course outcomes. | **Unit 0: Vision, Mission, Core Values, and Outcomes**<br>1. University vision, mission, core values and outcomes<br>2. College of Education outcomes<br>3. Degree outcomes<br>4. Course outcomes |

| | |
|---|---|
| **LO1: Discussed, summarized and explored the patterns of nature that could relate to mathematical concepts.**<br><br>➢ Identified patterns in nature and regularities in the world.<br><br>➢ Determined the patterns in the Fibonacci Sequence<br><br>➢ Reflected on the importance of Mathematics in nature and the world | **Unit 1. *Mathematics in our World***<br>1.1 Patterns and Numbers in Nature and the World<br>1.2 The Fibonacci Sequence<br>1.3 Mathematics for our World |
| **LO2. Discussed the language and symbols of mathematics.**<br>➢ Discussed the language, symbols and conventions of mathematics<br><br>**LO3. Translated about mathematics and mathematical concepts, ideas and principles using different types of reasoning.**<br>➢ Translated verbal phrase to mathematical phrase and vice versa. | **Unit II. *Mathematical Language and Symbols***<br>2.1 Expression Vs Sentences<br>2.2 The Language of Sets<br>2.3 The Language of Relations and Functions<br>2.4 Elementary Logic<br>2.5 Formality |
| **LO4. Solved real-life problems involving patterns, recreational and non-routine problems**<br>➢ Wrote clear and logical proofs.<br><br>➢ Solved problems involving patterns and recreational problems following Polya's four steps;.<br><br>**LO5. Organized methods and approaches for proving and solving problems.**<br>Used different types of reasoning to justify statements and arguments made about mathematics and mathematical concepts. | **Unit III. *Problem Solving and Reasoning***<br>3.1 Inductive and Deductive Reasoning<br>3.2 Intuition, Proof and Certainty<br>3.3 Polya's Four-Steps in Problem Solving<br>3.4 Problem Solving Strategies<br>3.5 Problem Solving with Patterns<br>3.6 Recreational and Non-Routine Problems |

| | |
|---|---|
| **At the end of the examination, the students must have: Achieved 75% from the Midterm examination** | ➢ **Midterm Examination** |
| **FINALS** | |
| **LO6. Applied variety of statistical tools to process and manage numerical data.**<br>➢ Solve for the measure of central tendency and dispersion<br>➢ Solve problems involving measures of central tendency and dispersion. | ***Unit IV. Descriptive Statistics***<br>1. Measures of central tendency and dispersion (Grouped and Ungrouped Data)<br>1.1 Mean<br>1.2 Median<br>1.3 Mode |
| ➢ Solve problems involving the normal curve distribution. | ***Unit V.*** Normal Distribution |
| ➢ Use a variety of statistical tools to process and manage numerical data<br>➢ Apply the use of statistical data in making important decision<br>➢ Make an interpretation, decision and conclusion of a research problem | ***Unit VI.***<br>1. Regression<br>2. Correlation<br>3. Chi-Square<br>4. Hypothesis Testing, |
| **At the end of the examination, the students must have: Achieved 75% from the Final examination** | ➢ **Final Examination** |

## VI. COURSE REQUIREMENTS:

1. Passed long exam (Midterm and Final Exam)
2. Passed Quizzes
3. Submitted work sheets (problem-based learning worksheets/performance task)
4. Submit problem sets and activities

## VII. GRADING SYSTEM:

**MIDTERM GRADE**

**Class Standing = 60 %**
Quizzes = 15%
Activities = 15%
Performance Task = 30%
**Mid-term exam = 40%**
100%

**TENTATIVE FINAL**

**Class standing = 60%**
Quizzes = 15%
Activities = 15%
Performance Task = 30%
**Final Exam =40%**
100%

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# Unit 1 MATHEMATICS IN OUR WORLD

## LEARNING OUTCOMES:

*At the end of this unit, the students must be able to:*

*1. identify patterns in nature and regularities in the world;*
*2. determine the patterns in numbers and in the Fibonacci sequence;*
*3. Solve problems involving different number patterns;*
*4. Create a synthesis about the importance of Mathematics in nature and the world.*

## *INTRODUCTION*

Mathematics exists everywhere and is applied from the simplest to the most complex activities or problem we encounter in our daily lives. For instances like paying for our bills, measuring the right amount of ingredients needed in cooking, constructing houses, buildings and bridges and etc. This is one subject thought as the sole objective language that we in the modern world understand each other.

Merriam dictionary defines mathematics as the science of numbers and their operations, interrelations, combinations, generalizations, and abstractions and of space configurations of their structure, measurement, transformations and generalizations.

Mathematics came from an ancient word ***manthanein*** means “to learn”. The Greek root *mathesis* means “knowledge” or its other form *máthema* meaning science, knowledge, or learning, and *mathematikos or mathemata* means “fond of learning”. These might be the theory of the early mathematicians and philosophers which lead them continue to seek for knowledge and truth.

Mathematics is not just about numbers. Much of it is problem solving and reasoning- inductive and deductive. It also discusses intuition, proof, and certainty. It utilizes Polya’s 4-step in problem solving, varied problem solving strategies, mathematical problems involving patterns and recreational problem using mathematics.

*"If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is." - John von Neumann*

# LESSON 1: Patterns in Nature and the World


## Patterns in Nature and the Regularities in the World

### *What is a Pattern?*



According to ***Ian Stewart*** *(1995),* we live in a universe of patterns. Everyday the sun moves in cycle from east to west. No two fingerprints are ever exactly the same. Snowflakes are unique but they all have six-fold symmetry. Tigers and Zebras are covered in patterns of stripes, leopards and hyenas are covered in patterns of spots. Intricate trains of waves march across the ocean; very similar trains of sand dunes march across the desert. Colored arcs of light adorn the sky in the form the rainbows, and a bright circular halo sometimes surrounds the moon on winter nights. Spherical drops of water fall from clouds. Mathematics is used in recognizing, classifying, and exploiting patterns. By using mathematics to organize and systematize our ideas about patterns we had concluded that nature's patterns are not just there to be admired, they are the critical clues to the concept that determine natural processes.


A ***regularity*** (Collins,2018), is the fact that the same thing always happens in the same circumstances.

***Pattern*** is a discernible regularity in the world or in a man-made design and the elements repeat in a predictable manner.

***Patterns in Nature*** (Wikipedia) are visible regularities of form found in the natural world and recur in different contexts and can sometimes be modeled mathematically.

## SYMMETRY

**Symmetry** (dictionary.com) a geometrical or other regularity that is possessed by a mathematical object and is characterized by the operations that leave the object invariant. In everyday language, it refers to a sense of harmonious and beautiful proportion and balance. It is an agreement in dimensions, due proportion, and an arrangement.

http://teakes.blogspot.com/2009/10/symmetry-in-and-of-nature.html

## SPIRAL

A **spiral** is a curve which turns around some central point, getting further away, or closer, as it goes. An example of a spiral found in nature is a shell, it shows a patterns that arranged in a approximately logarithmic spiral.

https://sarahbrown001.wordpress.com/2015/03/20/disasters/

## MEANDER

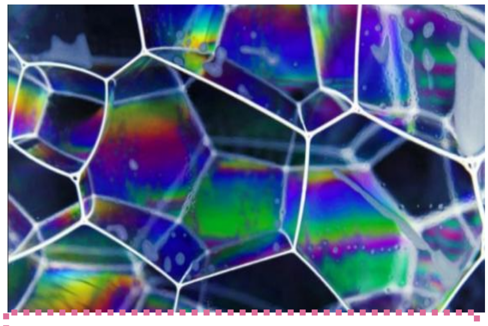
A **meander** is a series of regular sinuous curves, bends, loops, turns, or windings in the channel of a river, stream, or other watercourse. It is produced when a river shifts its channel within a valley or it swings from side to side as it flows across its floodplain.

https://laurasriverfeatures.weebly.com/meanders.html

## WAVE

A **wave** is a disturbance that transfers energy through matter or space, with little or no associated mass transport. It consist of oscillations or vibrations of a physical medium or a field,around relatively fixed locations. In water, surface waves show water ripples.

https://marshabalaeva.files.wordpress.com/2013/11/powerful-waves7.jpg



https://ecstep.com/wp-content/uploads/2017/12/Natural-P-Foam-of-soap-bubbles-2-705x445.jpg

## FOAM

**Foam** is a substance formed by trapping pockets gas in a liquid or solid. Examples are bath and the head on a glass of beers. In most foams, the volume of gas is large, with thin films of liquid or solid separating the regions of gas. Soap foams are also known as suds.

https://www.pinterest.ph/pin/275846164468076302/

## STRIPES

**Stripes** are made by a series of bands or strips, often of the same width and color along the length.

http://en.tessellations-nicolas.com/zoom/bird-hexagon-regular-tilings.php

## TESSELLATION

A **tessellation** of a flat surface is the tilling of a plane using one or more geometric shapes, called tiles, with no overlaps and no gasps. In mathematics, tessellations can be generalized to higher dimensions and a variety of geometries.

https://www.deviantart.com/tangledfrog/art/Cracked-Rocks-179913174

## FRACTURE OR CRACK

A **fracture or crack** is the separation of an object or material into more or more pieces under the action of stress. The fracture of a solid usually occurs due to the development of certain displacement discontinuity surfaces within the solid.

https://webecoist.momtastic.com/2008/09/07/17-amazing-examples-of-fractals-in-nature/

## FRACTAL

A **fractal** is a never ending pattern. Fractals are infinitely complex patterns that are self-similar across different scales. They are created by repeating a simple process over and over in an ongoing feedback loop. Driven be recursion, fractals are images of dynamic systems-the pictures of chaos.

https://www.maplesoft.com/fern/

## AFFINE TRANSFORMATION

**Affine Transformation** is a processes of rotation,reflection and scaling. Many plant forms utilize these processes to generate their structure.

One example is a plant wherein each branch appears to be a smaller version of the main plant and so on, at smaller scales.

**The following pictures show patterns and regularities in nature.**

Snail shell

Peacock 's Feathers

Cactus

Chameleon's tail

Leopard's spots

Spiderweb

## LEARNER'S ACTIVITY

A. **IDENTIFICATION:** Write the correct word on the blank in the right that is being referred to in the following:

1. An agreement in dimension, due proportion, an arrangement ____________________
2. The infinitely complex patterns that are self-similar across different scales ____________________
3. A series of bands of strips, often of the same width and color along the length ____________________
4. A curve which emanates from a point, moving farther away as it revolves around the point ____________________
5. A series of regular sinuous curves, bends, loops, turns, or windings in the channel of a river, stream, or other watercourse ____________________



**B. PERFORMANCE TASKS**

1. Go to your garden or front-yard, and start counting leaves and petals. Write the result of your observation on this form:

Plant name:________________________________________

Description: ________________________________________

________________________________________

________________________________________

A. What is the pattern of the leaves?

________________________________________

________________________________________

________________________________________

________________________________________

B. Are there flowers? (Yes/No) What is the arrangement of the petals?

________________________________________

________________________________________

________________________________________

________________________________________

________________________________________

________________________________________

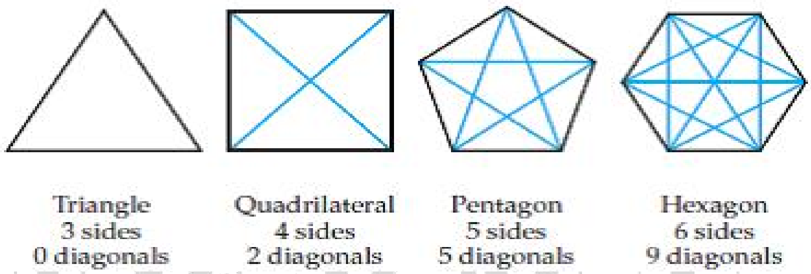
# LESSON 2: Number Patterns and Fibonacci sequence

## Common Patterns

### LOGIC PATTERNS

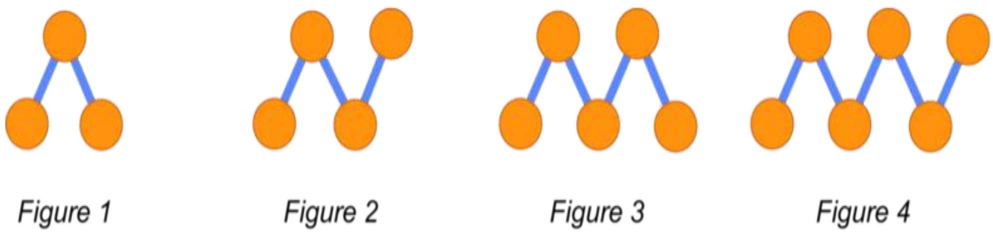
***Logic patterns*** are related to geometric patterns and number patterns. It helps us classify objects or figures. One kind of logic patterns deal with characteristics of various objects. Another kind deals with orders: there is a sequence of objects and a pattern in the attributes the objects possess.

***Examples 1*** : In a polygon, a diagonal connects two vertices that are not al




How many diagonals can be drawn in a heptagon? Illustrate your answer inside the rectangle below.

***Examples 2:*** Look at the following pattern.




How many circles are in figure 10?

A. 8 B. 10 C. 12 D. 14

# NUMBER PATTERNS

A ***number pattern*** is a list or set of numbers that follow a certain sequence or arrangement in either ascending or descending order. It includes basic operations of mathematics or a certain series of arithmetical operation like addition and multiplication repeatedly done.

## 1. Arithmetic Number Pattern

An ***arithmetic sequence*** is made by adding the same value each time. The value added is called the ***common difference.***

Examples: **a. 1, 4, 7, 10, 13, 16, 19, 22, 25, …**
*(each number is 3 larger than the number before it)*

**b. 25, 23, 21, 19, 17, 15, …**
*(The common difference is -2. The pattern is continued by **subtracting 2** each time)*



## 2. Geometric Number Pattern

Some other types are created in which every successive terms is multiplied or (dividing) the same value each time. This number pattern is called ***geometric sequence.***

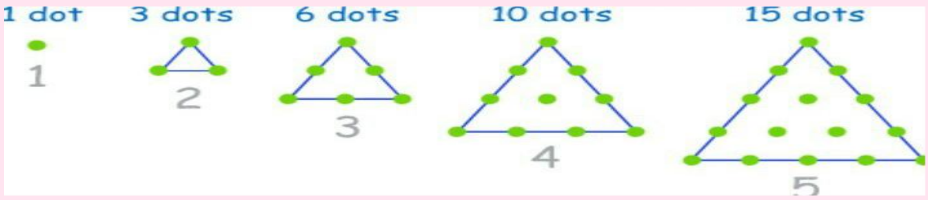
Examples: **a. 1, 2, 4, 8, 16, 32, 64, 128, …**
*(*This sequence has a factor of 2 between each number. Each term (except the first term) is found by **multiplying** the previous term by **2**.*)*

**b. 4, 2, 1, 0.5, 0.25, …**
(This sequence has a factor of 0.5(a half) between each number.)

## 3. Triangular Numbers

This ***Triangular Number Sequence*** is generated from a pattern of dots that form a triangle.

Example: 1, 3, 6, 10, 15, 21, 28, 36, 45, …




By adding another row of dots and counting all the dots we can find the next number of the sequence.

**4. Square Numbers**

0, 1, 4, 9, 16, 25, 36, 49, 64, 81, ...

They are the **squares** of whole numbers:
0 (=0×0)
1 (=1×1)
4 (=2×2)
9 (=3×3)
16 (=4×4)
etc...

**5. Cube Numbers**

**1, 8, 27, 64, 125, 216, 343, 512, 729, ...**

They are the cubes of the counting numbers (they start at 1):

1 (1×1×1)
8 (=2×2×2)
27 (=3×3×3)
64 (=4×4×4)
etc...



**6. Other Number Patterns**

Other number patterns are patterns in which the numbers are in the increasing form.

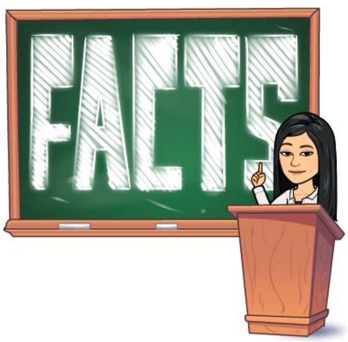
**4, 5, 7, 10, 14, 19….**

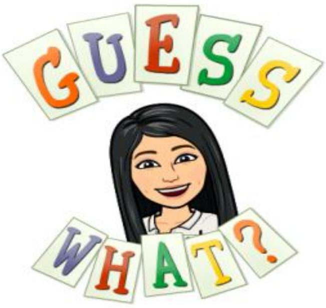
In this pattern, the amount that is added in the terms, changes every time in predictable manner.

The pattern starts with 4, add 1 and the amount which we can add increase by 1 every time.

## MATH TRIVIA

1. *The symbol for infinity (∞) was used by the Romans to represent 1000.*
2. *The number 365 is equal to the sum of three consecutive squares and two consecutive squares in which the five squares are also consecutive.*
   $365 = 10^2 + 11^2 + 12^2 = 13^2 + 14^2$
3. *169 is equal to $13^2$ and its reverse 961 is equal to $31^2$*






# Student's Activity

1. What is the next number in the pattern 45 ,48, 51, 54, 57, _______?

2. What is the missing number in the pattern 67, 74, 81, 88, ______, 102?

3. What is the next number in the pattern 108, 102, 96, ____?

4. What should come in place of the blank space in the following series of number? 2, 3, 6, 11, 18, 27, ______

5. What is the next number in the pattern? 1,- 3, 9, -27, 81 ______

## FIBONACCI SEQUENCE

**Fibonacci Numbers** is another one in this world that involves pattern (Grist, 2011). They are nature's numbering system and appear everywhere in nature, from the leaf arrangement in plants, to the pattern of florets of flower, the bracts of a pine-cone, or the scales of a pineapple. It is therefore applicable to the growth of every living thing, including a single cell, a grain of wheat, a hive of bees and even all mankind.

In mathematics,(Wikipedia), the **Fibonacci numbers**, commonly denoted $F_n$, form a sequence, called the **Fibonacci sequence**, such that each number is the sum of the two preceding ones, starting from 0 and 1. That is,

$$F_0 = 0 \text{ and } F_1 = 1 \text{ and } F_n = F_{n-1} + F_{n-2} \quad {\scriptstyle for\, n>1}$$

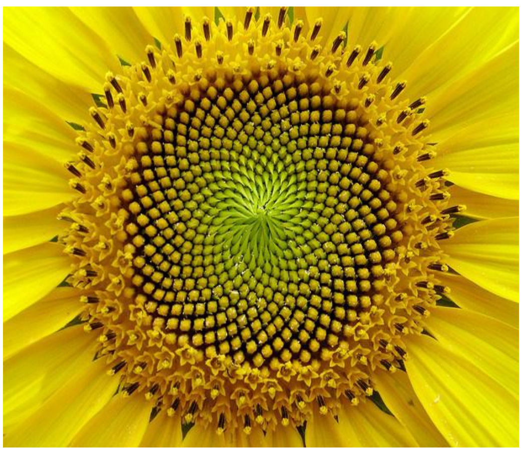
The beginning of the sequence is thus: **0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144,…**

According to George Dviorsky (2013), the famous Fibonacci sequence has captivated mathematicians, artists, designers, and scientists for centuries. Also known as the Golden Ratio, its ubiquity and astounding functionality in nature suggests its importance as a fundamental characteristic of the Universe. Leonardo Fibonacci came up with the sequence when calculating the ideal expansion pairs of rabbits over the course of one year. Today, its emergent patterns and ratios (phi = 1.61803...) can be seen from the micro scale to the macro scale, and right through to biological systems and inanimate objects. While the Golden Ratio doesn't account for *every* structure or pattern in the universe, it's certainly a major player.

## EXAMPLE OF FIBONACCI SEQUENCE IN NATURE

### Seed heads

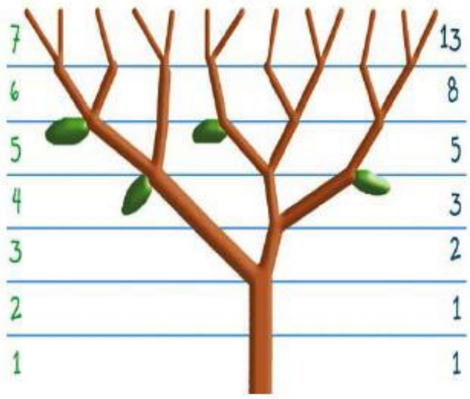
The head of a flower is also subject to Fibonaccian processes. Typically, seeds are produced at the center, and then migrate towards the outside to fill all the space. Sunflowers provide a great example of these spiraling



## Pine cones

Similarly, the seed pods on a pine cone are arranged in a spiral pattern. Each cone consists of a pair of spirals, each one spiraling upwards in opposing directions. The number of steps will almost always match a pair of consecutive Fibonacci numbers. For example, a 3-5 cone is a cone which meets at the back after three steps along the left spiral, and five steps along the right.

## Tree branches

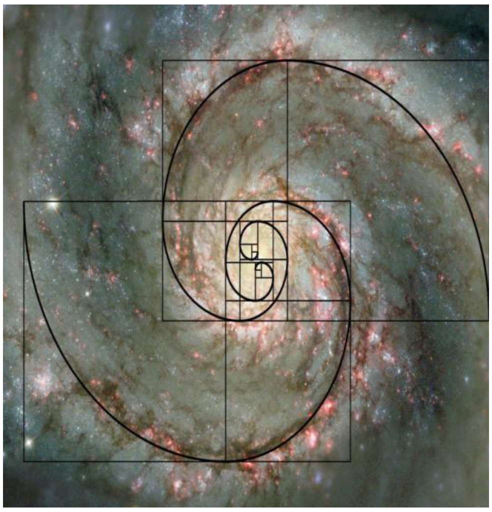



The Fibonacci sequence can also be seen in the way tree branches form or split. A main trunk will grow until it produces a branch, which creates two growth points. Then, one of the new stems branches into two, while the other one lies dormant. This pattern of branching is repeated for each of the new stems. A good example is the sneeze wort. Root systems and even algae exhibit this pattern.

## Shells

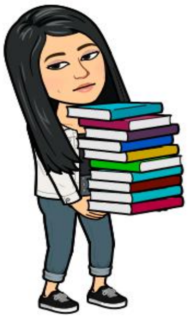
The unique properties of the Golden Rectangle provides another example. This shape, a rectangle in which the ratio of the sides a/b is equal to the golden mean (phi), can result in a nesting process that can be repeated into infinity — and which takes on the form of a spiral. It's call the logarithmic spiral, and it abounds in nature.

## Spiral Galaxies



Not surprisingly, spiral galaxies also follow the familiar Fibonacci pattern. The Milky Way has several spiral arms, each of them a logarithmic spiral of about 12 degrees. As an interesting aside, spiral galaxies appear to defy Newtonian physics. As early as 1925, astronomers realized that, since the angular speed of rotation of the galactic disk varies with distance from the center, the radial arms should become curved as galaxies rotate. Subsequently, after a few rotations, spiral arms should start to wind around a galaxy. But they don't — hence the so-called winding problem. The stars on the outside, it would seem, move at a velocity higher than expected — a unique trait of the cosmos that helps preserve its shape

# *Mathematical Problems Involving Number Patterns*



## ARITHMETIC SEQUENCE

For the arithmetic sequence 1, 3, 5, 7, each number is called a ***term*** of a sequence. The first number is called the first term, followed by the second term, the third term, and so on. Sometimes the words '**series**' or '**progression**' are used in place of the sequence. It is the indicated sum of the terms of a sequence. The arithmetic series of the sequence 1,3,5, 7 is 1+3+5+7. ***Subscripts*** are used to designate the ordinal of a term. It is customary to use $a_{1,} a_{2,} a_{3,\ldots,} a_n$ to designate the sequence of a number. The subscript n is understood to start from 1, then 2, then 3, and so on, unless otherwise stated.

The nth term $(a_n)$of an arithmetic sequence is designated by an algebraic formula

$$a_n = a_1 + (n-1)d$$

where $a_1 = the\ first\ term$

$a_n = the\ nth\ term$

$d = the\ common\ difference$



The sum of n terms is: $S_n = \frac{n}{2}(a_1 + a_n)$ or $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

## EXAMPLES

1.Rhea decided to save money for one week from her allowance. Each day she saves 15 pesos more than the previous day. If she started saving 10 pesos in the first day, how much will she set aside in the 6th day? After a week, how much would she saved?

**Solution:**

The sequence starts with 10 then add 15 until the sixth day.

$a_1$= 10 d = 15 $a_6$= 10+15(6-1) = 85

For the total savings after a week, use

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$S = \frac{7}{2}\ [2(10) + 15(6)] = 385\ pesos$$

2. Find the 14th term of the sequence 5, 7, 9, 11,….

**Solution:**

$$a_1 = 5; n = 4;\ \ d = 2$$

$$a_n = a_1 + (n-1)d$$

$$a_{14} = 5 + \ (14-1)12 = 31$$

Therefore, 31 is the 14th term in the given sequence.

# GEOMETRIC SEQUENCE

In a **Geometric Sequence** each term is found by **multiplying** the previous term by a **constant**.

**Example:1, 2, 4, 8, 16, 32, 64, 128, 256, ….**

This sequence has a factor of 2 between each number. Each term (except the first term) is found by **multiplying** the previous term by **2**.

**In General** we write a Geometric Sequence like this:

$\{a_1, a_1r, a_1r^2, a_1r^3, \ldots\}$

where:

- $a_1$ is the first term, and
- **r** is the factor between the terms (called the **"common ratio"**)

**Example:** **{1,2,4,8,...}**

The sequence starts at 1 and doubles each time, so

- **$a_1$=1** (the first term)
- **r=2** (the "common ratio" between terms is a doubling)

And we get:

$\{a, ar, ar^2, ar^3, \ldots\}$

$= \{1, 1\times2, 1\times2^2, 1\times2^3, \ldots\}$

$= \{1, 2, 4, 8, \ldots\}$



## THINGS TO REMEMBER

## GENERALIZATION

We can also calculate **nth term** of a geometric sequence using the Rule:

$$a_n = a_1r^{(n-1)}$$

(We use "n-1" because $ar^0$ is for the 1st term)

**Example:** Find the $10^{th}$ term in the sequence.

**5, 20, 80, 320, 1280 …**

This sequence has a factor of 5 between each number.

The values of **a** and **r** are:

- **$a_1$ = 5** (the first term)
- **r = 4** (the "common ratio")

The Rule for any term is:

$$a_n = 5 \times 4^{(n-1)}$$

So, the **4th** term is:

$$a_4 = 5\times4^{(4-1)} = 5\times4^3 = 5\times64 = 320$$

And the **10th** term is:

$$a_{10} = 5\times4^{(10-1)} = 5\times4^9 = 5\times262144 = 1310720$$

# Problems Set

***Instruction:*** Solve the following problems and show your solutions. Write your answers on the space provided.

1. Find the $10^{th}$ term in the sequence, 3, 11, 19, 27, 35, . . .
2. What is the common ratio and the $8^{th}$ term in the given sequence?

$$10, \frac{10}{3}, \frac{10}{9}, \frac{10}{27}, \frac{10}{81}, \ldots$$

3. A theater in the round has 70 seats in the first row, 78 seats in the second row, 86 seats in the third row, and so on in a sequence. If the theater has 24 rows of seats, find the total number of seats it contains.



**1**

**Solution:**

______________________________________________

______________________________________________

______________________________________________

**2**

**Solution:**

______________________________________________

______________________________________________

______________________________________________

**3**

**Solution:**

______________________________________________

______________________________________________

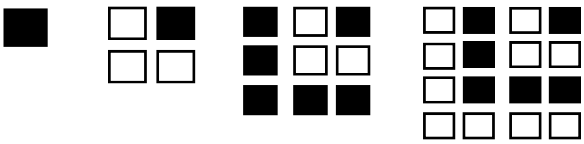
______________________________________________

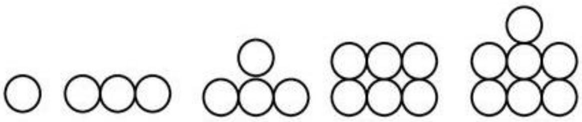
# *Geometric Patterns*

A **geometric pattern** is a kind of pattern formed or sequences of lines and curves to form geometric shapes and figures. A motif, pattern, or design depicting abstract, nonrepresentational shapes such as lines, circles, ellipses, triangles, rectangles, and polygons. It is typically repeated like a wallpaper design and tilling. Geometric pattern could also be related to number patterns and anticipate the next geometric shape that would follow the sequence.

## *EXAMPLES of GEOMETRIC PATTERNS*

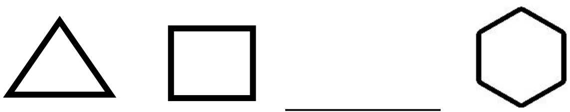
Example 1: Determine what shape, figure, or series of figures would follow the sequence.

a.  ____________________

b.  ____________________

c.  ____________________

Example 2. Draw the missing figure in the sequence



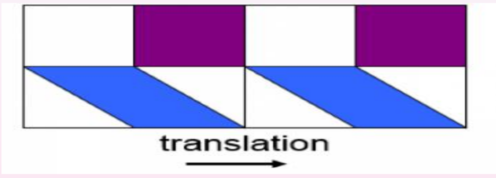
# *Isomerty*

Geometric figure is transformed in a way that the relative distance between any two points has not change.

## 4 TYPES OF ISOMETRY

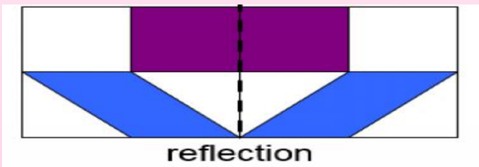
1. ***Translation***

   It is an isometry that moves very points in the plane in a fixed distance and in a fixed direction.



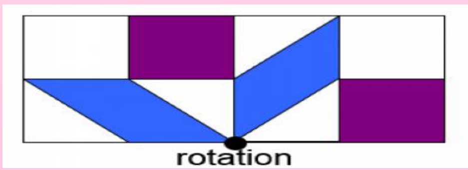

2. ***Reflection***

   It moves the object into a new position that is a mirror image of the original position



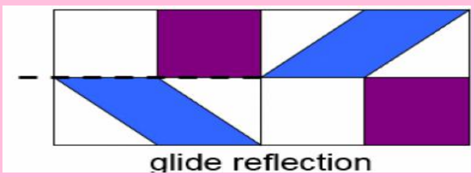

3. **Rotation**

   Is an isometry that moves each point in a fixed angle relative to a central point.



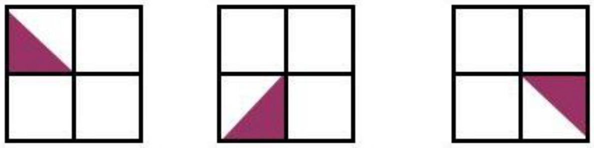

4. ***Glide Reflection***

   Is an isometry that consists of a translation followed by a reflection

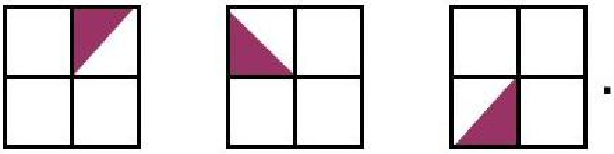


## EXAMPLES of ISOMETRY

a. Identify a possible pattern. Use it to draw the next three figures.



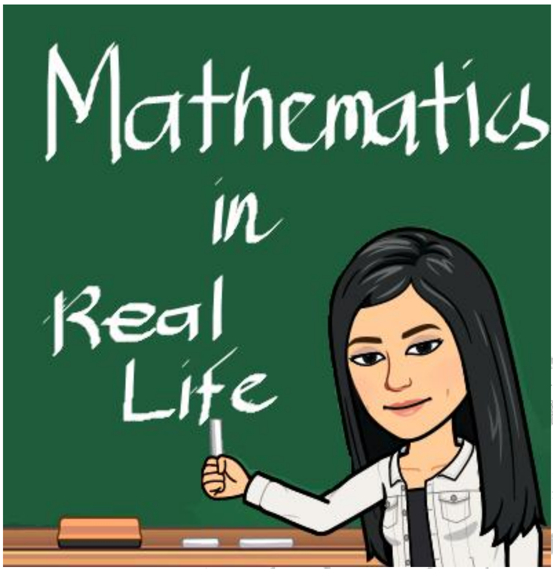
The pattern is to rotate the figure in a counterclockwise direction, so the next three figures would be



# LESSON 3: Mathematics for our World

## *Importance of Mathematics in Life*

Different questions arises through the four walls of math classroom. Students are oftentimes curious if, when, and how they will be able to use math in real-life situation.




The fact is that math is everywhere, we use it all the time! From calculating complex algorithms to counting down the days till the next episode of your favorite drama series. Mathematics is versatile and very important, no matter how difficult it is to deny. Thus, before you make up your mind to doze off in math class, here are the some reasons to consider why learning math is very important to you and the world.

1. **Restaurant Tipping**

   You need to have the most basic math skills to calculate how much a 15% or 20% tip would you pay your waiter after you have finished eating at a restaurant.

2. **Netflix Film Viewing**

   You need math to figure out how many 20 minutes Netflix episodes you will be able to watch and fit in one hour. As simple as it is, math just made your hour 100 times better.

3. **Calculating Bills**

   Math is required to calculate mandatory payments like taxes, mortgage, and insurance if you will be home-owners and car-owners one day and subtract them from your savings.

4. **Computing Test Scores**

   You need math to calculate that test score (and maybe even to ace the test) to know what your final grade can be.

5. **Tracking Career**

Math is needed if you want to be a CEO, a real estate agent, a biologist, or even a rocket scientist, it is without a doubt that numbers will be utilized.

6. **Doing Exercise**

You need math to know how many more reps to curl, or how many seconds to cut off your mile time, or how many more pounds to lose to achieve that goal.

7. **Handling Money**

Another aspect of growing up into a young adult is opening and managing a bank account. It is important to be accurate in math to care for your precious savings, making sure there are no mistakes.

8. **Making Countdowns**

For many, this will be the most important reason on this list to know math: so you can countdown the days until school is over and summer starts!

9. **Baking and Cooking**

In order to prevent unexpected results, you have to know the difference between a quarter of a cup from a quarter of a teaspoon. Baking + cooking=fractions=math!

10. **Surfing Internet**

Ultimately, without math, how would you be reading this article online at this exact moment? How would you be able to tweet to your friends or post an Instagram from last night? We have math to thank for establishing technology and the social media that consumes our lives.



## *Nature of Mathematics*

According to *American Association for the Advancement of Science (*1990), Mathematics relies on both logic and creativity, and it is pursued both for a variety of practical purposes and for its intrinsic interest. For some people, and not only professional mathematicians, the essence of mathematics lies in its beauty and its intellectual challenge.

### 1. PATTERNS AND RELATIONSHIPS

Mathematics is the science of patterns and relationships. As a theoretical discipline, mathematics explores the possible relationships among abstractions without concern for whether those abstractions have counterparts in the real world. The abstractions can be anything from strings of numbers to geometric figures to sets of equations.

### 2. MATHEMATICS, SCIENCE, AND TECHNOLOGY

Because of its abstractness, mathematics is universal in a sense that other fields of human thought are not. It finds useful applications in business, industry, music, historical scholarship, politics, sports, medicine, agriculture, engineering, and the social and natural sciences.

**3. MATHEMATICAL INQUIRY**

Using mathematics to express ideas or to solve problems involves at least three phases: (1) representing some aspects of things abstractly, (2) manipulating the abstractions by rules of logic to find new relationships between them, and (3) seeing whether the new relationships say something useful about the original things.

4. **ABSTRACTION AND SYMBOLIC REPRESENTATION**

Mathematical thinking often begins with the process of abstraction—that is, noticing a similarity between two or more objects or events. Aspects that they have in common, whether concrete or hypothetical, can be represented by symbols such as numbers, letters, other marks, diagrams, geometrical constructions, or even words. Whole numbers are abstractions that represent the size of sets of things and events or the order of things within a set. The circle as a concept is an abstraction derived from human faces, flowers, wheels, or spreading ripples; the letter A may be an abstraction for the surface area of objects of any shape, for the acceleration of all moving objects, or for all objects having some specified property; the symbol + represents a process of addition, whether one is adding apples or oranges, hours, or miles per hour. And abstractions are made not only from concrete objects or processes; they can also be made from other abstractions, such as kinds of numbers (the even numbers, for instance).



5. **MANIPULATING MATHEMATICAL STATEMENTS**

After abstractions have been made and symbolic representations of them have been selected, those symbols can be combined and recombined in various ways according to precisely defined rules. Typically, strings of symbols are combined into statements that express ideas or propositions. For example, the symbol $A$ for the area of any square may be used with the symbol $s$ for the length of the square's side to form the proposition $A = s^2$. This equation specifies how the area is related to the side—and also implies that it depends on nothing else.

**6. APPLICATION**

Mathematical processes can lead to a kind of model of a thing, from which insights can be gained about the thing itself. Any mathematical relationships arrived at by manipulating abstract statements may or may not convey something truthful about the thing being modeled. Sometimes common sense is enough to enable one to decide whether the results of the mathematics are appropriate.

## *The Role of Mathematics in some Disciplines*

Here are some main disciplines in which Role of Mathematics is widely accepted.

**1. Mathematics in Physical Science**

In Physics, every rule and principle takes the mathematical form ultimately. Mathematics gives a final shape to the rules of physics. It presents them in a workable form. Mathematical calculations occur at every step in physics.

The units of measurement are employed to substances in physics as frequently as in mathematics. The Charle's law of expansion of gases is based upon mathematical calculations. Graduation of the stem of thermometer and then the conversion of scales is also a mathematical work. The Concept is involved Fluid Dynamics, Computational Fluid Dynamics, Physical Oceanography.

2. **Mathematics in Chemistry**

Math is extremely important in physical chemistry especially advanced topics such as quantum or statistical mechanics. Quantum relies heavily on group theory and linear algebra and requires knowledge of mathematical/physical topics such as Hilbert spaces and Hamiltonian operators. Statistical mechanics relies heavily on probability theory. Other fields of chemistry also use a significant amount of math. For example, most modern IR and NMR spectroscopy machines use the Fourier transform to obtain spectra. Even biochemistry has important topics which rely heavily on math, such as binding theory and kinetics.



3. **Mathematics in Biological Sciences**

Bio-mathematics is a rich fertile field with open, challenging and fascination problems in the areas of mathematical genetics, mathematical ecology, mathematical neuron- physiology, development of computer software for special biological and medical problems, mathematical theory of epidemics, use of mathematical programming and reliability theory in bio sciences and mathematical problems in biomechanics, bioengineering and bio electronics.

4. **Mathematics in Engineering and Technology**

The use of mathematics in engineering is very well known. It is considered to be the foundation of engineering. Engineering deals with surveying, levelling, designing, estimating, construction etc., In all these processes, application of mathematics is very important. By the application of geometric principles to design and constructions, the durability of things constructed can be increased. With its help, results can often be verified in engineering.

Mathematics has played an important role in the development of mechanical, civil, aeronautical and chemical engineering through its contributions to mechanics of rigid bodies, hydro-dynamics, aero-dynamics, heat transfer, lubrication, turbulence, elasticity, and others.

**5. Mathematics and Agriculture**

Agriculture as a science is going to depend extensively on mathematics. It needs a direct application of mathematics, such as, measurement of land or area, average investment and expenditure, average return or income, production per unit area, cost of labour, time and work, seed rate etc., Progress of the farm can be judged by drawing graphs of different items of production.

6. **Mathematics and Economics**

The level of mathematical literacy required for personal and social activities is continually increasing. Mastery of the fundamental processes is necessary for clear thinking. The social sciences are also beginning to draw heavily upon mathematics.Mathematical language and methods are used frequently in describing economic phenomena. According to Marshall – “The direct application of mathematical reasoning to the discovery of economic truths has recently rendered great services in the hand of master mathematicians.” Another important subject for economics is Game theory. The whole economic situation is regarded as a game between consumers, distributors, and producers, each group trying to optimize its profits.

7. **Mathematics and Psychology**

The great educationist Herbart has said, “It is not only possible, but necessary that mathematics be applied to psychology”.

Now, experimental psychology has become highly mathematical due to its concern with such factors as intelligence quotient, standard deviation, mean, median, mode, correlation coefficients and probable errors. Statistical analysis is the only reliable method of attacking social and psychological phenomena. Until mathematicians entered into the field of psychology, it was nothing but a flight of imagination.

8. **Mathematics and Actuarial Science, Insurance and Finance**

Actuaries use mathematics and statistics to make financial sense of the future. For example, if an organization is embarking on a large project, an actuary may analyze the project, assess the financial risks involved, model the future financial outcomes and advise the organization on the decisions to be made. Much of their work is on pensions, ensuring funds stay solvent long into the future, when current workers have retired. They also work in insurance, setting premiums to match liabilities.Mathematics is also used in many other areas of finance, from banking and trading on the stock market, to producing economic forecasts and making government policy.

9. **Mathematics and Archaeology**

Archaeologists use a variety of mathematical and statistical techniques to present the data from archaeological surveys and try to distinguish patterns in their results that shed light on past human behavior. Statistical measures are used during excavation to monitor which pits are most successful and decide on further excavation. Finds are analyzed using statistical and numerical methods to spot patterns in the way the archaeological record changes over time, and geographically within a site and across the country. Archaeologists also use statistics to test the reliability of their interpretations.

10. **Mathematics and Logic**

D'Alembert says, "Geometry is a practical logic, because in it, rules of reasoning are applied in the most simple and sensible manner". Pascal says – "Logic has borrowed the rules of geometry, the method of avoiding error is sought by everyone. The logicians profess to lead the way, the geometers alone reach it, and aside form their science there is no true demonstration". C.J.Keyser – "Symbolic logic is mathematics, mathematics is symbolic logic".The symbols and methods used in the investigation of the foundation of mathematics can be transferred to the study of logic. They help in the development and formulation of logical laws.

11. **Mathematics in Music**

Leibritz, the great mathematician had said, - "Music is a hidden exercise in arithmetic of a mind unconscious of dealing with numbers".Pythogoras has said – "Where harmony is, there are numbers".Calculations are the root of all sorts of advancement in different disciplines. The rhythm that we find in all music notes is the result of innumerable permutations and combinations of SAPTSWAR. Music theorists often use mathematics to understand musical structure and communicate new ways of hearing music. This has led to musical applications of set theory, abstract algebra, and number theory. Music scholars have also used mathematics to understand musical scales, and some composers have incorporated the Golden ratio and Fibonacci numbers into their work.

12. **Mathematics in Arts**

"Mathematics and art are just two different languages that can be used to express the same ideas." It is considered that the universe is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures. The old Goethic Architecture is based on geometry. Even the Egyptian Pyramids, the greatest feat of human architecture and engineering, were based on mathematics. Artists who strive and seek to study nature must therefore first fully understand mathematics. Appreciation of rhythm, proportion, balance and symmetry postulates a mathematical mind.

13. **Mathematics in Philosophy**

The function of mathematics in the development of philosophical thought has been very aptly put by the great educationist Herbart, in his words. The real finisher of our education is Philosophy, but it is the office of mathematics to ward off the dangers of philosophy."Mathematics occupies a central place between natural philosophy and mental philosophy. It was in their search of distinction between fact and fiction that plato and other thinkers came under the influence of mathematics.

14. **Mathematics in Social Networks**

Graph theory, text analysis, multidimensional scaling and cluster analysis, and a variety of special models are some mathematical techniques used in analyzing data on a variety of social networks.

15. **Mathematics in Political Science**

In Mathematical Political Science, we analyze past election results to see changes in voting patterns and the influence of various factors on voting behavior, on switching of votes among political parties and mathematical models for Conflict Resolution. Here we make use of Game Theory.

16. **Mathematics in Linguistics**

The concepts of structure and transformation are as important for linguistic as they are for mathematics. Development of machine languages and comparison with natural and artificial language require a high degree of mathematical ability. Information theory, mathematical biology, mathematical psychology etc. are all needed in the study of Linguistics. Mathematics has had a great influence on research in literature. In deciding whether a given poem or essay could have been written by a particular poet or author, we can compare all the characteristics of the given composition with the characteristics of the poet or other works of the author with the help of a computer.

17. **Mathematics in Management**

Mathematics in management is a great challenge to imaginative minds. It is not meant for the routine thinkers. Different Mathematical models are being used to discuss management problems of hospitals, public health, pollution, educational planning and administration and similar other problems of social decisions. In order to apply mathematics to management, one must know the mathematical techniques and the conditions under which these techniques are applicable.

18. **Mathematics in Computers**

An important area of applications of mathematics is in the development of formal mathematical theories related to the development of computer science. Now most applications of Mathematics to science and technology today are via computers. The foundation of computer science is based only on mathematics. It includes, logic, relations, functions, basic set theory, countability and counting arguments, proof techniques, mathematical induction, graph theory, combinatorics, discrete probability, recursion, recurrence relations, and number theory, computer-oriented numerical analysis, Operation Research techniques, modern management techniques like Simulation, Monte Carlo program, Evaluation Research Technique, Critical Path Method, Development of new computer languages, study of Artificial Intelligence, Development of automata theory etc. Cryptography is the practice and study of hiding information. In modern times cryptography is considered a branch of both mathematics and computer science and is affiliated closely with information theory, computer security and engineering. It is the mathematics behind cryptography that has enabled the e-commerce revolution and information age. Pattern Recognition is concerned with training computers to recognize pattern in noisy and complex situations. e.g. in recognizing signatures on bank cheques, in remote sensing etc.

19. **Mathematics in Geography**

Geography is nothing but a scientific and mathematical description of our earth in its universe. The dimension and magnitude of earth, its situation and position in the universe the formation of days and nights, lunar and solar eclipses, latitude and longitude, maximum and minimum rainfall, etc are some of the numerous learning areas of geography which need the application of mathematics. The surveying instruments in geography have to be mathematically accurate. There are changes in the fertility of the soil, changes in the distribution of forests, changes in ecology etc., which have to be mathematically determined, in order to exercise desirable control over them.

# Student's Activity

Cite the mathematical application that you commonly do in each of the following stations and state your appreciation.

| Stations | Application of and Appreciation for Mathematics |
|---|---|
| ***1. Market*** | |
| ***2. Bus/ Jeepney*** | |
| ***3. Church*** | |
| ***4. Club meeting*** | |
| ***5. Clinic*** | |
| ***6. Court*** | |
| ***7. Laboratory*** | |
| ***8. Birthday Party*** | |
| ***9. Watching games*** | |
| ***10. Police Station*** | |


Submit 2-3-page synthesis paper on the following aspects of mathematics:

1. Mathematics helps predict the behavior of nature and phenomena in the world.



2. Mathematics has numerous applications in the world making it indispensable.

Use the rubric for grading the output.

| 0-point | 10-point | 20-point | 30-point | 40-point |
|---|---|---|---|---|
| The student is unable to elicit the ideas and concepts from the readings/ video indicating that s/he has not read the prescribed reading or watched the video. | The student is able to elicit the ideas and concepts from the readings/ videos but shows erroneous understanding of these. | The student is able to elicit the ideas and concepts from the readings/ video shows correct understanding of these. | The student not only elicits correct ideas from the readings/ video but also shows evidence of internalizing these. | The student elicits the correct ideas from the readings/ video, shows evidence of internalizing these, consistently contributes additional thoughts to the core idea. |

# U N I T  T E S T

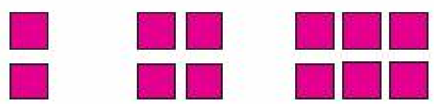
A. **Answer the following and show your solution.**

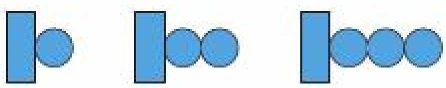
1. Find the common difference of the sequence $\frac{1}{5}, \frac{3}{5}, 1, \frac{7}{5}$.
2. What are the five terms of the geometric sequence whose first term is 2 and the common ratio is 3.
3. If $a_1 = 5$ and $a_7 = 17$, find the common difference.
4. Find the 18th term of the geometric sequence x, 2x, 4x, ….
5. What is the 11th term of the arithmetic sequence 3, 4, 5, ….15?

**B. Multiple Choice.**

1. Which number is next in the Fibonacci sequence of numbers:
   0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144,…
   A. 355 B. 434 C. 233 D. 177
2. Which of the following is NOT an example of Fibonacci numbers found in nature?
   A. spirals on a sunflower C. the number of petals on a daisy
   B. pinecone spirals D. a mountain range
3. What is the 10th number in the Fibonacci Sequence?
   A. 34 B. 55 C. 8 D. 1.62
4. What is one way to decide if two numbers follow a Fibonacci sequence?
   A. if their sum is the same as their difference
   B. if their ratio is approximately the golden ratio
   C. if each number is prime
   D. if their product is approximately the golden ratio
5. The length of a golden rectangle is approximately 8 cm. Which of the following measures could be the width of the rectangle?
   A. 4 cm B. 10 cm C. 5 cm D. 16 cm

**C. Draw the picture that comes next in each growing pattern.**

1.  ______________

2.  ______________

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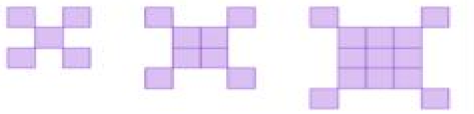
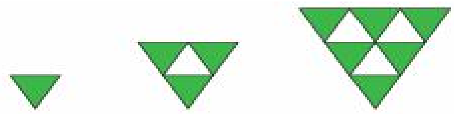
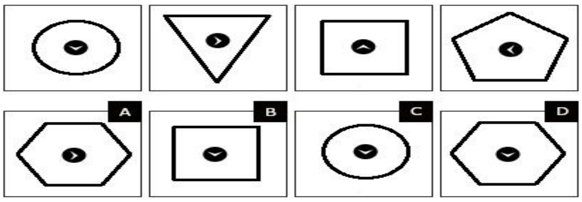
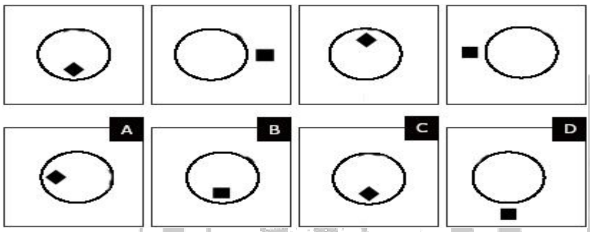
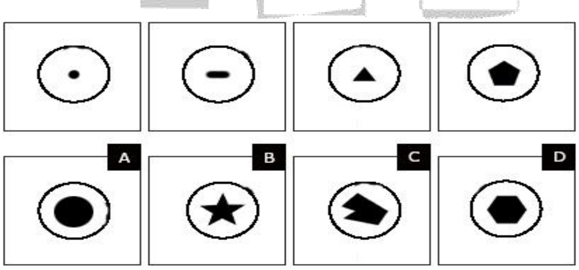
4.  __________

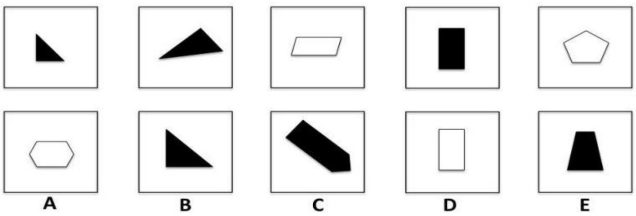
5.  __________

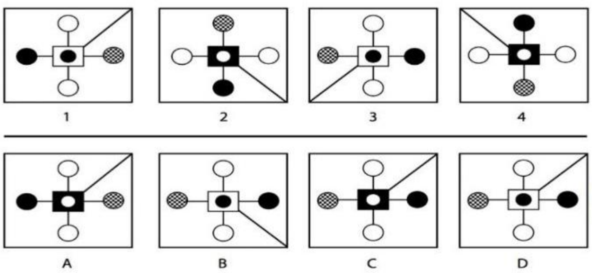
**D. Identify the next shape in the pattern.**

1. 


2. 


3. 


4. 


5. 

**E. EXPLANATION:**

1. Give for example where Fibonacci sequence or the Golden ratio is used in relation to your major field of specialization.
2. What new ideas about mathematics did you learn?

**F. What role is played by Mathematics in the following areas?**

a. Economics

_______________________________________________
_______________________________________________
_______________________________________________
_______________________________________________
_______________________________________________

b. Music

_______________________________________________
_______________________________________________
_______________________________________________
_______________________________________________
_______________________________________________

c. Psychology

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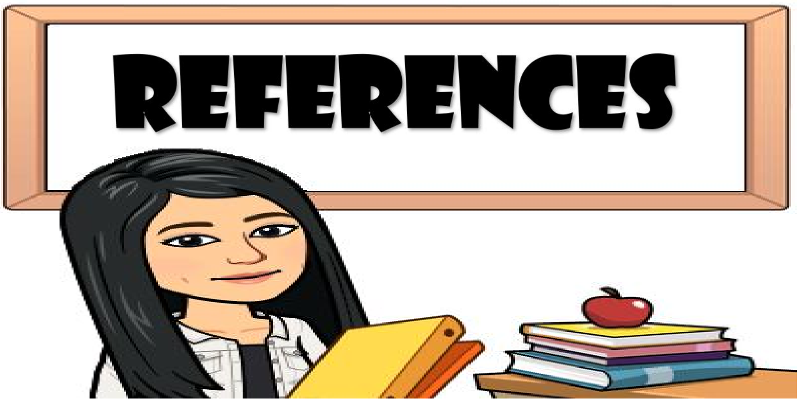



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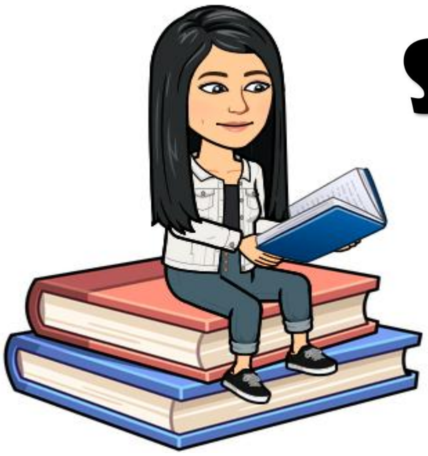

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