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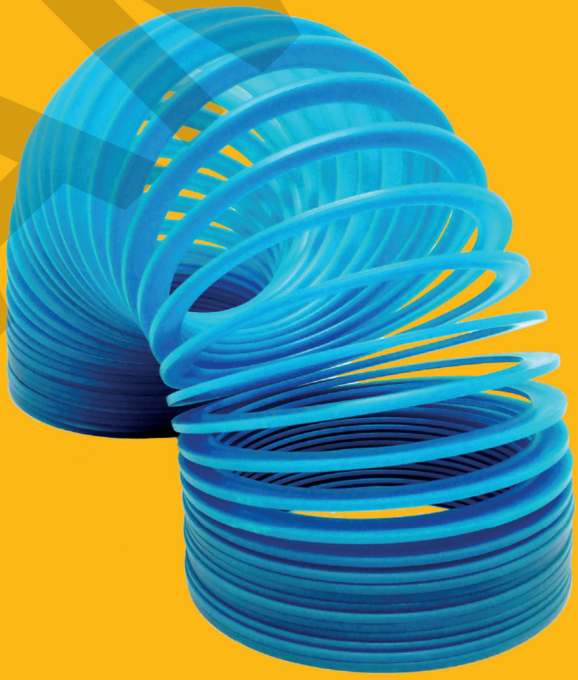
Executive
Preview

Cambridge IGCSE™

Mathematics
Core and Extended

MULTI-COMPONENT SAMPLE



Cambridge Assessment
International Education

Endorsed for full syllabus coverage

Dear Cambridge Teacher,

Welcome to the third edition of our *Cambridge IGCSE™ Mathematics Core and Extended* series, which supports the revised Cambridge IGCSE and IGCSE (9–1) Mathematics syllabuses (0580/0980) for examination from 2025. We have developed this new edition through extensive research with teachers around the world to provide you and your learners with the support you need, where you need it. You can be confident that this series supports all aspects of the revised syllabuses.

This Executive Preview contains sample content from the series, including:

- A guide explaining how to use the series
- A guide explaining how to use each resource
- The table of contents from each resource

We have updated the 'Worked examples' alongside engaging exercise sets, to include a step-by-step process of working through a question or problem. A 'Link' feature presents real-world examples to relate mathematics to everyday life. Further student support comes in the form of a 'Mathematical connections' feature, providing connections between coursebook topics and mathematical skills. With the move to a non-calculator paper in the assessment for both Core and Extended students, we have also flagged some questions with a non-calculator icon to provide learners with additional practice opportunities and help them feel prepared.

We are pleased to include a series of investigative projects authored by NRICH (a collaboration between the Faculties of Mathematics and Education at the University of Cambridge). Five of these projects appear in the coursebook to facilitate pair, small group and whole-class work. The digital teacher's resource provides full support and guidance on these projects. Our digital teacher's resource focuses on key syllabus points and pedagogical approaches. The practice book for learners is offered as a *Core Practice Book* and a separate *Extended Practice Book* to differentiate and offer sufficient challenge – find out more in our resource guide pages.

We are also delighted to present our new digital formative assessment tool. Accessed through your Cambridge GO subscription, this resource helps you gauge prior student knowledge and overcome student misconceptions.

Finally, we are happy to introduce Cambridge Online Mathematics, hosted on our Cambridge GO platform. Cambridge Online Mathematics provides enhanced teacher and student support; it can be used to create virtual classrooms allowing you to blend print and digital resources into your teaching, in the classroom or as homework. Cambridge Online Mathematics contains all coursebook content in a digital format, an additional 4,000 quiz questions, worksheets, guided walkthroughs of new skills and reporting functionality for teachers. The platform is easy to use, tablet-friendly and flexible.

We hope you enjoy this new series of resources. Visit our website to view the full series or speak to your local sales representative. You can find their details here:

cambridge.org/gb/education/find-your-sales-consultant

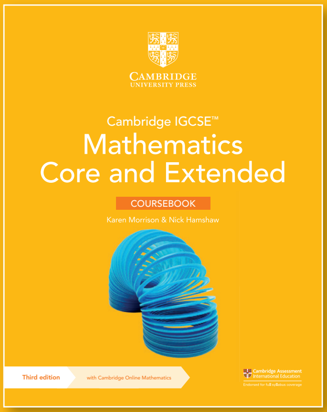
With best wishes from the Cambridge team,

Arifah A. Khan

Commissioning Editor for IGCSE™ Mathematics Core and Extended
Cambridge University Press and Assessment

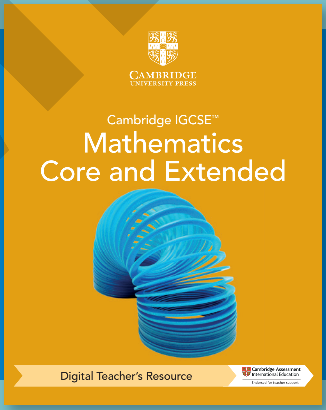
How to use this series

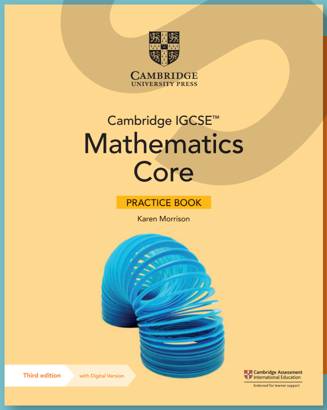
This suite of resources supports learners and teachers following the Cambridge IGCSE™ and IGCSE (9–1) Mathematics syllabuses (0580/0980). Up-to-date metacognition techniques have been incorporated throughout the resources to meet the changes in the syllabus and develop a complete understanding of mathematics for learners. All of the components in the series are designed to work together.



The coursebook contains six units that together offer complete coverage of the syllabus. We have worked with NRICH to provide a variety of new project activities, designed to engage learners and strengthen their problem-solving skills. A new Mathematical Connections feature creates a holistic view of mathematics to help learners identify links between themes and topics. Each chapter contains opportunities for formative assessment, differentiation and peer and self-assessment offering learners the support needed to make progress. Cambridge Online Mathematics is available through the digital/print bundle option or on its own without the print coursebook. Learners can review content digitally, explore worked examples and test their knowledge with practice questions and answers. Teachers benefit from the ability to set tests and tasks with the added auto-marking functionality and a reporting dashboard to help track learner progress quickly and easily.

The digital teacher's resource provides extensive guidance on how to teach the course, including suggestions for differentiation, formative assessment and language support, teaching ideas and PowerPoints. The Teaching Skills Focus shows teachers how to incorporate a variety of key pedagogical techniques into teaching, including differentiation, assessment for learning, and metacognition. Answers for all components are accessible to teachers for free on the Cambridge GO platform.

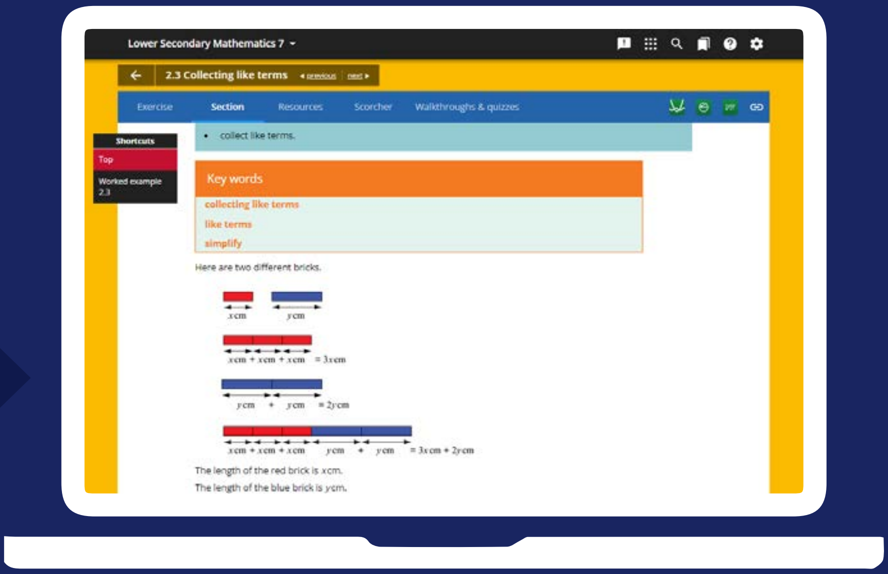




There are two practice books available, one for the core content of the syllabus and the other for learners studying extended content. These resources, which can be used in class or assigned as homework, provide a wide variety of extra maths activities and questions to help learners consolidate their learning and prepare for assessment. 'Tips' are also regularly featured to give learners extra advice and guidance on the different areas of maths they encounter. Access to the digital versions of the practice books is included, and answers can be found either here or in the back of the books.

Cambridge Online Mathematics

Discover our enhanced digital mathematics support for Cambridge Lower Secondary, Cambridge IGCSE™ and Cambridge International AS & A Level Mathematics – endorsed by Cambridge Assessment International Education.



- collect like terms.

Here are two different bricks.

x cm y cm

x cm + x cm + x cm = $3x$ cm

y cm + y cm = $2y$ cm

x cm + x cm + x cm y cm + y cm = $3x$ cm + $2y$ cm

The length of the red brick is x cm.

The length of the blue brick is y cm.

Available in 2023

New content to support the following syllabuses:

- Cambridge IGCSE Mathematics
- Cambridge IGCSE International Mathematics
- Cambridge IGCSE and O Level Additional Mathematics

Features can include:

- Guided walkthroughs of key mathematical concepts for students
- Teacher-set tests and tasks with auto-marking functionality
- A reporting dashboard to help you track student progress quickly and easily
- A test generator to help students practise and refine their skills – ideal for revision and consolidating knowledge

Free trials

A free trial will be available for Cambridge IGCSE Mathematics in 2023. In the meantime, please visit **https://bit.ly/3TUGI4l** for a free trial of our Cambridge Lower Secondary and Cambridge International AS & A Level Mathematics versions.

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Cambridge IGCSE™
Mathematics Core and Extended

COURSEBOOK

Karen Morrison & Nick Hamshaw

Third edition

Digital Access

Cambridge Assessment International Education
Endorsed for full syllabus coverage

Contents

Unit 6

How to use this book

Throughout this book, you will notice lots of different features that will help your learning. These are explained below.

IN THIS CHAPTER YOU WILL:

These set the scene for each chapter, help with navigation through the coursebook and indicate the important concepts in each topic.

Extended Content

Where content is intended for students who are studying the Extended content of the syllabus as well as the Core, this is indicated using the arrow and the bar, as on the left here.

GETTING STARTED

These boxes contain questions and activities on subject knowledge you will need before starting this chapter.

KEY WORDS

The key vocabulary appears in a box at the start of each chapter, and is highlighted in the text when it is first introduced. You will also find definitions of these words in the Glossary at the back of this Coursebook.

TIP

The information in this feature will help you complete the exercises, and give you support in areas that you might find challenging or confusing.

APPLY YOUR SKILLS

These activities give you an opportunity to apply your understanding of a concept to a real-world context. You can find answers to these questions in the digital version of the Coursebook.

Exercise 9.1

Appearing throughout the text, exercises give you a chance to check that you have understood the topic you have just learned about and practise the mathematical skills you have learned. You can find the answers to these questions in the digital version of the Coursebook on Cambridge GO.

INVESTIGATION/DISCUSSION

These boxes contain questions and activities that will allow you to extend your learning by investigating a problem, or by discussing it with classmates.

WORKED EXAMPLE 4

These boxes show you the step-by-step process to work through an example question or problem, giving you the skills to work through questions yourself.

LINK

This feature presents real-world examples and applications of the content in a chapter, encouraging you to look further into topics. Many of these examples, particularly ones that link to other syllabus subjects, extend beyond the syllabus and are presented solely for interest.

REFLECTION

These activities ask you to think about the approach that you take to your work, and how you might improve this in the future.

This icon shows you where you should complete an exercise without using your calculator.

MATHEMATICAL CONNECTIONS

This feature will help you to link content in the chapter to what you have already learned, and highlights where you will use your understanding again in the course.

Practice Questions

Questions at the end of each chapter provide more demanding practice, some of which may require use of knowledge from previous chapters. Answers to these questions can be found in the digital version of the Coursebook on Cambridge GO.

SUMMARY

There is a summary of key points at the end of each chapter.

SELF/PEER ASSESSMENT

At the end of some exercises you will find opportunities to help you assess your own work, or that of your classmates, and consider how you can improve the way you learn.

Projects from NRICH allow you to apply your learning from several chapters. They may give you the opportunity to extend your learning beyond the syllabus if you want to.

Past paper questions at the end of each unit give further practice in applying your learning from the previous chapters. Although all past paper questions were taken from calculator-based papers, we have marked some of the questions as non-calculator to indicate that you could try to answer these without your calculator for additional practice.

Answers to these questions can be found on Cambridge GO.

If a question asks you to complete a diagram/table/graph, you can find a printable copy of this in the Past Paper Questions Resource Sheets, which are available to download from Cambridge GO.

Chapter 1

Review of number concepts

IN THIS CHAPTER YOU WILL:

- identify and classify different types of numbers
- find common factors and common multiples of numbers
- write numbers as products of their prime factors
- work with integers used in real-life situations
- calculate with powers and roots of numbers
- understand the meaning of indices
- use the rules of indices
- revise the basic rules for operating with numbers
- perform basic calculations using mental methods and with a calculator
- round numbers in different ways to estimate and approximate answers.

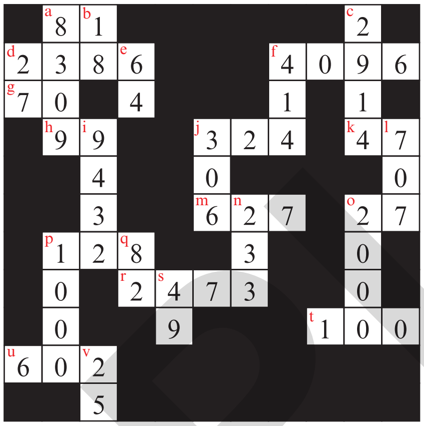
GETTING STARTED

1 A lot of the work in this chapter is revision. Look through the chapter to see what is covered.

- **a** Are there any parts of this chapter that you could confidently skip? Explain why.
- **b** If you only had to do three topics in this chapter, which would you choose? Why?

2 Look at this completed cross-number puzzle.

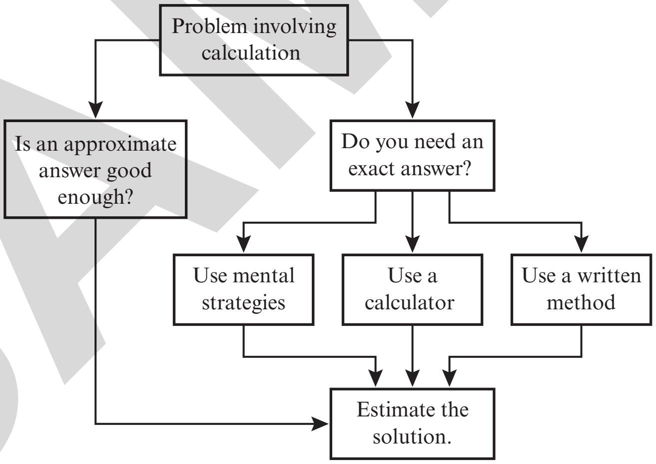
- **a** Write a set of clues for the puzzle. Each clue should include at least one of the concepts from this chapter.
- **b** Find the sum of the three greatest numbers. Write the answer in words.



3 Write each of the following using only numbers and brackets if needed.

- **a** nine cubed
- **b** twelve squared
- **c** seven to the power of five
- **d** the reciprocal of three to the power of two
- **e** the reciprocal of three-quarters to the power of zero
- **f** nine to the power of half
- **g** fourteen billion, ten thousand and nineteen

4 Look at this decision diagram for problems involving calculation.



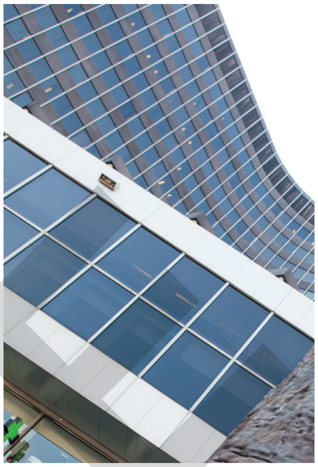
- **a** Give an example of a problem where an approximate answer is good enough.
- **b** How do you decide which method to use when an exact answer is needed?
- **c** Estimates are useful for all of the methods in this decision tree. How could you convince someone that it is important to estimate even if you can use a calculator?

KEY WORDS

base
composite number
cube
cube root
exponent
factor
index
index notation
integer
irrational number
multiple
power
prime factor
prime number
rational number
reciprocal
square number
square root

The statue shown in the photograph is a replica of a 22 000-year-old bone found in the Congo Basin. The real bone is only 10 cm long and it is carved with groups of notches that represent numbers. One column lists the prime numbers from 10 to 20. It is one of the earliest examples of a number system using tallies. What do you think ancient civilisations used tallies for?

Our modern number system is called the Hindu-Arabic system because it was developed by Hindus and spread by Arab traders who brought it with them when they moved to different places in the world. The Hindu-Arabic system is decimal. This means it uses place value based on powers of ten. Any number at all, including decimals and fractions, can be written using place value and the digits from 0 to 9.



1.1 Different types of numbers

Make sure you know the correct mathematical words for the types of numbers in the table.

Number	Definition	Example
Natural number	Any whole number from 1 to infinity, sometimes called 'counting numbers'. 0 is not included.	1, 2, 3, 4, 5, …
Odd number	A whole number that cannot be divided exactly by 2.	1, 3, 5, 7, …
Even number	A whole number that can be divided exactly by 2.	2, 4, 6, 8, …
Integer	Any of the negative and positive whole numbers, including zero.	… −3, −2, −1, 0, 1, 2, 3, …
Prime number	A whole number greater than 1 which has only two factors: the number itself and 1.	2, 3, 5, 7, 11, …
Square number	The product obtained when an integer is multiplied by itself.	1, 4, 9, 16, …
Fraction	A number representing part of a whole number, written in the form $\frac{a}{b}$, where a and b are non-zero integers.	$\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{8}, \frac{13}{3}$
Decimal	A number that used place value and a decimal point to show a fraction.	0.5, 0.2, 0.08, 1.7

TIP

'Find the product' means 'multiply'. So, the product of 3 and 4 is 12, i.e. 3 × 4 = 12.

The set of real numbers is made up of **rational numbers** and **irrational numbers**.

Rational numbers can be written as fractions in the form $\frac{a}{b}$ where a and b are non-zero integers. The set of rational numbers includes all integers, all fractions, all terminating decimals and all recurring decimals.

Irrational numbers cannot be written as fractions. The set of irrational numbers consists of non-terminating, non-recurring decimals. The square root of a non-square number (such as $\sqrt{2}$), the cube root of a non-cube number (such as $\sqrt[3]{12}$) and π are all irrational numbers.

MATHEMATICAL CONNECTIONS

You will deal with rational and irrational numbers in more detail in Chapter 9.

LINK

Some numbers, for example $\sqrt{-1}$ and other roots of negative numbers, are not real numbers. They are neither rational nor irrational. Mathematicians call these imaginary numbers and you may learn about them if you study maths beyond Cambridge IGCSE.

Exercise 1.1

1 Here is a set of numbers: $\{-4, -1, 0, \frac{1}{2}, 0.75, 3, 4, 6, 11, 16, 19, 25\}$

List the numbers from this set that are:

a natural numbers

b even numbers

c odd numbers

d integers

e negative integers

f fractions

g square numbers

h prime numbers

i neither square nor prime.

2 List:

a the next four odd numbers after 107

b four consecutive even numbers between 2008 and 2030

c all odd numbers between 993 and 1007

d the first five square numbers

e four decimal fractions that are smaller than 0.5

f four common fractions that are greater than $\frac{1}{2}$ but smaller than $\frac{3}{4}$.

3 State whether the following will be odd or even.

a the sum of two odd numbers

b the sum of two even numbers

c the sum of an odd and an even number

d the square of an odd number

e the square of an even number

f an odd number multiplied by an even number

INVESTIGATION

4 There are many other types of numbers. Find out what these numbers are and give an example of each.

a Perfect numbers

b Palindromic numbers

c Narcissistic numbers (in other words, numbers that love themselves!)

MATHEMATICAL CONNECTIONS

You will learn much more about sets in Chapter 9. For now, just think of a set as a list of numbers or other items that are often placed inside curly brackets.

TIP

Remember that a 'sum' is the result of an addition. The term is often used for *any* calculation in early mathematics, but its meaning is very specific at this level.

TIP

Being able to communicate information effectively is a key 21st-century skill. As you work, think about what you are being asked to do in this task and how best to present your answers.

Using symbols to link numbers

Mathematicians use numbers and symbols to write mathematical information in the shortest, clearest way possible.

Exercise 1.2

1 Rewrite each of these statements using mathematical symbols.

- **a** 19 is less than 45
- **b** 12 plus 18 is equal to 30
- **c** 0.5 is equal to $\frac{1}{2}$
- **d** 0.8 is not equal to 8.0
- **e** -34 is less than 2 times -16
- **f** therefore the number x equals the square root of 72
- **g** a number (x) is less than or equal to negative 45
- **h** π is approximately equal to 3.14
- **i** 5.1 is greater than 5.01
- **j** the sum of 3 and 4 is not equal to the product of 3 and 4
- **k** the difference between 12 and -12 is greater than 12
- **l** the sum of -12 and -24 is less than 0
- **m** the product of 12 and a number (x) is approximately -40

TIP

Remember:
$=$ is equal to
$\neq$ is not equal to
$<$ is less than
$\leqslant$ is less than or equal to
$>$ is greater than
$\geqslant$ is greater than or equal to
$\therefore$ therefore
$\sqrt{}$ the positive square root of

2 Say whether these mathematical statements are true or false.

- **a** $0.599 > 6.0$
- **b** 5×1999 is approximately equal to 10 000
- **c** $8.1 = 8\frac{1}{10}$
- **d** $6.2 + 4.3 = 4.3 + 6.2$
- **e** $20 \times 9 \geqslant 21 \times 8$
- **f** $6.0 = 6$
- **g** $-12 > -4$
- **h** $19.9 \leqslant 20$
- **i** $1000 > 199 \times 5$
- **j** $\sqrt{16} = 4$
- **k** $35 \times 5 \times 2 \neq 350$
- **l** $20 \div 4 = 5 \div 20$
- **m** $20 - 4 \neq 4 - 20$
- **n** $20 \times 4 \neq 4 \times 20$

INVESTIGATION

3 Work with a partner.

- **a** Look at the symbols used on the keys of your calculator. Say what each one means in words.
- **b** List any symbols that you do not know. Try to find out what each one means.

1.2 Multiples and factors

Multiples

A **multiple** of a number is found when you multiply that number by a positive integer. You can think of the multiples of a number as the 'times table' for that number. For example, the multiples of 3 are 3 × 1 = **3**, 3 × 2 = **6**, 3 × 3 = **9** and so on. The first multiple of any number is the number itself.

WORKED EXAMPLE 1

a What are the first three multiples of 12?

b Is 300 a multiple of 12?

Answers

a 12, 24, 36

Multiply 12 by 1, 2 and then 3.
$12 \times 1 = 12$
$12 \times 2 = 24$
$12 \times 3 = 36$

b Yes, 300 is a multiple of 12.

Divide 300 by 12. If it goes exactly, then 300 is a multiple of 12.
$300 \div 12 = 25$

Exercise 1.3

1 List the first five multiples of:
  a 12 **b** 3 **c** 5 **d** 8
  e 9 **f** 10 **g** 12 **h** 100

2 Use a calculator to find and list the first ten multiples of:
  a 29 **b** 44 **c** 75 **d** 114
  e 299 **f** 350 **g** 1012 **h** 9123

3 List:
  a the multiples of 4 between 29 and 53
  b the multiples of 50 less than 400
  c the multiples of 100 between 4000 and 5000.

4 Here are five numbers: 576, 396, 354, 792, 1164. Which of these are multiples of 12?

5 Which of the following numbers are not multiples of 27?

324 783 816 837 1116

The lowest common multiple (LCM)

The lowest common multiple of two or more numbers is the smallest number that is a multiple of all the given numbers.

WORKED EXAMPLE 2

Find the lowest common multiple of 4 and 7.

Answer

M_4 = 4, 8, 12, 16, 20, 24, **28**, 32	List several multiples of 4.
M_7 = 7, 14, 21, **28**, 35, 42	List several multiples of 7.
LCM = 28	Find the lowest number that appears in both sets. This is the LCM.

TIP

M_4 means the multiples of 4.

MATHEMATICAL CONNECTIONS

Later in this chapter you will see how prime factors can be used to find LCMs.

Exercise 1.4

1 Find the lowest common multiple of:

a 2 and 5 **b** 8 and 10 **c** 6 and 4
d 3 and 9 **e** 35 and 55 **f** 6 and 11

2 Is it possible to find the highest common multiple of two or more numbers? Give a reason for your answer.

Factors

A **factor** is a number that divides exactly into another number with no remainder. For example, 2 is a factor of 16 because it goes into 16 exactly 8 times. 1 is a factor of every number. The largest factor of any number is the number itself.

WORKED EXAMPLE 3

a 12 **b** 25 **c** 110

Answers

a F_{12} = 1, 2, 3, 4, 6, 12	Find pairs of numbers that multiply to give 12: 1 × 12 2 × 6 3 × 4
b F_{25} = 1, 5, 25	Write the factors in numerical order. 1 × 25 5 × 5 Do not repeat the 5.
c F_{110} = 1, 2, 5, 10, 11, 22, 55, 110	1 × 110 2 × 55 5 × 22 10 × 11

TIP

F_{12} means the factors of 12.

Exercise 1.5

1 List all the factors of:

a 4
b 5
c 8
d 11
e 18
f 12
g 35
h 40
i 57
j 90
k 100
l 132
m 160
n 153
o 360

2 Which number in each set is not a factor of the given number?

a 14 {1, 2, 4, 7, 14}
b 15 {1, 3, 5, 15, 45}
c 21 {1, 3, 7, 14, 21}
d 33 {1, 3, 11, 22, 33}
e 42 {3, 6, 7, 8, 14}

3 State true or false in each case.

a 3 is a factor of 313
b 9 is a factor of 99
c 3 is a factor of 300
d 2 is a factor of 300
e 2 is a factor of 122 488
f 12 is a factor of 60
g 210 is a factor of 210
h 8 is a factor of 420

4 What is the smallest factor and the largest factor of any number?

The highest common factor (HCF)

The highest common factor of two or more numbers is the highest number that is a factor of all the given numbers.

WORKED EXAMPLE 4

Find the highest common factor of 8 and 24.

Answer

$F_8 = \underline{1}, \underline{2}, \underline{4}, \underline{8}$	List the factors of each number.
$F_{24} = \underline{1}, \underline{2}, 3, \underline{4}, 6, \underline{8}, 12, 24$	Underline factors that appear in both sets.
HCF = 8	Pick out the highest underlined factor (HCF).

Exercise 1.6

1 Find the highest common factor of each pair of numbers.

a 3 and 6
b 24 and 16
c 15 and 40
d 42 and 70
e 32 and 36
f 26 and 36
g 22 and 44
h 42 and 48

2 Not including the factor provided, find two numbers less than 20 that have:

a an HCF of 2
b an HCF of 6

3 What is the highest common factor of two different prime numbers? Give a reason for your answer.

MATHEMATICAL CONNECTIONS

You will learn how to find HCFs using prime factors later in the chapter.

APPLY YOUR SKILLS

4 Simeon has two lengths of rope. One piece is 72 metres long and the other is 90 metres long. He wants to cut both lengths of rope into the longest pieces of equal length possible. How long will the pieces be?

5 Ms Sanchez has 40 canvases and 100 tubes of paint to give to the students in her art group. What is the largest number of students she can have if she gives each student an equal number of canvasses and an equal number of tubes of paint?

6 A jeweller has 300 blue beads, 750 red beads and 900 silver beads, which are used to make bracelets. Each bracelet must have the same number and colour of beads. What is the maximum number of bracelets that can be made with these beads?

TIP

Recognising the type of problem helps you to choose the correct mathematical techniques for solving it.

Word problems involving HCF usually involve splitting things into smaller pieces or arranging things in equal groups or rows.

1.3 Prime numbers

Prime numbers have exactly two different factors: one and the number itself.

Composite numbers have more than two factors.

The number 1 has only one factor so it is not prime and it is not composite.

MATHEMATICAL CONNECTIONS

Later in this chapter you will learn how to write integers as products of prime factors. One of the reasons why it is important for 1 to NOT be defined is prime is to make sure that the prime factorisation of any number is unique.

Finding prime numbers

Over 2000 years ago, a Greek mathematician called Eratosthenes made a simple tool for sorting out prime numbers. This tool is called the 'Sieve of Eratosthenes' and the diagram shows how it works for prime numbers up to 100.

~~1~~	②	③	~~4~~	⑤	~~6~~	⑦	~~8~~	~~9~~	~~10~~
⑪	~~12~~	⑬	~~14~~	~~15~~	~~16~~	⑰	~~18~~	⑲	~~20~~
~~21~~	~~22~~	㉓	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	㉙	~~30~~
㉛	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	㊲	~~38~~	~~39~~	~~40~~
㊶	~~42~~	㊸	~~44~~	~~45~~	~~46~~	㊼	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	(53)	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	(59)	~~60~~
(61)	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	(67)	~~68~~	~~69~~	~~70~~
(71)	~~72~~	(73)	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	(79)	~~80~~
~~81~~	~~82~~	(83)	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	(89)	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	(97)	~~98~~	~~99~~	~~100~~

Cross out 1, it is not prime.

Circle 2, then cross out other multiples of 2.

Circle 3, then cross out other multiples of 3.

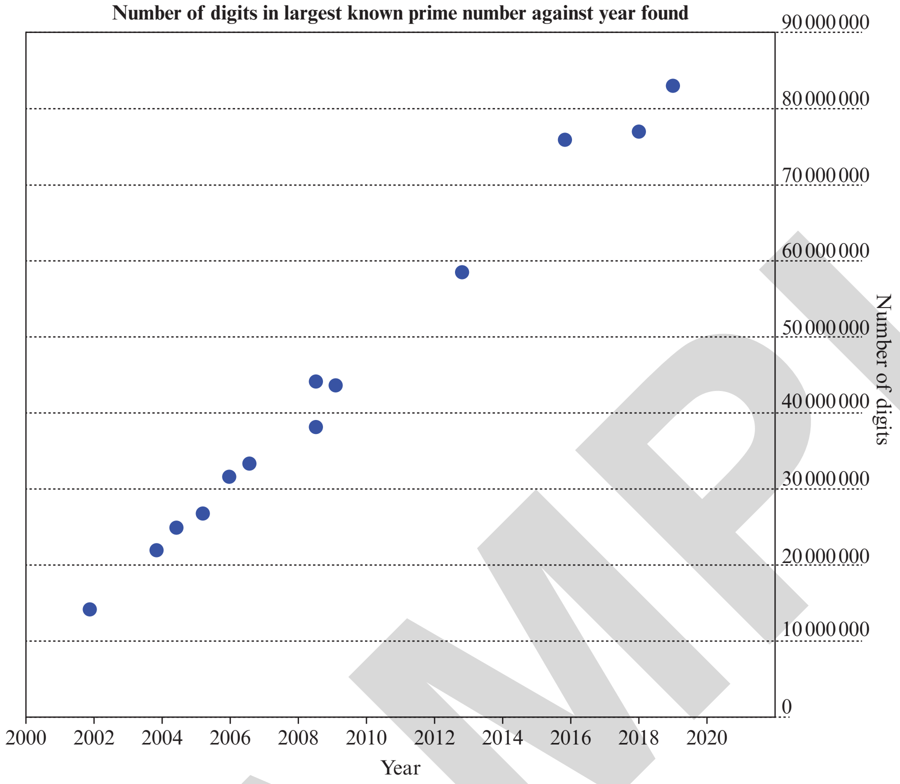
Circle the next available number then cross out all its multiples.

Repeat until all the numbers in the table are either circled or crossed out.

The circled numbers are the primes.

Other mathematicians over the years have developed ways of finding larger and larger prime numbers. Until 1955, the largest known prime number had less than 1000 digits. Since the 1970s and the invention of more and more powerful computers, more and more prime numbers have been found. The graph below shows the number of digits in the largest known primes since 2000.

You should try to memorise the prime numbers between 1 and 100.



Source: https://www.mersenne.org/primes/

LINK

Today anyone can join the search for Mersenne prime numbers. This project links thousands of home computers to search continuously for larger and larger prime numbers while the computer processors have spare capacity.

INVESTIGATION

Why do mathematicians find prime numbers exciting?

One reason why prime numbers are interesting and intriguing is because there is a lot about them that we don't know and that mathematicians have not been able to prove.

1 Goldbach's conjecture (1742) is one of the oldest and best-known unsolved problems in number theory.

  a What is Goldbach's strong conjecture?

  b A Peruvian mathematician, Harald Helfgott, has published a largely accepted proof of Goldbach's weak conjecture. Find out more about this.

LINK

Prime numbers are used in codes and codebreaking. The larger the prime you use, the harder it is to break the code. This is why it is more and more important to find larger and larger primes.

INVESTIGATION CONTINUED

2 The Mersenne prime number search relies on massive computing power to find large primes. There is no other way to work out where the *n*th prime number will be or what the distance between large primes will be. Riemann's hypothesis (1859) claims you can accurately pinpoint the distribution of prime numbers. An Indian mathematician, Dr Kumar Eswaran published a proof for this hypothesis in 2016, but it has received mixed responses and is not yet fully accepted.

- **a** Riemann built his ideas on the prime number theorem. Find out what this is and express it in simple language.
- **b** Is there a proof for the existence of infinitely-many prime numbers?

3 And just for fun … What is an emirp? Find some examples to show what these are.

Exercise 1.7

1 Which is the only even prime number?

2 How many odd prime numbers are there that are less than 50?

3 **a** List the composite numbers greater than four, but less than 30.

b Try to write each composite number on your list as the sum of two prime numbers. For example: 6 = 3 + 3 and 8 = 3 + 5.

4 Twin primes are pairs of prime numbers that differ by two. List the twin prime pairs up to 100.

5 Is 149 a prime number? Explain how you decided.

Prime factors

Prime factors are the factors of a number that are also prime numbers.

Every composite whole number can be broken down and written as the product of its prime factors. You can do this using tree diagrams or using division. Both methods are shown in Worked example 5.

TIP

Remember, a product is the answer to a multiplication. So to write a number as the product of its prime factors you write it like this:
12 = 2 × 2 × 3.

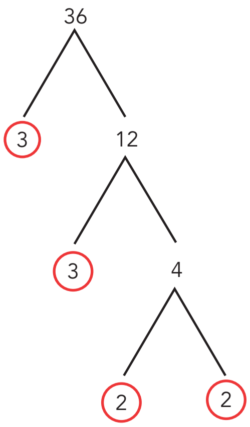
WORKED EXAMPLE 5

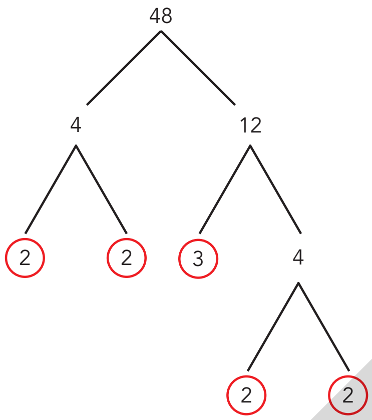
Write the following numbers as the product of prime factors.

a 36 **b** 48

Answers

Using a factor tree





Write the number as two factors.

If a factor is a prime number, circle it.

If a factor is a composite number, split it into two factors.

Keep splitting until you end up with two primes.

TIP

Prime numbers only have two factors: 1 and the number itself. As 1 is not a prime number, do not include it when expressing a number as a product of prime factors.

$36 = 2 \times 2 \times 3 \times 3$

$48 = 2 \times 2 \times 2 \times 2 \times 3$

Write the primes in ascending order with × signs.

Using division

2	36
2	18
3	9
3	3
	1

2	48
2	24
2	12
2	6
3	3
	1

Divide by the smallest prime number that will go into the number exactly.

Continue dividing, using the smallest prime number that will go into your new answer each time.

Stop when you reach 1.

$36 = 2 \times 2 \times 3 \times 3$

$48 = 2 \times 2 \times 2 \times 2 \times 3$

Write the prime factors in ascending order with × signs.

TIP

Choose the method that works best for you and stick to it. Always show your method when using prime factors.

Exercise 1.8

1 Express the following numbers as the product of prime factors.

a	30	**b**	24	**c**	100	**d**	225	**e**	360
f	504	**g**	650	**h**	1125	**i**	756	**j**	9240

TIP

When you write your number as a product of primes, group all occurrences of the same prime number together.

Using prime factors to find the HCF and LCM

When you are working with larger numbers you can determine the HCF or LCM by expressing each number as a product of its prime factors.

WORKED EXAMPLE 6

Find the HCF of 168 and 180.

Answer

$168 = \underline{2} \times \underline{2} \times 2 \times \underline{3} \times 7$	First express each number as a product of prime factors. Use tree diagrams or division to do this.
$180 = \underline{2} \times \underline{2} \times \underline{3} \times 3 \times 5$	Underline the factors common to both numbers.
$2 \times 2 \times 3 = 12$	Multiply these out to find the HCF.
HCF = 12	

WORKED EXAMPLE 7

Find the LCM of 72 and 120.

Answer

$72 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3}$	First express each number as a product of prime factors. Use tree diagrams or division to do this.
$120 = 2 \times 2 \times 2 \times 3 \times \underline{5}$	Underline the largest set of multiples of each factor.
$2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$	List these and multiply them out to find the LCM.
LCM = 360	

MATHEMATICAL CONNECTIONS

You can also use prime factors to find the square and cube roots of numbers if you don't have a calculator. You will deal with this in more detail later in this chapter.

Exercise 1.9

1 Find the HCF of these numbers by using prime factors.

a 48 and 108 **b** 120 and 216 **c** 72 and 90 **d** 52 and 78
e 100 and 125 **f** 154 and 88 **g** 546 and 624 **h** 95 and 120

2 Use prime factorisation to determine the LCM of the following numbers.

a 54 and 60 **b** 54 and 72 **c** 60 and 72 **d** 48 and 60
e 120 and 180 **f** 95 and 150 **g** 54 and 90 **h** 90 and 120

3 Determine both the HCF and LCM of the following numbers.

a 72 and 108 **b** 25 and 200 **c** 95 and 120 **d** 84 and 60

APPLY YOUR SKILLS

4 A radio station runs a phone-in competition for listeners. Every 30th caller gets a free airtime voucher and every 120th caller gets a free mobile phone. How many listeners must phone in before one receives both an airtime voucher *and* a free phone?

5 Li runs round a track in 12 minutes. Jaleel runs round the same track in 18 minutes. If they start together, how many minutes will pass before they both cross the start line together again?

TIP

Recognising the type of problem helps you to choose the correct mathematical techniques for solving it.

Word problems involving LCM usually include repeating events. You may be asked how many items you need to 'have enough' or when something will happen again at the same time.

6 The number p can be written as a product of the three prime numbers x, y and z, where x, y and z are all different.

a How many factors does the number p have?

Another number q can be written as the product of four different primes.

b How many factors does q have?

The number r can be written as a product of n different prime numbers.

c How many factors does r have?

1.4 Working with directed numbers

When you use numbers to represent real-life situations like temperatures, altitude, depth below sea level, profit or loss and directions (on a grid), you sometimes need to use the negative sign to indicate the direction of the number. For example, you can show a temperature of three degrees below zero as $-3\,°\text{C}$. Numbers like these, which have direction, are called directed numbers. So if a point 25 m above sea level is at +25 m, then a point 25 m below sea level is at −25 m.

TIP

Once a direction is chosen to be positive, the opposite direction is taken to be negative. So:

- if up is positive, down is negative
- if right is positive, left is negative
- if north is positive, south is negative
- if above 0 is positive, below 0 is negative.

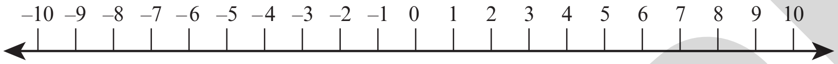
Exercise 1.10

1 Express each of these situations using a directed number.

a a profit of \$100
b 25 km below sea level
c a drop of 10 marks
d a gain of 2 kg
e a loss of 1.5 kg
f 8000 m above sea level
g a temperature of 10 °C below zero
h a fall of 24 m
i a debt of \$2000
j an increase of \$250
k a time two hours behind local time
l a height of 400 m

Calculating with directed numbers

In mathematics, directed numbers are also known as integers. You can represent the set of integers on a number line like this:



The further to the right a number is on the number line, the greater its value.

LINK

Directed numbers are important when describing temperatures. The Celsius (or centigrade) temperature scale places the temperature at which water freezes at zero. Positive temperatures indicate 'above freezing' and are warmer. Negative temperatures are 'below freezing' and are colder.

When you calculate with negative and positive integers, you need to pay attention to the signs and remember these rules:

- Adding a negative number is the same as subtracting the number. $3 + -5 = -2$
- Subtracting a negative number is the same as adding a positive number. $3 - -5 = 8$
- Multiplying or dividing the same signs gives a positive answer. $-3 \times -5 = 15$ and $-20 \div -4 = -5$
- Multiplying or dividing different signs gives a negative answer. $3 \times -5 = 15$ and $15 \div -3 = -5$.

MATHEMATICAL CONNECTIONS

You will use similar number lines when solving linear inequalities in Chapter 14.

TIP

Your calculator will have a [+/−] key that allows you to enter negative numbers. Make sure you know which key this is.

Exercise 1.11

1 Copy the numbers and fill in < or > to make a true statement.

a 2 ☐ 8
b 4 ☐ 9
c 12 ☐ 3
d 6 ☐ −4
e −7 ☐ 4
f −2 ☐ 4
g −2 ☐ −11
h −12 ☐ −20
i −8 ☐ 0
j −2 ☐ 2
k −12 ☐ −4
l −32 ☐ −3
m 0 ☐ −3
n −3 ☐ 11
o 12 ☐ −89
p −3 ☐ 0

2 Arrange each set of numbers in ascending order.

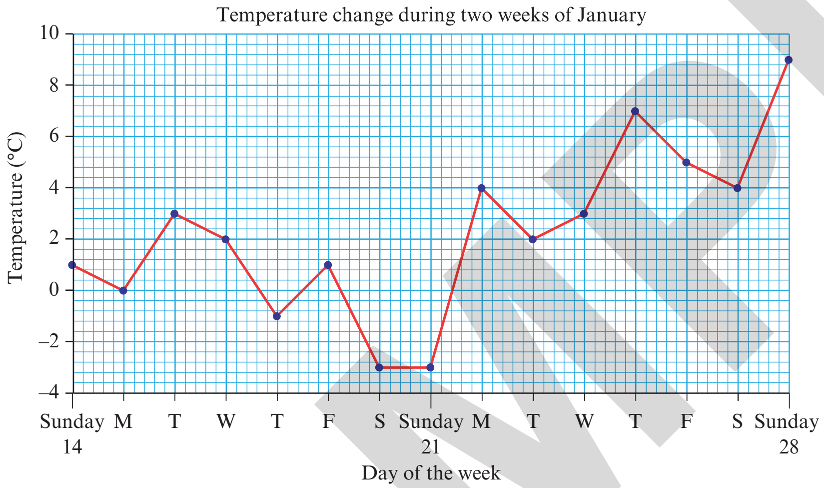
- **a** −8, 7, 10, −1, −12
- **b** 4, −3, −4, −10, 9, −8
- **c** −11, −5, −7, 7, 0, −12
- **d** −94, −50, −83, −90, 0

3 Write down the missing integer in each of these calculations.

- **a** $7 + \square = 3$
- **b** $-1.7 + \square = 8.3$
- **c** $-7 + \square = -21$
- **d** $8 - \square = 11$
- **e** $4 - \square = 6.7$
- **f** $-8 - \square = -13$
- **g** $12 \div \square = -2$
- **h** $-18 \div \square = 3$
- **i** $\square \div 3 = -9$
- **j** $-3 \times \square = 12$
- **k** $\square \times 4 = -16$
- **l** $\square \times -4 = 20$

APPLY YOUR SKILLS

4 Study the temperature graph carefully.



- **a** What was the temperature on Sunday 14 January?
- **b** By how much did the temperature drop from Sunday 14 to Monday 15?
- **c** What was the lowest temperature recorded?
- **d** What is the difference between the highest and lowest temperatures?
- **e** On Monday 29 January the temperature changed by −12 degrees. What was the temperature on that day?

5 Manu has a bank balance of \$45.50. He deposits \$15.00 and then withdraws \$32.00. What is his new balance?

6 A bank account is \$420 overdrawn.

- **a** Express this as a directed number.
- **b** How much money needs to be deposited for the account to have a balance of \$500?
- **c** \$200 is deposited. What is the new balance?

7 A diver 27 m below the surface of the water rises 16 m. At what depth is the diver now?

TIP

The difference between the highest and lowest temperature is also called the range of temperatures.

APPLY YOUR SKILLS CONTINUED

8 On a cold day in New York, the temperature at 6 a.m. was −5 °C. By noon, the temperature had risen by 8 °C. By 7 p.m. the temperature had dropped by 11 °C from its value at noon. What was the temperature at 7 p.m.?

9 Local time in Abu Dhabi is four hours ahead of local time in London. Local time in Rio de Janeiro is three hours behind local time in London.

- **a** If it is 4 p.m. in London, what time is it in Abu Dhabi?
- **b** If it is 3 a.m. in London, what time is it in Rio de Janiero?
- **c** If it is 3 p.m. in Rio de Janeiro, what time is it in Abu Dhabi?
- **d** If it is 8 a.m. in Abu Dhabi, what time is it in Rio de Janeiro?

10 A fuel tank at a workshop should be refilled when the gauge shows 0; however, there is a 100 litre reserve in the tank, so the level can drop below 0 if the tank is not filled on time.

- **a** On 3 March, the gauge indicated 412 litres above the 0 mark. On 31 March the level had dropped to −66 litres. Calculate the mean rate of fuel use per day.
- **b** On 1 April, the tank was topped up. The workshop owner estimates that this amount of fuel would be enough for 30 days, after which the level should be 0. How much fuel was added to the tank?

1.5 Powers, roots and laws of indices

You know that 2 × 2 × 2 × 2 = 16

You can write this in **index notation** as:

$2^4 = 16$

$$\text{base} \longrightarrow 2^{4} \longleftarrow \text{index}$$

2 is the **base**

4 is the **index**

The index is also called a **power** or an **exponent**.

Square numbers and square roots

A number is squared when it is multiplied by itself. For example, the square of 5 is $5 \times 5 = 25$. The symbol for squared is ^{2}. So you can write 5×5 as 5^2.

The **square root** of a number is the number that was multiplied by itself to get the square number. The symbol for square root is $\sqrt{\ }$.

You know that $25 = 5^2$, so $\sqrt{25} = 5$.

MATHEMATICAL CONNECTIONS

In Section 1.1 you learned that the product obtained when an integer is multiplied by itself is a square number.

You also know that −5 × −5 = 25. However, the mathematical convention is that the square root sign only refers to the positive square root. This is why if you enter $\sqrt{25}$ in your calculator you will always get the positive answer, 5.

If you want to indicate both the positive and negative square roots of 25 you need to write $\pm\sqrt{25}$.

MATHEMATICAL CONNECTIONS

To solve equations like $x^2 = 25$, you need to find both the positive and negative square roots, so if $x^2 = 25$, then $x = \pm\sqrt{25} = 5$ and −5.

Cube numbers and cube roots

A number is cubed when it is multiplied by itself and then multiplied by itself again. For example, the **cube** of 2 is 2 × 2 × 2 = 8. The symbol for cubed is ^{3}. So 2 × 2 × 2 can also be written as 2^3.

The **cube root** of a number is the number that was multiplied by itself to get the cube number. The symbol for cube root is $\sqrt[3]{\;}$. You know that $8 = 2^3$, so $\sqrt[3]{8} = 2$.

Finding powers and roots

You should know the squares of numbers from 1 to 15 (and their roots) and the cubes of numbers from 1 to 5 as well as they cube of 10. For other numbers, you can use your calculator to square or cube numbers quickly using the x^2 and x^3 keys or the $x^{\square}$ key. Use the $\sqrt{\;}$ or $\sqrt[3]{\;}$ keys to find the roots.

TIP

Not all calculators have exactly the same buttons. $x^{\square}$ x^y and ^ all mean the same thing on different calculators. Make sure you know how to find powers and roots on your calculator.

WORKED EXAMPLE 8

Use your calculator to find:

a 19^2 **b** 9^3 **c** $\sqrt{324}$ **d** $\sqrt[3]{512}$

Answers

a $19^2 = 361$ Enter 1 9 x^2 =

b $9^3 = 729$ Enter 9 x^3 =

c $\sqrt{324} = 18$ Enter $\sqrt{\;}$ 3 2 4 =

d $\sqrt[3]{512} = 8$ Enter $\sqrt[3]{\;}$ 5 1 2 =

If you don't have a calculator, you can use the product of prime factors method to find square and cube roots of numbers. This method is shown in Worked example 9.

WORKED EXAMPLE 9

Without using a calculator find:

a $\sqrt{324}$ **b** $\sqrt[3]{512}$

Answers

a $324 = \underbrace{2 \times 2}_{2} \times \underbrace{3 \times 3}_{3} \times \underbrace{3 \times 3}_{3}$

Group the factors into pairs, and write down the square root of each pair.

$2 \times 3 \times 3 = 18$

Multiply the roots together to get the square root of 324.

$\sqrt{324} = 18$

b $512 = \underbrace{2 \times 2 \times 2}_{2} \times \underbrace{2 \times 2 \times 2}_{2} \times \underbrace{2 \times 2 \times 2}_{2}$

Group the factors into threes, and write the cube root of each group.

$2 \times 2 \times 2 = 8$

Multiply together to get the cube root of 512.

$\sqrt[3]{512} = 8$

Exercise 1.12

1 Write down the value of:

a 3^2
b 7^2
c 11^2
d 12^2
e 100^2
f 14^2
g 1^3
h 3^3
i 4^3
j 10^3

2 Calculate:

a 21^2
b 19^2
c 32^2
d 68^2
e 6^3
f 9^3
g 100^3
h 18^3
i 30^3
j 200^3

3 Find a value of x to make each of these statements true.

a $x \times x = 25$
b $x \times x \times x = 8$
c $x \times x = 121$
d $x \times x \times x = 729$
e $x \times x = 324$
f $x \times x = 400$
g $x \times x \times x = 8000$
h $x \times x = 225$
i $x \times x \times x = 1$
j $\sqrt{x} = 9$
k $\sqrt{1} = x$
l $\sqrt{x} = 81$
m $\sqrt[3]{x} = 2$
n $\sqrt[3]{x} = 1$
o $\sqrt[3]{64} = x$

4 Use a calculator to find the following roots.

a $\sqrt{9}$
b $\sqrt{64}$
c $\sqrt{1}$
d $\sqrt{4}$
e $\sqrt{100}$
f $\sqrt{0}$
g $\sqrt{81}$
h $\sqrt{400}$
i $\sqrt{1296}$
j $\sqrt{1764}$
k $\sqrt[3]{8}$
l $\sqrt[3]{1}$
m $\sqrt[3]{-27}$
n $\sqrt[3]{64}$
o $\sqrt[3]{1000}$
p $\sqrt[3]{-216}$
q $\sqrt[3]{512}$
r $\sqrt[3]{729}$
s $\sqrt[3]{-1728}$
t $\sqrt[3]{5832}$

5 Use the given product of prime factors to find the square root of each number. Show your working.

- **a** $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$
- **b** $225 = 3 \times 3 \times 5 \times 5$
- **c** $784 = 2 \times 2 \times 2 \times 2 \times 7 \times 7$
- **d** $2025 = 3 \times 3 \times 3 \times 3 \times 5 \times 5$
- **e** $19\,600 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 7 \times 7$
- **f** $250\,000 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

6 Use the given product of prime factors to find the cube root of each number. Show your working.

- **a** $27 = 3 \times 3 \times 3$
- **b** $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$
- **c** $2197 = 13 \times 13 \times 13$
- **d** $1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$
- **e** $15\,625 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$
- **f** $32\,768 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

7 Calculate:

a $(\sqrt{25})^2$ **b** $(\sqrt{49})^2$ **c** $(\sqrt[3]{64})^3$ **d** $(\sqrt[3]{32})^3$

e $\sqrt{9} + \sqrt{16}$ **f** $\sqrt{9 + 16}$ **g** $\sqrt{36} + \sqrt{64}$ **h** $\sqrt{36 + 64}$

i $\sqrt{100 - 36}$ **j** $\sqrt{100} - \sqrt{36}$ **k** $\sqrt{25} \times \sqrt{4}$ **l** $\sqrt{25 \times 4}$

m $\sqrt{9 \times 4}$ **n** $\sqrt{9} \times \sqrt{4}$ **o** $\sqrt{\dfrac{36}{4}}$ **p** $\dfrac{\sqrt{36}}{4}$

TIP

Brackets act as grouping symbols. Work out any calculations inside brackets before doing the calculations outside the brackets.

Root signs work in the same way as a bracket. If you have $\sqrt{25 + 9}$, you must add 25 and 9 before finding the root.

8 Find the length of the edge of a cube with a volume of:

a $1000\,\text{cm}^3$ **b** $19\,683\,\text{cm}^3$ **c** $68\,921\,\text{mm}^3$ **d** $64\,000\,\text{cm}^3$

9 If the symbol ★ means 'add the square of the first number to the cube of the second number', calculate:

a 2 ★ 3 **b** 3 ★ 2 **c** 1 ★ 4 **d** 4 ★ 1 **e** 2 ★ 4

f 4 ★ 2 **g** 1 ★ 9 **h** 9 ★ 1 **i** 5 ★ 2 **j** 2 ★ 5

REFLECTION

You have covered many of the concepts in this chapter earlier in your study of mathematics.

- Which concepts did you remember really well?
- Why do you think you remembered these so well?
- Did you find any new ways of doing things or better ways of explaining things as you worked through this chapter? Share your ideas with a partner.

Other indices and roots

You have seen that square numbers are all raised to the power of two (5 squared = $5 \times 5 = 5^2$) and that cube numbers are all raised to the power of three (5 cubed = $5 \times 5 \times 5 = 5^3$). You can raise a number to any power. For example, $5 \times 5 \times 5 \times 5 = 5^4$. You read this as '5 to the power of 4'. The same principle applies to finding roots of numbers.

$5^2 = 25$ $\sqrt{25} = 5$

$5^3 = 125$ $\sqrt[3]{125} = 5$

$5^4 = 625$ $\sqrt[4]{625} = 5$

You can use your calculator to perform operations using any roots or squares.

The $\boxed{y^x}$ key calculates any power.

So, to find 7^5, you enter 7 $\boxed{y^x}$ 5 and get a result of 16 807.

The $\boxed{\sqrt[x]{\ }}$ key calculates any root.

So, to find $\sqrt[4]{81}$, you enter 4 $\boxed{\sqrt[x]{\ }}$ 81 and get a result of 3.

Make sure that you know which key is used for each function on your calculator and that you know how to use it. On some calculators these keys might be second functions.

MATHEMATICAL CONNECTIONS

You will work with higher powers and roots again when you deal with indices in algebra in Chapter 2, standard form in Chapter 5 and rates of growth and decay in Chapters 17 and 18.

Index notation and products of prime factors

Index notation is very useful when you have to express a number as a product of its prime factors because it allows you to write the factors in a short form.

WORKED EXAMPLE 10

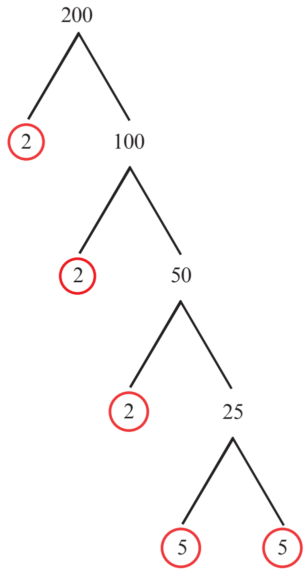
Express these numbers as products of their prime factors in index form.

a 200 **b** 19 683

Answers

These diagrams are a reminder of the factor tree and division methods for finding the prime factors.

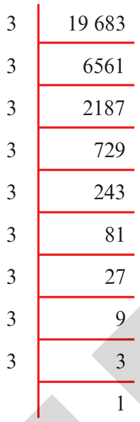
a



$= 2 \times 2 \times 2 \times 5 \times 5$

$200 = 2^3 \times 5^2$

b



$= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$19\,683 = 3^9$

Exercise 1.13

1 Evaluate.

a $2^4 \times 2^3$ **b** $3^5 \times \sqrt[6]{64}$ **c** $3^4 + \sqrt[4]{256}$

d $2^4 \times \sqrt[5]{7776}$ **e** $\sqrt[4]{625} \times 2^6$ **f** $8^4 \div (\sqrt[5]{32})^3$

2 Which is greater and by how much?

a $8^0 \times 4^4$ or $2^4 \times 3^4$ **b** $\sqrt[4]{625} \times 3^6$ or $\sqrt[6]{729} \times 4^4$

3 Express the following as products of prime factors, in index notation.

a 64 **b** 243 **c** 400 **d** 1600 **e** 16 384

f 20 736 **g** 59 049 **h** 390 625

4 Write several square numbers as products of prime factors, using index notation. What can you say about the index needed for each prime?

TIP

Remember that anything raised to the power of zero is equal to 1.

The laws of indices

The laws of indices are a set of mathematical rules that allow you to multiply and divide numbers written in index notation without having to write them in expanded form.

Make sure that you remember these three important rules.

To multiply different powers of the same number, add the indices.
For example, $3^2 \times 3^5 = 3^{2+5} = 3^7$ and $4^{-2} \times 4^3 = 4^{-2+3} = 4$.

To divide different powers of the same number, subtract the indices.

For example, $3^6 \div 3^2 = 3^{6-2} = 3^4$ and $\frac{4^3}{4^7} = 4^{3-7} = 4^{-4}$

To find the power of a power you multiply the indices.
For example, $(3^3)^2 = 3^{3\times 2} = 3^6$ and $(4^2)^{-3} = 4^{2\times -3} = 4^{-6}$

In general terms:

$a^m \times a^n = a^{m+n}$ $a^m \div a^n = a^{m-n}$ $(a^m)^n = a^{mn}$

Zero and negative indices

Do you remember how to work with zero and negative indices? Read through this information to refresh your memory.

In this table, each value is $\frac{1}{5}$ of the one to its left. For example, $5^4 \div 5 = 5^3$.

Power of 5	5^4	5^3	52	5^1	5^0	5^{-1}	5^{-2}	5^{-3}	5^{-4}
Value	625	125	25	5	1	$\frac{1}{5}$	$\frac{1}{25}$	$\frac{1}{125}$	$\frac{1}{625}$

÷5

The pattern in the table shows that $5^0 = 1$. This is true for any number to the power of 0.

We can say $a^0 = 1$ (where a ≠ 0, because 0^0 is undefined.).

You can also see from the table that a number with a negative index is equal to its **reciprocal** with a positive index. For example: $5^{-2} = \frac{1}{5^2}$.

This is true for all negative indices.

We can say $a^{-m} = \frac{1}{a^m}$ (where $a \neq 0$).

Exercise 1.14

1 Decide whether each statement is true or false. If it is false, work out the correct answer.

a $4^3 \times 4^5 = 4^8$ **b** $\frac{3^8}{3^2} = 3^4$ **c** $4^5 \div 4^2 = 4^3$ **d** $(8^3)^2 = 8^5$

e $34^0 = 1$ **f** $7^4 \times 7^3 = 7^7$ **g** $\frac{2^{10}}{2^5} = 2^5$ **h** $10^{10} \div 10^5 = 10^2$

i $(5^{-2})^4 = 5^2$ **j** $(-2^4)^2 = -2^6$ **k** $\frac{7^{-2}}{7^{-3}} = 7$ **l** $-(5^2)^0 = 1$

2 Simplify. Leave your answers in index notation.

a $10^3 \times 10^4$
b $3^{10} \times 3^{-5}$
c $2 \times 2^5 \times 2^{-1}$
d $10^0 \times 10^{-3}$
e $\frac{10^5}{10^4}$
f $\frac{12^6}{12^6}$
g $\frac{3^{-4}}{3^3}$
h $4^{-3} \div 4^4$
i $(3^4)^3$
j $(5^{-2})^2$
k $(4^2)^{-3}$
l $(4^3)^0$

3 Substitute $a = 2$, $b = 3$ and $c = \frac{1}{2}$ to find the value of each expression.

a $a^{-1} + b^{-1}$
b $(ab)^{-2}$
c $(a^2c)^{-1}$
d $a^{-1}b^{-1}c$

4 Evaluate.

a 3^{-1}
b 4^{-1}
c 2^{-1}
d 4^{-2}
e 2^{-4}

5 Express each value with a negative index.

a $\frac{1}{4}$
b $\frac{1}{5}$
c $\frac{1}{7}$
d $\frac{1}{3^3}$
e $\frac{1}{10^4}$
f $\frac{1}{2^8}$
g $\frac{1}{7^2}$
h $\frac{1}{2 \times 3^2}$

6 Evaluate.

a $\left(\frac{4}{9}\right)^{-2}$
b $8^0 \times 10^3$
c $12^2 \times 4^{-3}$
d $(2^3)^{-2}$
e $(-3)^2 \times \left(\frac{1}{2}\right)^{-2}$
f $\frac{(10-6)^3}{2^3}$
g $2^3 + \frac{3^2}{3} + 2$
h $(-3)^2 + \left(\frac{1}{2}\right)^{-3}$

7 Rewrite each expression in the form of 3^x (in other words, as a power of 3).

a 3
b 9
c 729
d $\frac{1}{27}$
e $\frac{1}{3}$
f 1
g $\frac{1}{243}$
h $-\sqrt{81}$

Fractional indices

Do you remember what a fractional index such as $5^{\frac{1}{2}}$ means?

You can use the laws of indices to show the meaning of fractional indices.

$5^{\frac{1}{2}} \times 5^{\frac{1}{2}} = 5^{\left(\frac{1}{2}+\frac{1}{2}\right)} = 5^1 = 5$

You also know that $\sqrt{5} \times \sqrt{5} = 5$

So, $5^{\frac{1}{2}} = \sqrt{5}$

$5^{\frac{1}{3}} \times 5^{\frac{1}{3}} \times 5^{\frac{1}{3}} = 5^{\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right)} = 5^1 = 5$

And $\sqrt[3]{5} \times \sqrt[3]{5} \times \sqrt[3]{5} = 5$

So, $5^{\frac{1}{3}} = \sqrt[3]{5}$

In general terms, for unit fractions:

$a^{\frac{1}{2}} = \sqrt{a}$ $a^{\frac{1}{3}} = \sqrt[3]{a}$ $a^{\frac{1}{n}} = \sqrt[n]{a}$

LINK

Fractional indices and roots are used in many different financial calculations involving investments, insurance policies and economic decisions.

You can use the rule for finding the power of a power to show the meaning of fractional indices where the numerator is not 1 (non-unit fractions).

$\left(4^{\frac{1}{4}}\right)^3 = 4^{\left(\frac{1}{4} \times \frac{3}{1}\right)} = 4^{\frac{3}{4}}$

This shows that a number such as $5^{\frac{2}{3}}$ can be written with a unit fraction index as $\left(5^{\frac{1}{3}}\right)^2$.

You already know that you can write a unit fraction (such as $\frac{1}{3}$) as a root.

So $5^{\frac{2}{3}} = \left(5^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{5}\right)^2$

It is simpler to input the value in root form into your calculator than to enter $5^{\frac{2}{3}}$.

In general terms, for non-unit fractions:

$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$

TIP

Multiplication is commutative, so $\left(a^{\frac{1}{n}}\right)^m$ is the same as $(a^m)^{\frac{1}{n}} = \sqrt[n]{a^m}$

WORKED EXAMPLE 11

Work out the value of:

a $27^{\frac{2}{3}}$ **b** $25^{1.5}$

Answers

a $27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$
$= (3)^2$
$= 9$

$\frac{2}{3} = 2 \times \frac{1}{3}$, so you square the cube root of 27.

b $25^{1.5} = 25^{\frac{3}{2}}$
$= \left(\sqrt{25}\right)^3$
$= (5)^3$
$= 125$

Change the decimal to a fraction.

$\frac{3}{2} = 3 \times \frac{1}{2}$, so you need to cube the square root of 25.

Exercise 1.15

1 Rewrite each expression using a root symbol.

a $25^{\frac{1}{2}}$ **b** $3^{\frac{1}{3}}$ **c** $40^{\frac{1}{5}}$ **d** $6^{\frac{1}{2}}$
e $3^{\frac{1}{8}}$ **f** $2^{\frac{3}{4}}$ **g** $12^{\frac{2}{3}}$ **h** $5^{\frac{2}{9}}$

2 Write each expression using index notation.

a $\sqrt{5}$ **b** $\sqrt[3]{8}$ **c** $\sqrt[3]{13}$ **d** $\sqrt[4]{11}$
e $\left(\sqrt[3]{9}\right)^2$ **f** $\left(\sqrt[3]{6}\right)^4$ **g** $\left(\sqrt[4]{32}\right)^3$ **h** $2\left(\sqrt[5]{12}\right)^7$

3 Use a calculator to evaluate.

a $25^{\frac{1}{2}}$ **b** $27^{\frac{1}{3}}$ **c** $8^{\frac{2}{3}}$ **d** $16^{\frac{3}{4}}$
e $216^{\frac{2}{3}}$ **f** $0.125^{\frac{1}{3}}$ **g** $46^{\frac{1}{2}}$ **h** $125^{-\frac{4}{3}}$
i $32^{-\frac{1}{5}}$ **j** $8^{\frac{4}{3}}$ **k** $216^{\frac{2}{3}}$ **l** $256^{0.75}$

APPLY YOUR SKILLS

4 The number of calories a mammal uses when they are at rest can be worked out using the formula $C = 70 \times m^{\frac{3}{4}}$, where m is the mass of the animal in kilograms.

a Express the formula using a root sign.

b A cat has a mass of 5.5 kilograms. Work out how many calories it consumes while it is at rest.

c How many calories would a 5000 kg elephant consume at rest?

1.6 Order of operations

At this level of mathematics you are expected to carry out calculations involving more than one operation (+, −, × and ÷). When you do this you have to follow a sequence of rules so that there is no confusion about what operations you should do first. The rules for the order of operations are:

- complete operations in grouping symbols first
- deal with powers and roots next
- do division and multiplication next, working from left to right
- do addition and subtraction last, working from left to right.

Many people use the letters BODMAS to remember the order of operations.
The letters stand for:

Brackets
Orders
Divide **M**ultiply
Add **S**ubtract

BODMAS indicates that indices (powers of) are considered after brackets but before all other operations.

MATHEMATICAL CONNECTIONS

You will apply the order of operation rules to fractions, decimals and algebraic expressions as you progress through the course.

Grouping symbols

The most common grouping symbols in mathematics are brackets. Here are some examples of the different kinds of brackets used in mathematics:

$(4 + 9) \times (10 \div 2)$

$[2(4 + 9) - 4(3) - 12]$

$\{2 - [4(2 - 7) - 4(3 + 8)] - 2 \times 8\}$

When you have more than one set of brackets in a calculation, you work out the innermost set first.

Other symbols used to group operations are:

- fraction bars, e.g. $\frac{5 - 12}{3 - 8}$
- root signs, such as square roots and cube roots, e.g. $\sqrt{9 + 16}$

WORKED EXAMPLE 12

Simplify.

a $7 \times (3 + 4)$ **b** $(10 - 4) \times (4 + 9)$ **c** $45 - [20 \times (4 - 3)]$

Answers

a $7 \times 7 = 49$ **b** $6 \times 13 = 78$ **c** $45 - [20 \times 1] = 45 - 20 = 25$

WORKED EXAMPLE 13

Calculate.

a $\frac{4 + 28}{17 - 9}$ **b** $\sqrt{36 \div 4} + \sqrt{100 - 36}$

Answers

a $(4 + 28) \div (17 - 9)$

$= 32 \div 8$

$= 4$

b $\sqrt{36 \div 4} + \sqrt{100 - 36}$

$= \sqrt{9} + \sqrt{64}$

$= 3 + 8$

$= 11$

Now that you know what to do with grouping symbols, you can apply the rules for order of operations to perform calculations with numbers.

Exercise 1.16

1 Calculate. Show the steps in your working.

a $(4 + 7) \times 3$ **b** $(20 - 4) \div 4$ **c** $50 \div (20 + 5)$

d $6 \times (2 + 9)$ **e** $(4 + 7) \times 4$ **f** $(100 - 40) \times 3$

g $16 + (25 \div 5)$ **h** $19 - (12 + 2)$ **i** $40 \div (12 - 4)$

j $100 \div (4 + 16)$ **k** $121 \div (33 \div 3)$ **l** $15 \times (15 - 15)$

2 Calculate.

a $(4 + 8) \times (16 - 7)$ **b** $(12 - 4) \times (6 + 3)$ **c** $(9 + 4) - (4 + 6)$

d $(33 + 17) \div (10 - 5)$ **e** $(4 \times 2) + (8 \times 3)$ **f** $(9 \times 7) \div (27 - 20)$

g $(105 - 85) \div (16 \div 4)$ **h** $(12 + 13) \div 5^2$ **i** $(56 - 6^2) \times (4 + 3)$

3 Simplify. Show the steps in your working.

- **a** $5 \times 10 + 3$
- **b** $5 \times (10 + 3)$
- **c** $2 + 10 \times 3$
- **d** $(2 + 10) \times 3$
- **e** $23 + 7 \times 2$
- **f** $6 \times 2 \div (3 + 3)$
- **g** $\frac{15 - 5}{2 \times 5}$
- **h** $(17 + 1) \div 9 + 2$
- **i** $\frac{16 - 4}{4 - 1}$
- **j** $17 + 3 \times 21$
- **k** $48 - (2 + 3) \times 2$
- **l** $12 \times 4 - 4 \times 8$
- **m** $15 + 30 \div 3 + 6$
- **n** $20 - 6 \div 3 + 3$
- **o** $10 - 4 \times 2 \div 2$

TIP

A bracket 'type' is always twinned with another bracket of the same type or shape. This helps mathematicians to understand the order of calculations even more easily.

4 Simplify.

- **a** $18 - 4 \times 2 - 3$
- **b** $14 - (21 \div 3)$
- **c** $24 \div 8 \times (6 - 5)$
- **d** $42 \div 6 - 3 - 4$
- **e** $5 + 36 \div 6 - 8$
- **f** $(8 + 3) \times (30 \div 3) \div 11$

5 Simplify. Remember to work from the innermost grouping symbols to the outermost.

- **a** $4 + [12 - (8 - 5)]$
- **b** $6 + [2 - (2 \times 0)]$
- **c** $8 + [60 - (2 + 8)]$
- **d** $200 - [(4 + 12) - (6 + 2)]$
- **e** $200 \times \{100 - [4 \times (2 + 8)]\}$
- **f** $\{6 + [5 \times (2 + 30)]\} \times 10$
- **g** $[(30 + 12) - (7 + 9)] \times 10$
- **h** $1000 - [6 \times (4 + 20) - 4 \times (3 + 0)]$

6 Calculate.

- **a** $20 - 4 \div 2$
- **b** $\frac{31 - 10}{14 - 7}$
- **c** $\frac{100 - 40}{5 \times 4}$
- **d** $\sqrt{100 - 36}$
- **e** $\sqrt{8 + 8}$
- **f** $\sqrt{90 - 9}$

7 State whether the following are true or false.

- **a** $(1 + 4) \times 20 + 5 = 1 + (4 \times 20) + 5$
- **b** $6 \times (4 + 2) \times 3 > (6 \times 4) \div 2 \times 3$
- **c** $8 + (5 - 3) \times 2 < 8 + 5 - (3 \times 2)$
- **d** $100 + 10 \div 10 > (100 + 10) \div 10$

8 Insert brackets into the following calculations to make them true.

- **a** $3 \times 4 + 6 = 30$
- **b** $25 - 15 \times 9 = 90$
- **c** $40 - 10 \times 3 = 90$
- **d** $14 - 9 \times 2 = 10$
- **e** $12 + 3 \div 5 = 3$
- **f** $19 - 9 \times 15 = 150$
- **g** $10 + 10 \div 6 - 2 = 5$
- **h** $3 + 8 \times 15 - 9 = 66$
- **i** $9 - 4 \times 7 + 2 = 45$
- **j** $10 - 4 \times 5 = 30$
- **k** $6 \div 3 + 3 \times 5 = 5$
- **l** $15 - 6 \div 2 = 12$
- **m** $1 + 4 \times 20 \div 5 = 20$
- **n** $8 + 5 - 3 \times 2 = 20$
- **o** $36 \div 3 \times 3 - 3 = 6$
- **p** $3 \times 4 - 2 \div 6 = 1$
- **q** $40 \div 4 + 1 = 11$
- **r** $6 + 2 \times 8 + 2 = 24$

9 Place the given numbers in the correct spaces to make a correct number sentence.

- **a** 0, 2, 5, 10 $\square - \square \div \square = \square$
- **b** 9, 11, 13, 18 $\square - \square \div \square = \square$
- **c** 1, 3, 8, 14, 16 $\square \div (\square - \square) - \square = \square$
- **d** 4, 5, 6, 9, 12 $(\square + \square) - (\square - \square) = \square$

Using your calculator

A calculator with algebraic logic will apply the rules for order of operations automatically. So, if you enter $2 + 3 \times 4$, your calculator will do the multiplication first and give you an answer of 14. (Check that your calculator does this!)

When the calculation contains brackets you must enter these to make sure your calculator does the grouped sections first.

WORKED EXAMPLE 14

Use a calculator to find:

a $3 + 2 \times 9$ **b** $(3 + 8) \times 4$ **c** $(3 \times 8 - 4) - (2 \times 5 + 1)$

Answers

a 21 Enter $\boxed{3}\ \boxed{+}\ \boxed{2}\ \boxed{\times}\ \boxed{9}\ \boxed{=}$

b 44 Enter $\boxed{(}\ \boxed{3}\ \boxed{+}\ \boxed{8}\ \boxed{)}\ \boxed{\times}\ \boxed{4}\ \boxed{=}$

c 9 Enter $\boxed{(}\ \boxed{3}\ \boxed{\times}\ \boxed{8}\ \boxed{-}\ \boxed{4}\ \boxed{)}\ \boxed{-}$
$\boxed{(}\ \boxed{2}\ \boxed{\times}\ \boxed{5}\ \boxed{+}\ \boxed{1}\ \boxed{)}\ \boxed{=}$

Experiment with your calculator by carrying out several calculations, with and without brackets. For example: $3 \times 2 + 6$ and $3 \times (2 + 6)$. Do you understand why these are different?

TIP

Your calculator might only have one type of bracket $\boxed{(}$ and $\boxed{)}$. If there are two different shaped brackets in the calculation, such as $[4 \times (2 - 3)]$, enter the calculator bracket symbol for each type.

Exercise 1.17

1 Use your calculator to find the answers.

a $10 - 4 \times 5$
b $12 + 6 \div 7 - 4$
c $3 + 4 \times 5 - 10$
d $18 \div 3 \times 5 - 3 + 2$
e $5 - 3 \times 8 - 6 \div 2$
f $7 + 3 \div 4 + 1$
g $(1 + 4) \times 20 \div 5$
h $36 \div 6 \times (3 - 3)$
i $(8 + 8) - 6 \times 2$
j $100 - 30 \times (4 - 3)$
k $24 \div (7 + 5) \times 6$
l $[(60 - 40) - (53 - 43)] \times 2$
m $[(12 + 6) \div 9] \times 4$
n $[100 \div (4 + 16)] \times 3$
o $4 \times [25 \div (12 - 7)]$

2 Use your calculator to check whether the following answers are correct. If the answer is incorrect, work out the correct answer.

a $12 \times 4 + 76 = 124$
b $8 + 75 \times 8 = 698$
c $12 \times 18 - 4 \times 23 = 124$
d $(16 \div 4) \times (7 + 3 \times 4) = 76$
e $(82 - 36) \times (2 + 6) = 16$
f $(3 \times 7 - 4) - (4 + 6 \div 2) = 12$

3 Each ★ represents a missing operation. Work out what they are.

a 12 ★ (28 ★ 24) = 3
b 84 ★ 10 ★ 8 = 4
c 3 ★ 7(0.7 ★ 1.3) = 17
d 23 ★ 11 ★ 22 ★ 11 = 11
e 40 ★ 5 ★ (7 ★ 5) = 4
f 9 ★ 15 ★ (3 ★ 2) = 12

TIP

Some calculators have two '–' buttons: $\boxed{-}$ and $\boxed{(-)}$. The first means 'subtract' and is used to subtract one number from another. The second means 'make negative'. Experiment with the buttons and make sure that your calculator is doing what you expect it to do!

4 Calculate.

a $\dfrac{7 \times \sqrt{16}}{2^3 + 7^2 - 1}$ **b** $\dfrac{5^2 \times \sqrt{4}}{1 + 6^2 - 12}$ **c** $\dfrac{2 + 3^2}{5^2 + 4 \times 10 - \sqrt{25}}$

d $\dfrac{6^2 - 11}{2(17 + 2 \times 4)}$ **e** $\dfrac{3^2 - 3}{2 \times \sqrt{81}}$ **f** $\dfrac{3^2 - 5 + 6}{\sqrt{4} \times 5}$

g $\dfrac{36 - 3 \times \sqrt{16}}{15 - 3^2 \div 3}$ **h** $\dfrac{-30 + [18 \div (3 - 12) + 24]}{5 - 8 - 3^2}$

5 Use your calculator to find the answer. Give your answers to 3 significant figures.

a $\dfrac{0.345}{1.34 + 4.2 \times 7}$ **b** $\dfrac{12.32 \times 0.0378}{\sqrt{16 + 8.05}}$ **c** $\dfrac{\sqrt{16} \times 0.087}{2^2 - 5.098}$

6 Use your calculator to evaluate. Give your answers to 3 significant figures.

a $\sqrt{64 \times 125}$ **b** $\sqrt{2^3 \times 3^2 \times 6}$ **c** $\sqrt[3]{8^2 + 19^2}$

d $\sqrt{41^2 - 36^2}$ **e** $\sqrt{3.2^2 - 1.17^3}$ **f** $\sqrt[3]{1.45^3 - 0.13^2}$

g $\dfrac{1}{4}\sqrt{\dfrac{1}{4} + \dfrac{1}{4} + \sqrt{\dfrac{1}{4}}}$ **h** $\sqrt[3]{2.75^2 + \dfrac{1}{2} \times 1.7^3}$

7 Evaluate. Give your answer to 2 decimal places if necessary.

a $\sqrt[3]{8} - \sqrt{1}$ **b** $\sqrt[4]{16} \times 8^{-\frac{2}{3}}$ **c** $(-3)^3 + 2^{-4}$

d $\dfrac{15}{48 + 2\sqrt{7}}$ **e** $\dfrac{77}{14} \times \dfrac{29}{11}$ **f** $(0.467)^2 \times \sqrt{900}$

g $\left(\dfrac{5}{6}\right)^2 + (\sqrt{144})^3$ **h** $\sqrt[3]{205379} - 6(\sqrt{343})^2$ **i** $\dfrac{19.23 \times 0.087}{2.45^2 - 1.03^2}$

SELF ASSESSMENT

Draw up a flow chart like this one to assess your own learning.

Some sentence stems are provided below each box to help you get started.

I understood this easily because …
I struggled a bit with __ because …
I am still not sure of …
I am confident that I can …
I would give myself [ ] out of ten for this work.

I was very good at …
I was proud of …
My best work was …

To improve I can …
Next time I will …
I need to revise …

TIP

If you have forgotten how to round to significant figures, read through Worked example 16 in Section 1.7.

MATHEMATICAL CONNECTIONS

When you work with indices and standard form in Chapter 5, you will need to apply these skills and use your calculator effectively to solve problems involving any powers or roots.

1.7 Rounding and estimating

In many calculations, particularly with decimals, you will not need to find an exact answer. Instead, you will be asked to give an answer to a stated level of accuracy. For example, you may be asked to give an answer correct to 2 decimal places, or to 3 significant figures.

LINK

We use 'rounding' in all subjects where numerical data is collected. Masses in physics, temperatures in biology, prices in economics: these all need to be recorded sensibly and will be rounded to a degree of accuracy appropriate for the situation.

WORKED EXAMPLE 15

Round 64.839906 to:

a the nearest whole number

b 1 decimal place

c 3 decimal places.

Answers

a	64.839906	4 is in the units place.
	64.839906	The next digit is 8, so you will round up to get 5.
	= 65 (to nearest whole number)	To the nearest whole number.
b	64.839906	8 is in the first decimal place.
	64.839906	The next digit is 3, so the 8 will remain unchanged.
	= 64.8 (1 d.p.)	Correct to 1 decimal place.
c	64.839906	9 is in the third decimal place.
	64.839906	The next digit is 9, so you need to round up.
	= 64.840 (3 d.p.)	When you round 9 up, you get 10, so carry one to the previous digit and write 0 in the place of the 9.
		Correct to 3 decimal places.

When a number has many digits or decimal places it is useful to round it to significant figures (s.f.). The first significant digit of a number is the first *non-zero* digit, when reading from left to right. The next digit is the second significant digit, the next the third significant and so on. All zeros *after* the first significant digit are considered significant.

If you are rounding to a whole number, write the appropriate number of zeros after the last significant digit as place holders to keep the number the same size.

TIP

Rounding to 1 significant figure does not mean you will only have one digit. When 13 432 is rounded to 1 significant figure it is 10 000 and not 1.

WORKED EXAMPLE 16

Round:

a 1.076 to 3 significant figures
b 0.00736 to 1 significant figure
c 23 512 435 to 2 significant figures

Answers

a	1.076	The third significant figure is the 7. The next digit is 6, so round 7 up to get 8.
	= 1.08 (3 s.f.)	Correct to 3 significant figures.
b	0.00736	The first significant figure is the 7. The next digit is 3, so 7 will not change.
	= 0.007 (1 s.f.)	Correct to 1 significant figure.
c	23 512 475	The second significant figure is 3. The next digit is 5, so 3 will round up to 4.
	= 24 000 000 (2 s.f.)	Include the zeros and state the level of accuracy.

Exercise 1.18

1 Round each number to 2 decimal places.

a 3.185 **b** 0.064 **c** 38.3456 **d** 2.149 **e** 0.999

2 Round each number to the nearest 100.

a 456 **b** 53 438 **c** 3012.567 **d** 38.299 **e** 10 060

3 Round each number to the nearest 10 000.

a 629 534 **b** 100 999 **c** 9016 **d** 12 064 **e** 155 179

4 Express each number correct to:

i 4 significant figures **ii** 3 significant figures **iii** 1 significant figure

a 4512 **b** 12 305 **c** 65 238 **d** 320.55
e 25.716 **f** 0.000765 **g** 1.0087 **h** 7.34876
i 0.00998 **j** 0.02814 **k** 31.0077 **l** 0.0064735

5 Change $2\frac{5}{9}$ to a decimal using your calculator. Express the answer correct to:

a 3 decimal places **b** 2 decimal places

c 1 decimal place **d** 3 significant figures

e 2 significant figures **f** 1 significant figure

Estimating to get an approximate answer

To estimate the answer to a calculation, you need to round the numbers before you do the calculation. Although you can use any accuracy, often the numbers in the calculation are rounded to 1 significant figure:

3.9×2.1 is approximately equal to $4 \times 2 = 8$

Notice that $3.9 \times 2.1 = 8.19$, so the estimated value of 8 is not too far from the real value!

WORKED EXAMPLE 17

Estimate the value of:

a $\dfrac{4.6 + 3.9}{\sqrt{398}}$ **b** $\sqrt{42.2 - 5.1}$

Answers

a $\dfrac{4.6 + 3.9}{\sqrt{398}}$ is approximately equal to $\dfrac{5 + 4}{\sqrt{400}}$

Round the numbers to 1 significant figure.

$$= \frac{9}{20} = \frac{4.5}{10} = 0.45$$

Check the estimate:

$\dfrac{4.6 + 3.9}{\sqrt{398}} = 0.426$ (3 s.f.)

If you use a calculator you will find the exact value and see that the estimate was good.

b $\sqrt{42.2 - 5.1}$ is approximately equal to $\sqrt{40 - 5}$

$= \sqrt{35}$

is approximately equal to $\sqrt{36}$

$= 6$

Begin by rounding each value to 1 significant figure.

Notice that if you round 35 up to 36 you get a square number and you can easily take the square root.

A good starting point for the questions in the Exercise 1.19 is to round the numbers to 1 significant figure. Remember that you can sometimes make your calculation even simpler by modifying your numbers again.

Exercise 1.19

1 The calculator displays show the answers that a student got for each calculation. Write an estimate for each calculation and say whether the calculator answer is sensible or not.

a	$(7.1)^2 \div 9.9$	0.509191919192
b	$4 \times \pi \times 3^2$	75.39822369
c	5×7.9	395
d	50×7.9	395
e	3×292.5	87.75
f	$6.28 \times \sqrt{\dfrac{9.78}{0.53}}$	26.97684374

2 Estimate the value of each of the following. Show the rounded values that you use.

a $\frac{23.6}{6.3}$ **b** $\frac{4.3}{0.087 \times 3.89}$ **c** $\frac{7.21 \times 0.46}{9.09}$

d $\frac{4.82 \times 6.01}{2.54 + 1.09}$ **e** $\frac{\sqrt{48}}{2.54 + 4.09}$ **f** $(0.45 + 1.89)(6.5 - 1.9)$

g $\frac{23.8 + 20.2}{4.7 + 5.7}$ **h** $\frac{109.6 - 45.1}{19.4 - 13.9}$ **i** $(2.52)^2 \times \sqrt{48.99}$

j $\sqrt{223.8 \times 45.1}$ **k** $\sqrt{9.26} \times \sqrt{99.87}$ **l** $(4.1)^3 \times (1.9)^4$

3 Work out the actual answer for each part of question 2, using a calculator. How good were your estimates? How could you improve them?

TIP

When you are asked to estimate values, always show the rounded values that you use so anyone looking at your work knows what you have done.

INVESTIGATION

Making decisions about accuracy

There will be times when you have to decide how to round values to estimate. The place that you round to depends on the level of accuracy needed to solve each problem.

1 What would you round to in the following situations? Give reasons for your answers.
  a A real-life problem involving whole numbers, for example bricks or numbers of people.
  b Problems involving money amounts.
  c Calculations using numbers in the millions.
  d Scientific calculations with original values to four places.
  e Problems involving irrational numbers (such as π).

2 What have these students done to estimate?

Zaf 7.6×0.518 is approximately equal to $8 \times \frac{1}{2} = 4$

Marwan $\frac{\overset{2}{\cancel{17.73}} \times 5.7}{{}_{1}\cancel{8.7}}$ is approximately equal to $\frac{2 \times 6}{1} = 12$

  a Why is each strategy useful?
  b Why do you use the = symbol in some parts of the estimation but state 'is approximately equal to' in others?

3 What situations can you think of where it is helpful to make sure your estimate is:
  a an overestimate
  b an underestimate?

PEER ASSESSMENT

Tell … Ask … Give … (TAG) feedback is a way of assessing each other's work.

To use this method, read through your partner's answers to Exercise 1.19.

Use the guidelines in the table to help you give a TAG feedback on their work.

Tell your partner something they did well	**Ask a constructive or thoughtful question**	**Give them a positive suggestion for improvement**
I liked the way you … I could easily understand because you … The strongest part of your work was … You did … really well.	Why did you … Did you consider … Would it help if you … When does … Have you thought about …	One suggestion would be … Remember to … Think about … I'm confused by … If you … it might …

SUMMARY

Do you know …?
Numbers can be classified as natural numbers, integers, prime numbers and square numbers.
A multiple is obtained by multiplying a number by a natural number. The LCM of two or more numbers is the lowest multiple found in all the sets of multiples.
A factor of a number divides into it exactly. The HCF of two or more numbers is the highest factor found in all the sets of factors.
Prime numbers have only two factors, 1 and the number itself. The number 1 is not a prime number.
A prime factor is a number that is both a factor and a prime number.
All natural numbers that are not prime can be expressed as a product of prime factors.
Integers are also called directed numbers. The sign of an integer (− or +) indicates whether its value is above or below 0.
When you multiply an integer (a) by itself you get a square number (a^2). If you multiply it by itself again you get a cube number (a^3).
The number you multiply to get a square is called the square root and the number you multiply to get a cube is called the cube root. The symbol for square root is $\sqrt{\ }$. The symbol for cube root is $\sqrt[3]{\ }$.
You can express numbers as powers of their factors using index notation. For example, 2^3 means $2 \times 2 \times 2$. The base is 2 and the index is 3.
Any number to the power of 0 is equal to 1: $a^0 = 1$.
A negative index can be written as a reciprocal fraction with a positive index: $a^{-m} = \frac{1}{a^m}$.
Fractional indices can be rewritten as roots: $a^{\frac{1}{n}} = \sqrt[n]{a}$. For non-unit fractional indices: $a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$.
The laws of indices are: $a^m \times a^n = a^{m+n}$; $\frac{a^m}{a^n} = a^{m-n}$ and $(a^m)^n = a^{mn}$.

SUMMARY CONTINUED

Mathematicians apply a standard set of rules to decide the order in which operations must be carried out. Operations in grouping symbols are worked out first, then powers, then division and multiplication, then addition and subtraction.

Are you able to …?
identify rational numbers, irrational numbers, integers, square numbers and prime numbers
find multiples and factors of numbers and identify the LCM and HCF
write numbers as products of their prime factors using division and factor trees
work with integers used in real-life situations
apply the basic rules for operating with positive and negative numbers
perform basic calculations using mental methods and with a calculator
calculate squares, square roots, cubes and cube roots of numbers
apply the laws of indices to find the values of numbers written in index notation
round numbers to specified place to estimate and approximate answers.

Practice questions

1 Find the difference between the sum of the three largest prime numbers smaller than 20 and the product of the three smallest prime numbers. [3]

2 The product of two numbers is -36 and difference between the same two numbers is 13. Find the two possible pairs of numbers. [3]

3 Find the number that is one fifteenth of its own square. [2]

4 Find the highest common factor of

$$2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 7 \times 7 \times 11 \times 13$$

and

$$2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 11 \times 11 \times 13$$ [3]

5 The number 154.45ABC, where A, B and C represent the third, fourth and fifth decimal places in the number, is rounded to 4 decimal places and the answer is 154.4574. None of A, B or C are zero.

List all the possible sets of values of A, B and C. [4]

6 By expressing 1080 as a product of prime factors, determine whether 1080 is a cube number. Explain your answer. [4]

7 **a** Find two numbers that have a sum of 94 and a product of 2013. [2]

b Find two numbers have a difference of 19 and a product of 1170. [2]

8 Simplify.

a $6 \times 2 + 4 \times 5$ [2]

b $4 \times (100 - 15)$ [2]

c $(5 + 6) \times 2 + (15 - 3 \times 2) - 6$ [2]

d $-3 \times 5 - 6 \times -8$ [2]

e $-3 \times (-5 - 6) + 4 \times -6$ [2]

f $(-8 + 4)^3 + (-2)^4$ [2]

9 Insert +, −, × or ÷ into each blank square, to make the calculation work.

a $5 \square 7 - 3 \square 8 = 11$ [2]

b $(5 \square 3^2) \times 6 + 8 \square (-2) = -28$ [2]

10 Add brackets to this statement to make it true.

$7 + 14 \div 4 - 1 \times 2 = 14$ [2]

11 Use your calculator to find

$$\frac{5^3 - 3^2}{2^3 + 3^2 \times 11 - 2\sqrt{11}}$$

Round your answer to 3 significant figures. [2]

12 **a** Without using a calculator, estimate the value of

$$\frac{4.8 - 5.1^2}{\sqrt{24.6}}$$ [3]

b Use your calculator to find the difference between your estimate and the exact answer. [2]

13 Arrange the following numbers in order, starting with the smallest.

A $4 \times (4 + 4 \times 4)$

B $\frac{4^3}{4 \times 4} + 4$

C $\frac{4^2 - 4}{4} - 4$

D $4^2 - 4^2 \times 4 + \frac{4}{4}$ [3]

14 Find the exact values of

a $\sqrt{98} + \sqrt{72}$ [2]

b $(3^{-2} + 2^{-3}) \times 216^{\frac{2}{3}}$ [3]

c $\left((\sqrt{2})^2 + 23\right)^{\frac{1}{2}}$ [2]

d $\left(\frac{36}{25}\right)^{\frac{3}{2}}$ [2]

e $\left(\frac{16}{81}\right)^{-\frac{1}{4}}$ [2]

15 **a** Express 60 and 36 as products of primes. [2]

b Hence find the LCM of 60 and 36. [2]

c Planet Carceron has two moons, Anderon and Barberon. Anderon completes a full orbit of Carceron every 60 days, and Barberon completes a full single orbit of Carceron in 36 days. If Anderon, Barberon and Carceron lie on a straight line on 1 March 2023, on which date will this next be true? [2]

16 A code is developed as follows. Each letter of the alphabet is given a number, in order from 1 to 26. So A = 1, B = 2, C = 3, …, Z = 26.

For any word with three letters, the numbers corresponding to its letters are written as powers of the prime numbers 2, 3 and 5 in order and the answers are multiplied together.

Find the word with code 7500. [4]

SELF ASSESSMENT

Use your answers to the practice questions to assess what you already know and to analyse your strengths and weaknesses.

What areas are you good at?

Which areas require more work?

What work will you do?

Past paper questions

TIP

Unit 1 Past Paper Questions Resource Sheet is available on Cambridge GO.

1 Find the highest **odd** number that is a factor of 60 and a factor of 90. [1]

Cambridge IGCSE Mathematics (0580) Paper 11 Q8, June 2020

2 By rounding each number in the calculation correct to 1 significant figure, estimate the value of $\frac{38.7 \times 3.115}{20.3 - 4.1^2}$.

You must show all your working. [2]

Cambridge IGCSE Mathematics (0580) Paper 11 Q20, June 2021

3 **a** Write down the mathematical name for a polygon with 5 sides. [1]

b Work out the interior angle of a regular 18-sided polygon. [2]

Cambridge IGCSE Mathematics (0580) Paper 11 Q11, June 2021

4 **a** Write $\frac{1}{2 \times 2 \times 2 \times 2 \times 2}$ as a power of 2. [1]

b **i** $3^{18} \div 3^{t} = 3^{6}$

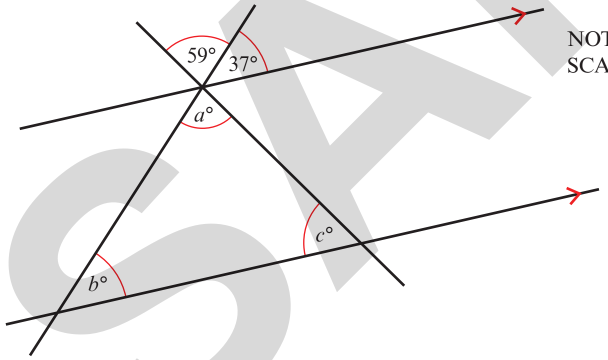
Find the value of t. t = …………………………………… [1]

ii Simplify.

$8w^{10} \times 6w^{5}$ [2]

Cambridge IGCSE Mathematics (0580) Paper 11 Q17, June 2021

5



NOT TO SCALE

The diagram shows two parallel lines intersected by two straight lines.

Find the values of a, b and c

a = ……………………………………

b = ……………………………………

c = …………………………………… [3]

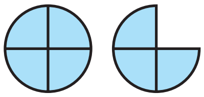
Cambridge IGCSE Mathematics (0580) Paper 11 Q10, June 2021

6 Zachary asks the 30 students in his class which is their favourite sport.

The table shows the results.

Netball	Football	Hockey	Tennis
7	12	6	5

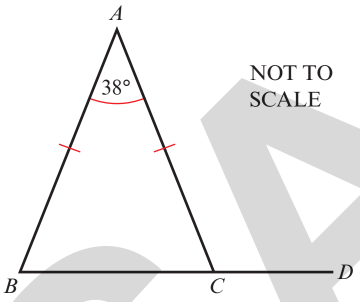
Complete the pictogram. [Using Figure 1 in the Unit 1 Past Paper Questions Resource Sheet.]

Netball	
Football	
Hockey	
Tennis	

Key: ⊕ represents 4 people [2]

Cambridge IGCSE Mathematics (0580) Paper 11 Q1, June 2021

7



In the triangle ABC, $AB = AC$ and angle $BAC = 38°$.

BCD is a straight line.

Work out angle ACD.

Angle ACD = ... [3]

Cambridge IGCSE Mathematics (0580) Paper 11 Q5, June 2020

8 Simplify.

$4p^5q^3 \times p^2q^{-4}$ [2]

Cambridge IGCSE Mathematics (0580) Paper 11 Q15, June 2020

9 **a** Using the integers from 60 to 75 only, find

  i a multiple of 17, [1]

  ii the prime numbers. [2]

b Find

  i the square root of 4489, [1]

  ii 4^3, [1]

  iii $\sqrt[3]{274625}$, [1]

  iv $2^{-3} \times 24^2$. [1]

c Write down the reciprocal of 7. [1]

d Write 3.72194 correct to 3 decimal places. [1]

e Find the lowest common multiple (LCM) of 8 and 14. [2]

f The average temperature at the North Pole is −23 °C in January and −11 °C in March.

  i Find the difference between these temperatures.

……………………………………. °C [1]

  ii The average temperature in July is 28 °C higher than the average temperature in March.
  Find the average temperature in July.

……………………………………. °C [1]

Cambridge IGCSE Mathematics (0580) Paper 31 Q5, June 2020

10 **a** Simplify $(81y^{16})^{\frac{3}{4}}$. [2]

b $2^3 = 4^p$

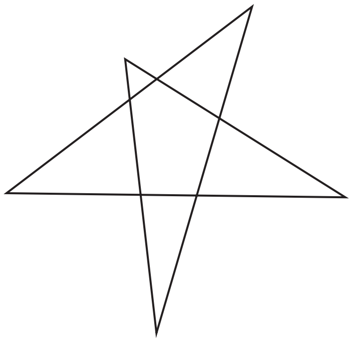
Find the value of p. $p =$ ……………………………………. [1]

Cambridge IGCSE Mathematics (0580) Paper 21 Q18, June 2019

Unit 1 Project

Star polygons

Here is a five-pointed star:



TIP

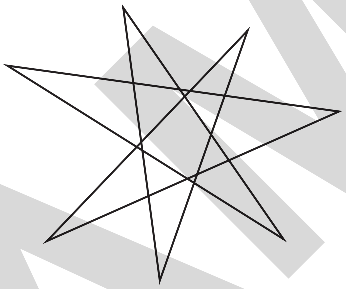
If you have access to the internet, you can find information to help you explain your findings on the NRICH website.

Go to the NRICH website to do this.

Draw some five-pointed stars of your own. Make sure your lines are straight.

Measure the interior angles at the five points and add them together.
What do you notice?

Here is a seven-pointed star:



Draw some seven-pointed stars of your own.

Measure the interior angles at the seven points and add them together.

What do you notice? How can you explain your findings?

Try to state and prove similar results for a star with any number of points.

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Cambridge IGCSE™
Mathematics
Core

PRACTICE BOOK

Karen Morrison

Third edition

Digital Acess

Cambridge Assessment
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Endorsed for learner support

Contents

How to use this book

Throughout this book, you will notice lots of different features that will help your learning. These are explained below.

Exercises

These help you to practise skills that are important for studying Cambridge IGCSE™ Mathematics.There are two types of exercise:

- Exercises which let you practise the mathematical skills you have learned.
- Review exercises which bring together all the mathematical concepts in a chapter, pushing your skills further.

KEY LEARNING STATEMENT

This will remind you of what you should already know from your previous study in order to complete the exercises in this section.

KEY CONCEPTS

These summarise the important concepts that are covered in each section.

REFLECTION

At the end of some exercises you will find opportunities to think about the approach that you take to your work, and how you might improve this in the future.

TIP

The information in these boxes will help you complete the exercises, and give you support in areas that you might find difficult.

SELF ASSESSMENT

At the end of some exercises, you will find opportunities to help you assess your own work, and consider how you can improve the way you learn.

This icon shows you where you should complete an exercise without using your calculator.

Chapter 1: Review of number concepts

1.1 Different types of numbers

KEY LEARNING STATEMENTS

- Real numbers are either rational or irrational.
- You can write rational numbers as fractions in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$. (Integers are negative and positive whole numbers, and zero.)
- Rational numbers include integers, fractions, recurring and terminating decimals and percentages.

KEY CONCEPTS

- Classifying and using different types of numbers.
- Interpreting and using the symbols $=$, $\neq$, $<$, $>$, $\leqslant$ and $\geqslant$.

1 Copy and complete this table by writing a definition and giving an example of each type of number.

Mathematical name	Definition	Example
Natural numbers		
Integers		
Prime numbers		
Square numbers		
Fractions		

TIP

Knowing the correct mathematical terms is important for understanding questions and communicating mathematically.

2 Give an example to show what each of the following symbols means.

a $>$ **b** $\leqslant$ **c** $\therefore$ **d** $\sqrt{}$

e $\neq$ **f** $\geqslant$ **g** $<$

3 Look at this set of numbers.

3, –2, 0, 1, 9, 15, 4, 5, –7, 10, 32, –32, 21, 23, 25, 27, 29, $\frac{1}{2}$

a Which of these numbers are **not** natural numbers?

b Which of these numbers are **not** integers?

c Which of these numbers are prime numbers?

d Which of these numbers are square numbers?

4 List:

- **a** four square numbers greater than 100
- **b** four rational numbers smaller than $\frac{1}{3}$
- **c** two prime numbers that are >80
- **d** the prime numbers <10.

5 Write each amount as a number.

- **a** Three hundred and sixty-five thousand, two hundred and eighty-nine.
- **b** One billion, seven hundred and three million, four hundred and seventy-three thousand, two hundred and twelve.

1.2 Multiples and factors

KEY LEARNING STATEMENTS

- When you multiply a number by another number you get a multiple of the original number.
- The lowest common multiple (LCM) of two numbers is the lowest number that is a multiple of both numbers.
- Any number that will divide into a number exactly is a factor of that number.
- The highest common factor (HCF) of two numbers is the highest number that is a factor of both numbers.

KEY CONCEPT

Finding the highest common factor and lowest common multiple of two numbers.

1 Find the LCM of the given numbers.

a 9 and 18 **b** 12 and 18 **c** 15 and 18 **d** 24 and 12

e 36 and 9 **f** 12 and 8 **g** 9 and 24 **h** 12 and 32

2 Find the HCF of the given numbers.

a 12 and 18 **b** 18 and 36 **c** 27 and 90 **d** 12 and 15

e 20 and 30 **f** 19 and 45 **g** 60 and 72 **h** 250 and 900

3 Amira has two rolls of cotton fabric. One roll has 72 metres on it and the other has 90 metres on it. She wants to cut the fabric to make as many equal length pieces as possible of the longest possible length. How long should each piece be?

TIP

To find the LCM of a set of numbers, you can list the multiples of each number until you find the first multiple that is in the lists for all of the numbers in the set.

TIP

You need to work out whether to use LCM or HCF to find the answers. Problems involving LCM usually include repeating events. Problems involving HCF usually involve splitting things into smaller pieces or arranging things in equal groups or rows.

4 In a shopping mall promotion every 30th shopper gets a $10 voucher and every 120th shopper gets a free meal. How many shoppers must enter the mall before one receives both a voucher and a free meal?

5 Amanda has 40 pieces of fruit and 100 sweets to share with the students in her class. She is able to give each student an equal number of pieces of fruit and an equal number of sweets. What is the largest possible number of students in her class?

6 Sam has sheets of green and yellow plastic that he wants to use to make a square chequerboard pattern on a coffee-table top. Each sheet measures 210 cm by 154 cm. The squares are to be the maximum size possible. What will be the length of the side of each square and how many will he be able to cut from each sheet?

REFLECTION

Read these problems carefully. How can they help you to recognise similar problems in future, even if you are not told to use HCF and LCM?

1.3 Prime numbers

KEY LEARNING STATEMENTS

- Prime numbers only have two factors: 1 and the number itself.
- Prime factors are factors of a number that are also prime numbers.
- You can write any number as a product of prime factors. Remember the number 1 itself is *not* a prime number, so you cannot use it to write a number as the product of its prime factors.
- You can use the product of prime factors to find the HCF or LCM of two numbers.

KEY CONCEPT

Prime numbers and prime factors.

1 Identify the prime numbers in each set.

a 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

b 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60

c 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105

2 Express the following numbers as a product of their prime factors.

a 36 **b** 65 **c** 64 **d** 84

e 80 **f** 1000 **g** 1270 **h** 1963

3 Find the LCM and the HCF of the following numbers by using prime factors.

a 27 and 14 **b** 85 and 15 **c** 96 and 27 **d** 53 and 16

e 674 and 72 **f** 234 and 66 **g** 550 and 128 **h** 315 and 275

TIP

You can use a tree diagram or division to find the prime factors of a composite whole number.

1.4 Working with directed numbers

KEY LEARNING STATEMENTS

- Integers are directed whole numbers.
- You write negative integers with a minus (–) sign. Positive integers may be written with a plus (+) sign, but usually they are not.
- In real life, negative numbers are used to represent temperatures below zero; movements downwards or left; depths; distances below sea level; bank withdrawals and overdrawn amounts, and many more things.

KEY CONCEPTS

- Using directed numbers in practical situations.
- Basic calculations with positive and negative numbers.

1 If the temperature is 4 °C in the evening and it drops 7 °C overnight, what will the temperature be in the morning?

2 Which is colder in each pair of temperatures?

a 0 °C or −2 °C **b** 9 °C or −9 °C **c** −4 °C or −12 °C

3 An office block has three basement levels (−1, −2 and −3), a ground floor (0) and 15 floors above the ground floor (1 to 15). Where will the lift be in the following situations?

a Starts on the ground floor and goes down one floor then up five?

b Starts on level −3 and goes up ten floors?

c Starts on floor 12 and goes down 13 floors?

d Starts on floor 15 and goes down 17 floors?

e Starts on level −2, goes up seven floors and then down eight?

4 Write the number that is 12 less than:

a 9 **b** −14 **c** −2 **d** 12

5 Calculate:

a $-400 \div 80$ **b** $-54 + 120 + (-25)$

c $-3 \times (14 - (-12))$ **d** $\frac{-18}{6} \times 3$

e $13 + (-7) + 25 + (-15)$

6 The table shows how much the value of a rupee changed in comparison to the euro over a period of five days. The rate was 80.72 rupees : 1 euro before any changes were recorded.

Day	1	2	3	4	5
Change	−0.25	+0.14	−0.27	−2.08	−3.04

a What was the value of the rupee compared to the euro and the end of day 3?

b What was the total change over the period of five days? Give your answer as a directed number.

1.5 Powers, roots and laws of indices

KEY LEARNING STATEMENTS

- Index notation is a way of writing repeated multiplication. For example, you can write 2 × 2 × 2 as 2^3. 2 is the base and 3 is the index that tells you how many times 2 is multiplied by itself.
- The $\sqrt[x]{n}$ of a number is the value that is multiplied by itself x times to reach that number.
- Any number to the power of 0 is equal to 1: $a^0 = 1$.
- Negative indices are used to write reciprocals. a^{-m} is the reciprocal of a^m because $a^{-m} \times a^m = 1$.
- To multiply numbers with the same base you add the indices. In general terms $a^m \times a^n = a^{m+n}$.
- To divide numbers with the same base you subtract the indices. In general terms $\frac{a^m}{a^n} = a^{m-n}$.
- To raise a power to another power you multiply the indices. In general terms $(a^m)^n = a^{mn}$.

KEY CONCEPTS

- Calculating with squares, square roots, cubes, cube roots and other powers and roots of numbers.
- The meaning of zero and negative indices.
- The laws of indices.

1 Calculate.

a 3^2 **b** 18^2 **c** 21^2 **d** 25^2

e 6^3 **f** 15^3 **g** 18^3 **h** 35^3

2 Find these roots.

a $\sqrt{121}$ **b** $\sqrt[3]{512}$ **c** $\sqrt{441}$

d $\sqrt[3]{1331}$ **e** $\sqrt[3]{46656}$ **f** $\sqrt{2601}$

g $\sqrt{3136}$ **h** $\sqrt{729}$ **i** $\sqrt[4]{1296}$

3 Find all the square and cube numbers between 100 and 300.

4 Which of the following are square numbers and which are cube numbers?

1, 24, 49, 64, 256, 676, 625, 128

5 Simplify.

a $\sqrt{9} + \sqrt{16}$ **b** $\sqrt{9 + 16}$ **c** $\sqrt{64} + \sqrt{36}$

d $\sqrt{64 + 36}$ **e** $\sqrt{\frac{36}{4}}$ **f** $(\sqrt{25})^2$

g $\frac{\sqrt{9}}{\sqrt{16}}$ **h** $\sqrt{169 - 144}$ **i** $\sqrt[3]{27} - \sqrt[3]{1}$

j $\sqrt{100 \div 4}$ **k** $\sqrt{1} + \sqrt{\frac{9}{16}}$ **l** $\sqrt{16} \times \sqrt[3]{27}$

TIP

If you don't have a calculator, you can use the product of prime factors to find the square root or cube root of a number.

6 Evaluate.

a 4^3
b 7^4
c 16^4
d 12^3
e 20^3
f 10^5
g $13^3 - 3^5$
h $3^3 + 2^7$
i $\sqrt[3]{64} + 4^5$
j $(2^4)^3$

7 Rewrite each of the following using only positive indices.

a 4^{-1}
b 5^{-1}
c 8^{-1}
d 5^{-2}
e 3^{-3}
f 2^{-5}
g 3^{-4}
h 8^{-6}
i 23^{-3}
j 12^{-4}

8 Express each term using a negative index.

a $\frac{1}{2}$
b $\frac{1}{6}$
c $\frac{1}{3^2}$
d $\frac{1}{2^3}$
e $\frac{1}{3^3}$
f $\frac{1}{2^4}$
g $\frac{1}{11^2}$
h $\frac{1}{4^3}$
i $\frac{2}{10}$
j $\frac{3}{9}$

9 Simplify. Leave your answers in index form.

a $3^2 \times 3^6$
b $10^{-2} \times 10^4$
c $3^8 \times 3^{-5}$
d $5^0 \times 3^2$
e $2^{-3} \times 2^{-4}$
f $3 \times 3^2 \times 3^{-2}$
g $4^0 \times 4^{-2} \times 4$
h $10^2 \times 10^3 \times 10^{-2}$
i $(3^2)^0$
j $(4^3)^4$
k $(3^{-2})^{-3}$
l $(4^{-3} \times 4^2)^{-2}$
m $\frac{10^6}{10^{-3}}$
n $\frac{10^0}{10^4}$
o $\frac{2^{-4}}{2^{-5}}$
p $\frac{4^3}{4^{-3}}$

1.6 Order of operations

KEY LEARNING STATEMENTS

- When there is more than one operation to be done in a calculation you must work out the parts in brackets first. Next deal with powers and roots. Then do any division or multiplication (from left to right) before adding and subtracting (from left to right).
- Long fraction lines and square or cube root signs act like brackets, indicating parts of the calculation that have to be done first.
- Scientific calculators apply the rules for order of operations automatically. If there are brackets, fractions or roots in your calculation you need to enter these correctly on the calculator. When there is more than one term in the denominator, the calculator will divide by the first term only unless you enter brackets.

KEY CONCEPT

Calculating using the correct order of operations.

1 Calculate and give your answer correct to two decimal places.

a $8 + 3 \times 6$

b $(8 + 3) \times 6$

c $8 \times 3 - 4 + 5$

d $12.64 + 2.32 \times 1.3$

e $6.5 \times 1.3 - 5.06$

f $(6.7 \div 8) + 1.6$

g $1.453 + \frac{7.6}{3.2}$

h $\frac{5.34 + 3.315}{4.03}$

i $\frac{6.54}{2.3} - 1.08$

j $\frac{5.27}{1.4 \times 1.35}$

k $\frac{11.5}{2.9 - 1.43}$

l $\frac{0.23 \times 4.26}{1.32 + 3.43}$

m $8.9 - \frac{8.9}{10.4}$

n $\frac{12.6}{8.3} - \frac{1.98}{4.62}$

o $12.9 - 2.03^2$

p $(9.4 - 2.67)^3$

q $12.02^2 - 7.05^2$

r $\left(\frac{16.8}{9.3} - 1.01\right)^2$

s $\frac{4.07^2}{8.2 - 4.09}$

t $6.8 + \frac{1.4}{6.9} - \frac{1.2}{9.3}$

u $4.3 + \left(1.2 + \frac{1.6}{5}\right)^2$

v $\frac{6.1}{2.8} + \left(\frac{2.1}{1.6}\right)^2$

w $6.4 - (1.2^2 + 1.9^2)^2$

x $\left(4.8 - \frac{1}{9.6}\right) \times 4.3$

TIP

Remember the order of operations using BODMAS:

Brackets
Orders
Divide
Multiply
Add
Subtract

Some people remember the order of operations as BIDMAS – I stands for indices.

1.7 Rounding and estimating

KEY LEARNING STATEMENTS

- You may be asked to round numbers to a given number of decimal places or to a given number of significant figures.
- To round to a decimal place:
  - look at the value of the digit to the right of the place you are rounding to
  - if this value is ⩽5 then you round up (add 1 to the digit you are rounding to)
  - if this value is ⩽4 then leave the digit you are rounding to as it is.
- To round to a significant figure:
  - the first *non*-zero digit (before or after the decimal place in a number) is the first significant figure
  - find the correct digit and then round off from that digit using the rules above.
- Estimating involves rounding values in a calculation to numbers that are easy to work with (usually without the need for a calculator).
- An estimate allows you to check that your calculations make sense.

KEY CONCEPTS

- Rounding numbers to a given number of decimal places or significant figures.
- Estimating an approximate answer.

TIP

If you are told what degree of accuracy to use, it is important to round to that degree. If you are not told, you can round to three significant figures.

1 Round these numbers to:

i two decimal places
ii one decimal place
iii the nearest whole number.

a 5.6543 **b** 9.8774 **c** 12.8706 **d** 0.009
e 10.099 **f** 45.439 **g** 13.999 **h** 26.001

2 Round each of these numbers to three significant figures.

a 53 217 **b** 712 984 **c** 17.364 **d** 0.007279

3 Round the following numbers to two significant figures.

a 35.8 **b** 5.234 **c** 12 345 **d** 0.00875
e 432 128 **f** 120.09 **g** 0.00456 **h** 10.002

4 Use whole numbers to show why these estimates are correct.

a 3.9×5.1 is approximately equal to 20
b 68×5.03 is approximately equal to 350
c 999×6.9 is approximately equal to 7000
d $42.02 \div 5.96$ is approximately equal to 7

5 Estimate the answers to each of these calculations to the nearest whole number.

a $5.2 + 16.9 - 8.9 + 7.1$ **b** $(23.86 + 9.07) \div (15.99 - 4.59)$

c $\dfrac{9.3 \times 7.6}{5.9 \times 0.95}$ **d** $8.9^2 \times \sqrt{8.98}$

REVIEW EXERCISE

1 List the integers in the following set of numbers.

$\frac{3}{4}$ 24 0.65 −12 $3\frac{1}{2}$ 0 −15 0.66 −17

2 List the first five multiples of 15.

3 Find the lowest common multiple of 12 and 15.

4 Write each number as a product of its prime factors.

a 196 **b** 1845 **c** 8820

5 Find the HCF of 28 and 42.

6 Simplify:

a $\sqrt{100} \div \sqrt{4}$ **b** $\sqrt{100 \div 4}$ **c** $\left(\sqrt[3]{64}\right)^3$ **d** $4^3 + 9^2$

e $23 \times \sqrt[4]{1296}$ **f** $-24 \times \sqrt[3]{343}$ **g** $\left(\frac{1}{2}\right)^{-2} + \sqrt[5]{1}$ **h** $\left(\frac{1}{2}\right)^{-4} - \sqrt[6]{46\,656}$

7 Calculate. Give your answer correct to two decimal places.

a $\frac{5.4 \times 12.2}{4.1}$ **b** $\frac{12.2^2}{3.9^2}$ **c** $\frac{12.65}{2.04} + 1.7 \times 4.3$

d $\frac{3.8 \times 12.6}{4.35}$ **e** $\frac{2.8 \times 4.2^2}{3.3^2 \times 6.2^2}$ **f** $2.5 - \left(3.1 + \frac{0.5}{5}\right)^2$

8 Write each of the following in the form of 3^x.

a 1 **b** 27 **c** $\frac{1}{9}$ **d** $\frac{1}{3}$

e $3^4 \times 3^{-2}$ **f** $\frac{3^8}{3^8}$ **g** $(3^2)^4$ **h** $(3^{-2})^2$

9 Simplify. Leave your answers in index notation.

a $\frac{3^4 \times 3^7}{3^4}$ **b** $\frac{2^5 \times 2^4}{2^3}$ **c** $\frac{2^3 \times 2^{-4}}{2^2 \times 2^{-2}}$ **d** $\frac{4 \times 4^{-3}}{4^{-2} \times 4^0}$

10 Round each number to three significant figures.

a 1235.6 **b** 0.76513 **c** 0.0237548 **d** 31.4596

11 Naresh has 6400 square tiles. Is it possible for him to arrange these to make a perfect square? Justify your answer.

12 Ziggy has a square sheet of fabric with sides 120 cm long. Is this big enough to cover a square table of area 1.4 m^2? Explain your answer.

13 A cube has a volume of 3.375 m^3. How high is it?

14 Estimate the answer to each of these calculations to the nearest whole number.

a 9.75×4.108 **b** $0.0387 \div 0.00732$

c $\frac{36.4 \times 6.32}{9.987}$ **d** $\sqrt{64.25} \times 3.098^2$

TIP

If there are brackets, fractions or roots in your calculation you need to enter these correctly on the calculator. When there is more than one term in the denominator, the calculator will divide by the first term only unless you enter brackets.

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Cambridge IGCSE™

Mathematics Extended

PRACTICE BOOK

Karen Morrison

Third edition

Digital Acess

Cambridge Assessment
International Education

Endorsed for learner support

Contents

Unit 3

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How to use this book

This resource supports learners who are studying the Extended syllabus, for which knowledge of Core content is also required, and therefore includes questions on both the Core and Extended syllabus content. (Note that in the Coursebook, Extended-only content is flagged. No such flagging has been included in this resource as all the material is relevant for students studying the Extended syllabus.)

Throughout this book, you will notice lots of different features that will help your learning. These are explained below.

Exercises

These help you to practise skills that are important for studying Cambridge IGCSE™ Mathematics.There are two types of exercise:

- Exercises which let you practise the mathematical skills you have learned.
- Review exercises which bring together all the mathematical concepts in a chapter, pushing your skills further.

KEY LEARNING STATEMENT

This will remind you of what you should already know from your previous study in order to complete the exercises in this section.

REFLECTION

At the end of some exercises you will find opportunities to think about the approach that you take to your work, and how you might improve this in the future.

SELF ASSESSMENT

At the end of some exercises, you will find opportunities to help you assess your own work, and consider how you can improve the way you learn.

This icon shows you where you should complete an exercise without using your calculator.

KEY CONCEPTS

These summarise the important concepts that are covered in each section.

TIP

The information in these boxes will help you complete the exercises, and give you support in areas that you might find difficult.

Chapter 1: Review of number concepts

1.1 Different types of numbers

KEY LEARNING STATEMENTS

- Real numbers are either rational or irrational.
- You can write rational numbers as fractions in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$. (Integers are negative and positive whole numbers, and zero.)
- Rational numbers include integers, fractions, recurring and terminating decimals and percentages.

KEY CONCEPTS

- Classifying and using different types of numbers.
- Interpreting and using the symbols $=$, $\neq$, $<$, $>$, $\leqslant$ and $\geqslant$.

1 Use the numbers in the box. List the numbers that are:

a natural **b** integers **c** prime **d** fractions.

$-0.2 \quad -57 \quad 3.142 \quad 0 \quad 0.\dot{3} \quad 1 \quad 51 \quad 10\,270 \quad -\frac{1}{4} \quad \frac{2}{7} \quad 11 \quad \sqrt[3]{512}$

2 List:

a four square numbers greater than 100.

b four rational numbers smaller than $\frac{1}{3}$.

c two prime numbers that are > 80.

d the prime numbers < 10.

3 What number is halfway between:

a 6.2 and 6.5 **b** 4 and 1.2 **c** −3 and 7 **d** 39 and 40.1

4 Two of the highest earning films of all time are *Avatar*, with gross earnings of two billion, eight hundred and forty-seven million, three hundred and seventy-nine thousand, seven hundred and ninety-four dollars and *Avengers: Endgame* with gross earnings of two billion, seven hundred and ninety-seven million, five hundred and one thousand, three hundred and twenty-eight dollars.

a Write each amount in digits.

b What is the difference between the two amounts?
Give your answer in digits and in words.

1.2 Multiples and factors

KEY LEARNING STATEMENTS

- When you multiply a number by another number you get a multiple of the original number.
- The lowest common multiple (LCM) of two or more numbers is the lowest number that is a multiple of both (or all) of the numbers.
- Any number that will divide into a number exactly is a factor of that number.
- The highest common factor (HCF) of two or more numbers is the highest number that is a factor of all the given numbers.

KEY CONCEPT

Finding the highest common factor and lowest common multiple of two or more numbers.

1 Find the LCM of the given numbers.

a 9 and 18 **b** 12 and 18 **c** 15 and 18

d 24 and 12 **e** 36 and 9 **f** 12 and 8

2 Find the HCF of the given numbers.

a 12 and 18 **b** 18 and 36 **c** 27 and 90

d 12 and 15 **e** 20 and 30 **f** 19 and 45

3 Amira has two rolls of cotton fabric. One roll has 72 metres on it and the other has 90 metres on it. She wants to cut the fabric to make as many equal length pieces as possible of the longest possible length. How long should each piece be?

4 In a shopping mall promotion every 30th shopper gets a $10 voucher and every 120th shopper gets a free meal. How many shoppers must enter the mall before one receives both a voucher and a free meal?

5 Amanda has 40 pieces of fruit and 100 sweets to share with the students in her class. She is able to give each student an equal number of pieces of fruit and an equal number of sweets. What is the largest possible number of students in her class?

6 The Smit family want to tile their rectangular veranda with dimensions 3.2 metres × 6.72 metres with a whole number of identical square tiles. They want the tiles to be as large as possible.

a Find the area of the largest possible tiles in cm^2.

b How many of these tiles will they need to tile the veranda?

TIP

To find the LCM of a set of numbers, you can list the multiples of each number until you find the first multiple that is in the lists for all of the numbers in the set.

TIP

You need to work out whether to use LCM or HCF to find the answers. Problems involving LCM usually include repeating events. Problems involving HCF usually involve splitting things into smaller pieces or arranging things in equal groups or rows.

REFLECTION

Read through the problems in the exercise carefully.

How can they help you to recognise similar problems in future, even if you are not told to use HCF and LCM?

1.3 Prime numbers

KEY LEARNING STATEMENTS

- Prime numbers only have two factors: 1 and the number itself.
- Factors of a number that are also prime numbers are called prime factors.
- You can write any number as a product of prime factors. Remember the number 1 itself is *not* a prime number, so you cannot use it to write a number as the product of its prime factors.
- You can use the product of prime factors to find the HCF or LCM of two numbers.

KEY CONCEPT

Prime numbers and prime factors.

1 Identify the prime numbers in each set.

a 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

b 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60

c 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105

2 Express the following numbers as a product of their prime factors.

a 36 **b** 65 **c** 64 **d** 84

e 80 **f** 1000 **g** 1270 **h** 1963

3 Find the LCM and the HCF of the following numbers by using prime factors.

a 27 and 14 **b** 85 and 15 **c** 96 and 27

d 53 and 16 **e** 674 and 72 **f** 270 and 234

TIP

You can use a tree diagram or division to find the prime factors of a composite whole number.

1.4 Working with directed numbers

KEY LEARNING STATEMENTS

- Integers are directed whole numbers.
- You write negative integers with a minus (−) sign. Positive integers may be written with a plus (+) sign, but usually they are not. Zero (0) is an integer because it is a whole number but it is neither negative nor positive.
- In real life, negative numbers are used to represent temperatures below zero, movements downwards or left, depths, distances below sea level, bank withdrawals and overdrawn amounts, and many more things.

KEY CONCEPTS

- Using directed numbers in practical situations.
- Basic calculations with positive and negative numbers.

1 If the temperature is 4 °C in the evening and it drops 7 °C overnight, what will the temperature be in the morning?

2 Which is colder in each pair of temperatures?

a 0 °C or −2 °C **b** 9 °C or −9 °C **c** −4 °C or −12 °C

3 An office block has three basement levels (−1, −2 and −3), a ground floor (0) and 15 floors above the ground floor (1 to 15). Where will the lift be in the following situations?

a Starts on the ground floor and goes down one floor then up five?

b Starts on level −3 and goes up ten floors?

c Starts on floor 12 and goes down 13 floors?

d Starts on floor 15 and goes down 17 floors?

e Starts on level −2, goes up seven floors and then down eight?

4 Write the number that is 12 less than:

a 9 **b** −14 **c** −2 **d** 12

5 Calculate:

a $-400 \div 80$ **b** $-54 + 120 + (-25)$

c $-3 \times (14 - (-12))$ **d** $\frac{-18}{6} \times 3$

e $13 + (-7) + 25 + (-15)$

6 The table shows how much the value of a rupee changed in comparison to the euro over a period of five days. The rate was 80.72 rupees : 1 euro before any changes were recorded.

Day	1	2	3	4	5
Change	−0.25	+0.14	−0.27	−2.08	−3.04

a What was the value of the rupee compared to the euro and the end of day 3?

b What was the total change over the period of five days? Give your answer as a directed number.

1.5 Powers, roots and laws of indices

KEY LEARNING STATEMENTS

- Index notation is a way of writing repeated multiplication. For example, you can write $2 \times 2 \times 2$ as 2^3. 2 is the base and 3 is the index that tells you how many times 2 is multiplied by itself.
- The $\sqrt[x]{n}$ of a number is the value that is multiplied by itself x times to reach that number.
- Any number to the power of 0 is equal to 1: $a^0 = 1$.
- Negative indices are used to write reciprocals. a^{-m} is the reciprocal of a^m because $a^{-m} \times a^m = 1$.
- You can use fractional indices to express the roots of numbers. $\sqrt{a} = a^{\frac{1}{2}}$, $\sqrt[3]{a} = a^{\frac{1}{3}}$ and $\sqrt[n]{a} = a^{\frac{1}{n}}$. For non-unit fractions, $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$.
- To multiply numbers with the same base you add the indices. In general terms $a^m \times a^n = a^{m+n}$.
- To divide numbers with the same base you subtract the indices. In general terms $\frac{a^m}{a^n} = a^{m-n}$.
- To raise a power to another power you multiply the indices. In general terms $(a^m)^n = a^{mn}$.

KEY CONCEPTS

- Calculating with squares, square roots, cubes, cube roots and other powers and roots of numbers.
- The meaning of zero, negative and fractional indices.
- The laws of indices.

1 Find all the square and cube numbers between 100 and 300.

2 Simplify.

a $\sqrt{9} + \sqrt{16}$ **b** $\sqrt{9 + 16}$ **c** $\sqrt{64} + \sqrt{36}$

d $\sqrt{64 + 36}$ **e** $\sqrt{\frac{36}{4}}$ **f** $(\sqrt{25})^2$

g $\frac{\sqrt{9}}{\sqrt{16}}$ **h** $\sqrt{169 - 144}$ **i** $\sqrt[3]{27} - \sqrt[3]{1}$

j $\sqrt{100 \div 4}$ **k** $\sqrt{1} + \sqrt{\frac{9}{16}}$ **l** $\sqrt{16} \times \sqrt[3]{27}$

m $\sqrt{(-5)^2} \times \sqrt[3]{-1}$ **n** $\sqrt{\frac{1}{4}} + \sqrt{\left(\frac{1}{3}\right)^2}$ **o** $\sqrt[3]{1} - \sqrt[3]{-125}$

3 Find the value of the following.

a $13^3 - 3^5$ **b** $3^3 + 2^7$ **c** $\sqrt[3]{64} + 4^5$

d $(2^4)^3$ **e** $5^4 \times \sqrt[5]{32}$ **f** $\sqrt[6]{729} \times 5^4$

g $\sqrt[4]{625} + 5^5$

4 A cube has a volume of 12 167 cm^3. Calculate:

a the height of the cube **b** the area of one face of the cube.

5 Rewrite each of the following using only positive indices.

a 4^{-1} **b** 5^{-1} **c** 8^{-1} **d** 5^{-2} **e** 3^{-3}

f 2^{-5} **g** 3^{-4} **h** 8^{-6} **i** 23^{-3} **j** 12^{-4}

6 Express each term using a negative index.

a $\frac{1}{2}$ **b** $\frac{1}{6}$ **c** $\frac{1}{3^2}$ **d** $\frac{1}{2^3}$ **e** $\frac{1}{3^3}$

f $\frac{1}{2^4}$ **g** $\frac{1}{11^2}$ **h** $\frac{1}{4^3}$ **i** $\frac{2}{10}$ **j** $\frac{3}{9}$

7 Simplify. Leave your answers in index form.

a $3^2 \times 3^6$ **b** $10^{-2} \times 10^4$ **c** $3^8 \times 3^{-5}$ **d** $5^0 \times 3^2$

e $2^{-3} \times 2^{-4}$ **f** $3 \times 3^2 \times 3^{-2}$ **g** $4^0 \times 4^{-2} \times 4$

h $10^2 \times 10^3 \times 10^{-2}$

i $(3^2)^0$ **j** $(4^3)^4$ **k** $(3^{-2})^{-3}$ **l** $(4^{-3} \times 4^2)^{-2}$

m $\frac{10^6}{10^{-3}}$ **n** $\frac{10^0}{10^4}$ **o** $\frac{2^{-4}}{2^{-5}}$ **p** $\frac{4^3}{4^{-3}}$

8 Write each value using a root sign.

a $3^{\frac{1}{2}}$ **b** $4^{\frac{1}{3}}$ **c** $5^{\frac{1}{9}}$ **d** $4^{\frac{3}{8}}$ **e** $6^{\frac{4}{9}}$

9 Write in index notation.

a $\sqrt{7}$ **b** $\sqrt[3]{6}$ **c** $\left(\sqrt[3]{8}\right)^5$ **d** $\left(\sqrt[4]{9}\right)^3$ **e** $\left(\sqrt[6]{5}\right)^5$

TIP

Apply the index laws and work in this order:

- simplify any terms in brackets
- apply the multiplication law to numerators and then to denominators
- cancel numbers if you can
- apply the division law if the same letter appears in the numerator and denominator
- express your answer using positive indices.

10 Evaluate.

a 5^{-2} **b** $81^{\frac{1}{2}}$ **c** $\left(\frac{2}{3}\right)^{-1}$ **d** $7^{-\frac{2}{3}}$ **e** $\left(\frac{5}{2}\right)^{-2}$

f $64^{\frac{1}{6}}$ **g** $81^{\frac{3}{4}}$ **h** $\left(0.64^{\frac{1}{2}}\right)^2$ **i** $3 \times 36^{\frac{1}{2}}$ **j** $\left(3^{\frac{1}{2}}\right)^{-4}$

11 Evaluate.

a $(-3^4) \times (-4)^2$

b $\dfrac{-2^4}{(-2)^4}$

c $\dfrac{6^3}{(-3)^4}$

d $8^{\frac{1}{3}}$

e $256^{-\frac{1}{4}}$

f $125^{-\frac{4}{3}}$

g $\left(\dfrac{1}{4}\right)^{-\frac{5}{2}}$

h $\left(\dfrac{1}{8}\right)^{-\frac{2}{3}}$

i $\left(\dfrac{8}{27}\right)^{-\frac{1}{3}}$

j $\left(\dfrac{8}{18}\right)^{-\frac{1}{2}}$

12 Calculate.

a $5 - 7(23 - 5^2) - 16 \div 2^3$

b $3(5^2) - 6(-3^2 - 4^2) \div -15$

c $-2(-3^2) + 24 \div (-2)^3$

d $-2(3)^4 - (6 - 7)^6$

1.6 Order of operations

KEY LEARNING STATEMENTS

- When there is more than one operation to be done in a calculation you must work out the parts in brackets first. Then do any division or multiplication (from left to right) before adding and subtracting (from left to right).
- Long fraction lines and square or cube root signs act like brackets, indicating parts of the calculation that have to be done first.
- Scientific calculators apply the rules for order of operations automatically. If there are brackets, fractions or roots in your calculation you need to enter these correctly on the calculator. When there is more than one term in the denominator, the calculator will divide by the first term only unless you enter brackets.

KEY CONCEPT

Calculating using the correct order of operations.

1 Calculate and give your answer correct to two decimal places.

a $8 + 3 \times 6$

b $(8 + 3) \times 6$

c $8 \times 3 - 4 \div 5$

d $12.64 + 2.32 \times 1.3$

e $6.5 \times 1.3 - 5.06$

f $(6.7 \div 8) + 1.6$

g $1.453 + \dfrac{7.6}{3.2}$

h $\dfrac{5.34 + 3.315}{4.03}$

i $\dfrac{6.54}{2.3} - 1.08$

j $\dfrac{5.27}{1.4 \times 1.35}$

k $\dfrac{11.5}{2.9 - 1.43}$

l $\dfrac{0.23 \times 4.26}{1.32 + 3.43}$

m $8.9 - \dfrac{8.9}{10.4}$

n $\dfrac{12.6}{8.3} - \dfrac{1.98}{4.62}$

o $12.9 - 2.03^2$

p $(9.4 - 2.67)^3$

q $12.02^2 - 7.05^2$

r $\left(\dfrac{16.8}{9.3} - 1.01\right)^2$

s $\dfrac{4.07^2}{8.2 - 4.09}$

t $6.8 + \dfrac{1.4}{6.9} - \dfrac{1.2}{9.3}$

u $4.3 + \left(1.2 + \dfrac{1.6}{5}\right)^2$

v $\dfrac{6.1}{2.8} + \left(\dfrac{2.1}{1.6}\right)^2$

w $6.4 - (1.2^2 + 1.9^2)^2$

x $\left(4.8 - \dfrac{1}{9.6}\right) \times 4.3$

TIP

Remember the order of operations using BODMAS:

Brackets
Orders
Divide
Multiply
Add
Subtract

Some people remember the order of operations as BIDMAS – I stands for indices.

1.7 Rounding and estimating

KEY LEARNING STATEMENTS

- You may be asked to round numbers to a given number of decimal places or to a given number of significant figures.
- To round to a decimal place:
  - look at the value of the digit to the right of the place you are rounding to
  - if this value is $\geqslant 5$ then you round up (add 1 to the digit you are rounding to)
  - if this value is $\leqslant 4$ then leave the digit you are rounding to as it is.
- To round to a significant figure:
  - the first non-zero digit (before or after the decimal place in a number) is the first significant figure
  - find the correct digit and then round off from that digit using the rules above.
- Estimating involves rounding values in a calculation to numbers that are easy to work with (usually without the need for a calculator).
- An estimate allows you to check that your calculations make sense.

KEY CONCEPTS

- Rounding numbers to a given number of decimal places or significant figures.
- Estimating an approximate answer.

1 Round these numbers to:

i two decimal places
ii one decimal place
iii the nearest whole number.

a 5.6543 **b** 9.8774 **c** 12.8706 **d** 0.0098
e 10.099 **f** 45.439 **g** 13.999 **h** 26.001

2 Round each of these numbers to three significant figures.

a 53 217 **b** 712 984 **c** 17.364 **d** 0.007279

3 Round the following numbers to two significant figures.

a 35.8 **b** 5.234 **c** 12 345 **d** 0.00875
e 432 128 **f** 120.09 **g** 0.00456 **h** 10.002

4 Use whole numbers to show why these estimates are correct.

a 3.9×5.1 is approximately equal to 20
b 68×5.03 is approximately equal to 350
c 999×6.9 is approximately equal to 7000
d $42.02 \div 5.96$ is approximately equal to 7

TIP

If you are told what degree of accuracy to use, it is important to round to that degree. If you are not told, you can round to 3 significant figures.

5 Estimate the answers to each of these calculations to the nearest whole number.

a $5.2 + 16.9 - 8.9 + 7.1$ **b** $(23.86 + 9.07) \div (15.99 - 4.59)$

c $\dfrac{9.3 \times 7.6}{5.9 \times 0.95}$ **d** $8.9^2 \times \sqrt{8.98}$

REVIEW EXERCISE

1 State whether each number is natural, rational, an integer and/or a prime number.

$-\frac{3}{4}$ 24 0.65 −12 $3\frac{1}{2}$ 0 0.66 17

2 **a** List the factors of 36.

b How many of these factors are prime numbers?

c Express 36 as the product of its prime factors.

d List two numbers that are factors of both 36 and 72.

e What is the highest number that is a factor of both 36 and 72?

3 Write each number as a product of its prime factors.

a 196 **b** 1845 **c** 8820

4 Amira starts a new exercise routine on 3 March. She decides she will swim every three days and cycle every four days. On which dates in March will she swim and cycle on the same day?

5 State whether each equation is true or false.

a $18 \div 6 + (5 + 3 \times 4) = 20$ **b** $6 \times (5 - 4) + 3 = 9$

c $\dfrac{30 + 10}{30} - 10 = 1$ **d** $(6 + 3)^2 = 45$

6 Simplify:

a $\sqrt{100} \div \sqrt{4}$ **b** $\sqrt{100 \div 4}$

c $(\sqrt[3]{64})^3$ **d** $4^3 + 9^2$

e $2^3 \times \sqrt[4]{1296}$ **f** $(-2)^4 \times \sqrt[3]{343}$

g $\left(\dfrac{1}{2}\right)^{-2} + \sqrt[5]{1}$ **h** $\left(\dfrac{1}{2}\right)^{-4} - \sqrt[6]{46\,656}$

CONTINUED

7 Calculate. Give your answer correct to two decimal places.

a $\frac{5.4 \times 12.2}{4.1}$ **b** $\frac{12.2^2}{3.9^2}$ **c** $\frac{12.65}{2.04} + 1.7 \times 4.3$

d $\frac{3.8 \times 12.6}{4.35}$ **e** $\frac{2.8 \times 4.2^2}{3.3^2 \times 6.2^2}$ **f** $2.5 - \left(3.1 + \frac{0.5}{5}\right)^2$

8 Write each of the following in the form of 3^x.

a 1 **b** 27 **c** $\frac{1}{9}$ **d** $\frac{1}{3}$

e $\sqrt{27}$ **f** $3^4 \times 3^{-2}$ **g** $\frac{3^8}{3^8}$ **h** $\frac{3^2}{3^4}$

i $(3^2)^4$ **j** $(3^{-2})^2$

9 Simplify. Leave your answers in index notation.

a $\frac{3^4 \times 3^7}{3^4}$ **b** $\frac{2^5 \times 2^4}{2^3}$ **c** $\frac{2^3 \times 2^{-4}}{2^2 \times 2^{-2}}$ **d** $\frac{4 \times 4^{-3}}{4^{-2} \times 4^0}$

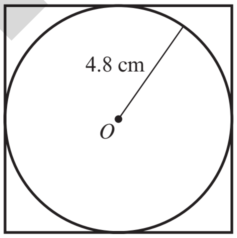
10 Determine the value of x in each equation.

a $\frac{2^2}{2^5} = 2^x$ **b** $2 \times 2^x = \frac{2^3}{2^5}$ **c** $\frac{3^x}{3} = \frac{3^2}{3^5}$ **d** $2^2 \times 2^{-x} = \frac{2^2}{2^6}$

11 Round each number to three significant figures.

a 1235.6 **b** 0.76513 **c** 0.0237548 **d** 31.4596

12 The diagram shows a design for a square tile with a circle printed on it. The circle has a radius of 4.8 cm.



a What is the area of the square tile?

b How much of the square is not covered by the circle? Give your answer correct to two decimal places.

TIP

You can find the area of a circle using the formula $A = \pi r^2$.

13 Ziggy has a square sheet of fabric with sides 120 cm long. Is this big enough to cover a square table of area 1.4 m²? Explain your answer.

14 A cube has a volume of 3.375 m³. How high is it?

15 Estimate the answer to each of these calculations to the nearest whole number.

a 9.75×4.108 **b** $0.0387 \div 0.00732$

c $\frac{36.4 \times 6.32}{9.987}$ **d** $\sqrt{64.25} \times 3.098^2$

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Cambridge IGCSE™
Mathematics Core and Extended

TEACHERS RESOURCE

Karen Morrison & Nick Hamshaw

Third edition

Digital Access

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Contents

Additional downloadable resources

The following items are available on Cambridge GO.

Syllabus correlation grid

Lesson plan template

Letter to parents

Approaches to learning and teaching

Active learning

Assessment for learning

Developing language skills

Differentiation

Language awareness

Metacognition

Skills for Life

Glossary

Coursebook answers

Core Practice Book answers

Extended Practice Book answers

How to use this Teacher's Resource

This Teacher's Resource contains both general guidance and teaching notes that help you to deliver the content in our Cambridge resources.

There are **teaching notes** for each chapter of the Coursebook. Each set of teaching notes contains the following features to help you deliver the unit.

At the start of each chapter there is a **learning plan**. This summarises the syllabus learning objectives and the related Coursebook learning intentions. Where parts of syllabus learning objectives are not covered in that specific chapter, they are shown in grey text.

Syllabus learning objectives	Learning intentions	Resources

Background knowledge gives guidance on what prerequisite knowledge and skills students should have before they start each chapter.

BACKGROUND KNOWLEDGE

A **common misconceptions** table identifies common misconceptions that students may have about the topic and suggests how you can overcome these.

Misconception	How to identify	How to overcome

Language support provides suggestions for supporting your EAL students with the mathematical language they encounter.

LANGUAGE SUPPORT

Useful **links to digital resources** give further engaging resources that you could use with your classes.

Links to digital resources

1. This Venn diagram gives a representation of the various types of numbers and their relationships.

Real-life contexts help you bring the subject of each chapter to life for your students.

REAL-LIFE CONTEXT

For each chapter, there are teaching ideas including **starter ideas**, **main activities, plenary ideas** and **homework ideas**. The activities include suggestions for how they can be differentiated depending on the needs of your students. The teaching ideas are intended to provide inspiration for delivering a key concept in the chapter, and can be used flexibly within your own teaching plan.

Starter ideas

1 Classified numbers (10–15 minutes)

Resources: A list of various types of number; the table in Section 1.1 of the Coursebook showing the

You can download additional resources including **presentations** for each chapter, **problem solving** and **calculator use** support, **answers** to exercises in the Coursebook, Core Practice Book and Extended Practice Book, the Coursebook **glossary**, a **syllabus correlation grid**, **lesson plan template** and more detailed guidance around **approaches to learning and teaching**.

1 Reviewing number concepts

Learning plan

Suggested learning hours: 5

Syllabus learning objectives	Learning intentions	Resources
C1.1/E1.1 Types of number Identify and use natural numbers, integers (positive, negative and zero), prime numbers, square numbers, cube numbers, common factors and common multiples, rational and irrational numbers, and reciprocals.	• identify and classify different types of numbers • find common factors and common multiples of numbers • write numbers as products of their prime factors • work with integers used in real-life situations	Coursebook Sections 1.1, 1.2 and 1.3 Practice Books Sections 1.1, 1.2 and 1.3 Teacher's Resource Starter ideas 1 and 2 Teacher's Resource Main teaching idea 1
C1.3/E1.3 Powers and roots Calculate with squares, square roots, cubes and cube roots of numbers and other powers and roots of numbers.	• calculate with powers and roots of numbers	Coursebook Section 1.5 Practice Books Section 1.5 Teacher's Resource Main teaching idea 2
C1.6/E1.6 The four operations Use the four operations for calculations with integers, fractions and decimals, including correct ordering of operations and use of brackets.	• revise the basic rules for operating with numbers	Coursebook Sections 1.4 and 1.6 Practice Books Sections 1.4 and 1.6 PowerPoints 1.1 and 1.2 Teacher's Resource Plenary ideas 1 and 2
C1.7 Indices I 1 Understand and use indices (positive, zero and negative integers). 2 Understand and use the rules of indices.	• understand the meaning of indices • use the rules of indices	Coursebook Section 1.5 Core Practice Book Section 1.5

Syllabus learning objectives	Learning intentions	Resources
E1.7 Indices I 1 Understand and use indices (positive, zero, negative, and fractional). 2 Understand and use the rules of indices.	• understand the meaning of indices • use the rules of indices	Coursebook Section 1.5 Extended Practice Book Section 1.5
C1.9/E1.9 Estimation 1 Round values to a specified degree of accuracy. 2 Make estimates for calculations involving numbers, quantities and measurements. 3 Round answers to a reasonable degree of accuracy in the context of a given problem.	• round numbers in different ways to estimate and approximate answers	Coursebook Section 1.7 Practice Books Section 1.7 Teacher's Resource Homework ideas 1 and 2
C1.14/E1.14 Using a calculator 1 Use a calculator efficiently. 2 Enter values appropriately on a calculator. 3 Interpret the calculator display appropriately.	• perform basic calculations using mental methods and with a calculator	Coursebook Exercise 1.17 Practice Books Section 1.6 PowerPoint 1.2

BACKGROUND KNOWLEDGE

- Learners will already be familiar with most, if not all, of the concepts in this chapter from their work in Primary and Lower Secondary Mathematics. This chapter provides opportunities to identify any learners who still struggle with basic arithmetic and give support so they are able to make progress with the more advanced concepts that will be covered in later chapters.

Common misconceptions

Misconception	How to identify	How to overcome
Learners may often think that fractions and rational numbers are the same.	Ask learners to give an example of a fraction that is not a rational number, or ask them if $\frac{\pi}{2}$ is both a fraction and a rational number.	Remind learners that for a fraction $\frac{a}{b}$, there is no restriction on the type of numbers represented by *a* and *b* (except that *b* must not be zero). However, for a rational number $\frac{a}{b}$, *a* and *b* must be non-zero integers.

Common misconceptions continued		
Learners may think that the number zero is odd, or neither odd nor even.	Ask learners whether zero is odd or even.	Remind learners of the definition of an even number: a number that is divisible by 2. Zero can be divided by 2, and so is even.
Learners may think that irrational numbers such as π are rational with terminating decimals because they are often presented as an approximation (for example, $\pi \approx 3.14$).	Look out for learners who write or use the value of π or another irrational number) in its approximate form $\left(3.14 \text{ or } \frac{22}{7}\right)$ but use the equals sign (=) rather than the approximately equal to sign (≈).	Remind learners that when they write an answer in terms of π they can use the equals sign, but otherwise, they need to use the approximately equal to symbol.

TEACHING SKILLS FOCUS

Area of focus: Metacognition

Specific focus: Thinking aloud

Benefits: *Thinking aloud*, when used by the teacher, makes the thinking process clear to the learners. This is particularly helpful when learning mathematics because learners can often find the steps taken to solve a problem to be mysterious or even magical. By uncovering the thinking process behind each of the problem-solving steps, teachers model the kind of thinking process that learners need to follow and understand. When *Thinking aloud* is used by learners, it makes their thinking clear for the teacher who can then formatively assess whether the learners are on the right track or whether there are misconceptions that need to be addressed immediately.

Practise: You can use this strategy when explaining how to find the highest common factor and the lowest common multiple in this chapter. For example, while writing each step on the board, explain verbally the thinking process that is taking place while you work through each step. The decision diagram for problems involving calculation shown in the Getting started activity in the Coursebook is a good example of a thinking process made explicit. Model *Thinking aloud* as you work through questions with the class, and also ask learners to use it when they are explaining what they have done to solve a problem.

Reflect: After modelling the *Thinking aloud* strategy, ask yourself if you made **all** of the thinking process explicit or whether there were particular parts you left out. When asking learners to use the strategy, did you try to help them by telling them your own thinking process or putting your words in their mouth, or did you encourage them by asking questions to reveal their thinking? What questions could you prepare in advance in order to support learners in thinking aloud?

LANGUAGE SUPPORT

There are differences in the terms used in some countries. For example, the mnemonic to help remember the order of operations used in this book is BODMAS/BIDMAS. This may be different in some countries, particularly the terms 'brackets' versus 'parentheses', and 'indices' versus 'exponents' which leads to the alternative mnemonic, PEMDAS.

Links to digital resources

- This Venn diagram gives a representation of the various types of numbers and their relationships.
- In this Factors and Multiples Game, learners can compete or collaborate to find the longest chain possible.
- This social media thread illustrates the problem of ambiguity when it comes to order of operations.

REAL-LIFE CONTEXT

Numbers are used all around us when dealing with quantities and they are involved in almost all our day-to-day activities. For example:

- When shopping, we use numbers when we think about the price per unit of an item or the mass of a product. We also use numbers to calculate any discounts and to estimate if we have enough money for the purchase.
- In baking and cooking, we measure ingredients, often using fractions.
- When traveling, we need to consider the distance, time (departure, arrival, duration) and cost (fuel, road tolls).

Starter ideas

1 Classified numbers (10–15 minutes)

Resources: A list of various types of number; the table in Section 1.1 of the Coursebook showing the different types of number

Description and purpose: In this activity learners will classify numbers according to their type. Briefly review the different types of numbers using the table in Section 1.1 of the Coursebook and also show learners a visual representation using a Venn diagram. For example this Venn diagram shows various types of numbers and their relationships.

Divide the class into two groups. Group A will go first. Write a number or show a prepared number card, for example $\sqrt{2}$ and ask Group A to determine what type of number it is. If Group A picks a type of number which is not the smallest subset the number belongs to, for example if Group A says 'real number', then Group B has the chance to steal a point by saying 'irrational number'. Each member of the group takes a turn at answering, but is allowed to pass if they are unsure. The game continues, alternating between the two groups. A bonus point can be given if a group is able in addition to identify any special types of number such as odd, even, prime, square, etc.

What to do next: Based on the learners' responses, you may want to take some time to review specific types of numbers.

2 Factors and multiples game (15–20 minutes)

Resources: Factors and multiples game

Description and purpose: The purpose of this activity is for learners to review factors and multiples in a fun and engaging way. Instead of using a 100 square grid as in the online version of the factors and multiples game, use a six by eight grid showing the numbers 1 to 48. The first player circles a number, then the second player crosses out a number that is either a factor or a multiple of the first number. Players continue circling (player 1) and crossing out (player 2) numbers. The player who can no longer circle or cross out a number has 10 points deducted and the game ends. Find the sum of the numbers each player gets (has either circled or crossed out) and the player with the largest sum after any deduction is the winner.

What to do next: While learners are playing the game in pairs, check to see if any of them are struggling to find factors or multiples of numbers. You may want to remind them about the concepts and how to determine them or ask their partners to teach them. If necessary, model playing the game with one of the learners and use the *Thinking aloud* strategy to demonstrate the thinking process.

Main teaching ideas

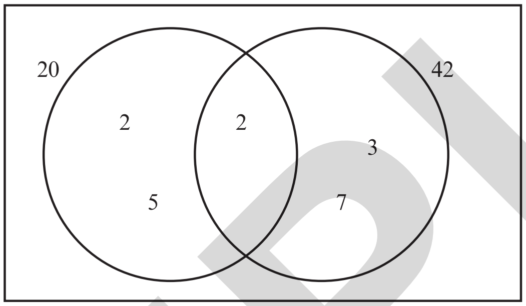
1 Prime factorisation (20–25 minutes)

Learning intention: Learners will make use of prime factorisation to solve problems.

Description and purpose: In this activity learners will find the prime factorisation of numbers using a prime factor tree and use it to find the highest common factor (HCF) and the lowest common multiple (LCM) in order to solve problems.

Demonstrate to learners how to find prime factors using a prime factor tree as shown in Worked Example 5 in the Coursebook and show how they can use these to find the HCF and LCM of the numbers.

An alternative method is to use a Venn diagram to show the HCF and LCM visually. For example, the diagram shows the prime factors of 20 and 42. The HCF is equal to the product of factors in the intersection (2) and the LCM is the product of all the factors shown in the Venn diagram (2 × 2 × 3 × 5 × 7 = 420).



Show learners that they can also write a number as a product of prime factors using index notation. For example, 168 = 2 × 2 × 3 × 7 = 23 × 31 × 71. (Note that the power of 1 is usually not written.)

Present the following problems and ask learners to solve them using what they know about prime factorisation.

a If a positive integer, *n*, is divisible by both 225 and 216, and $n = 2^x \times 3^y \times 5^z$, where *x*, *y* and *z* are positive integers, what are the least values of *x*, *y* and *z*?

b What is the smallest possible integer *n* such that the product 84 × *n* is a perfect square?

Answers:

a 3, 3, 2

b 21

Differentiation ideas:

Support: Ask the following questions to guide learners in their thinking:

- What does it mean that *n* is divisible by 225 and divisible by 216?
- What are the prime factorisations of 225 and 216, and how may they help you find the solution?
- What does the phrase 'least values' remind you of?

Challenge: Ask learners to come up with a proof that they are indeed the least values. They may use proof by contradiction, i.e., assume that there are smaller integers that are the smallest values, and show that it is a false assumption. Alternatively, learners may show and argue visually using a Venn diagram.

2 Power patterns (25–30 minutes)

Learning intention: Learners will explore patterns in the units (ones) digit of numbers raised to a power and use the pattern to find the units digit when the number is raised to a high power.

Description and purpose: The purpose of this activity is for learners to explore patterns in the units digit of numbers when they are raised to a power. Ask learners to start with the number 1 and find its value when it is raised to different powers (1, 2, 3 …). They should then repeat this starting with each of the numbers from 2 to 10. Ask them to see if they notice any patterns in the units digit each time. For example, the pattern for the exponents of the number 1 is always 1, for 2, it is 2, 4, 8, 6, 2, 4, 8, 6, … and continues in cycle of 4.

Complete the first two rows of this table and ask learners to complete the remaining rows. (Answers are shown in red.)

Units digit	Pattern of units digit	Length of cycle of pattern
1	1	1
2	2, 4, 8, 6	4
3	3, 9, 7, 1	4
4	4, 6	2
5	5	1
6	6	1
7	7, 9, 3, 1	4
8	8, 4, 2, 6	4
9	9, 1	2
0	0	1

Learners may also notice a pattern for the 'Length of cycle' column: 1, 4, 4, 2, 1, … .

Now, ask learners to use what they have found to help them determine the units digit for the following numbers, and explain their reasoning.

a 2^{71}

b 3^{123}

c 19^{99}

d $128^{57} \times 4^{29}$

e $11^{11} + 13^{13} + 15^{15} + 17^{17}$

Answers:

a 8

b 7

c 1

d 8

e 6

Differentiation ideas:

Support: Work through one or two problems together with the learners, to guide them step-by-step paying particular attention to the thinking process. Remind them that they need only to consider the units digit.

Challenge: Ask learners to explain why numbers raised to powers follow the particular patterns and also if they notice a pattern with the lengths of cycle (1, 4, 4, 2, 1 and repeats 1, 4, 4, 2, 1) and why this is so.

Plenary ideas

1 (Dis)order of operations (5–10 minutes)

Resources: A social media thread showing the problem of ambiguity when it comes to order of operations.

Description and purpose: The purpose of this activity is for learners to think about situations where there may be some ambiguity when using the order of operations and what they need to do.

Present this arithmetic problem to learners and ask them to solve it: $6 \div 2(1 + 2)$

Learners may disagree about the answer. Some may think that the answer is one while others think it is nine. Ask learners to try to identify what the problem is. The difficulty might become more obvious if the multiplication symbol is included between the two and the brackets to give $6 \div 2 \times (1 + 2)$, and learners may find that their calculators give different answers to the calculation if it they input it with and without this multiplication symbol.

Explain to learners that without a specific context, the order here is not entirely clear and so both answers can be defended. However, when a context is given, the order will become clear. You can also explain that including a second set of brackets to indicate either $6 \div (2(1 + 2))$ or $(6 \div 2)(1 + 2)$ will make the calculation unambiguous.

2. Operations with directed numbers (5–10 minutes)

Resources: Masking tape

Description and purpose: In this activity learners will explain the rules for operations involving directed numbers. Ask learners to study the rules for operating with directed numbers shown before Exercise 1.11 in the Coursebook. Then ask them to explain the reason behind the rules using an analogy.
For example, they may use the analogy of addition as a person moving forward on the number line, subtracting as moving backward. The positive and negative sign shows which direction the person is facing, positive is facing to the right on the number line and negative as facing to the left. So, for the calculation $5 - (-2)$ the person starts at five, faces left because the two is negative and moves backwards because they are subtracting. They will land on seven.

For multiplication, $a \times b$, encourage learners to imagine a person starting at zero and to think of b as how many steps to jump and the direction to be facing (positive to the right and negative to the left) and to think of a as how many times the person must jump either moving forwards (for positive a) or backwards (for negative a). So the calculation -4×-3 means jumping in steps of three while facing left, and jumping backwards four times, landing on 12.

Learners may demonstrate the operations using a physical number line made using masking tape on the floor.

Homework ideas

1 Rounding

Description: Ask learners to compare rounding to significant figures and decimal places. Both methods are used to round to specific levels of accuracy, but ask learners to explain the difference between them and how each method is used.

2 Accuracy decisions

Description: Ask learners to create a decision diagram (such as the one found in Question 4 of the Getting started section in Chapter 1 in the Coursebook) for making decisions about accuracy, i.e. determining when to round values to estimate.

Unit 1 project guidance

Star polygons

Why do this problem?

This problem offers a nice opportunity for students to practise measuring angles in a situation that leads them to make a discovery that they can then try to prove.

Possible approach

Show the image of the five-pointed star from the problem or draw one, and invite students to draw stars of their own. Ask them to measure the angles at the points as accurately as they can, and then add the five angles together. Collect some of their results on the board.

"It looks like the answers are all around 180°. I wonder if we can prove it?"

There are a number of options for proving the result. Students could create a diagram in GeoGebra and use it to gain insights about how the different angles change when they change the star.

Students who are fluent in algebra could be encouraged to label the angles in their diagram and use angle rules to write down relationships between the angles.

Some students may wish to consider the special case of a "regular" star whose points lie at five equally spaced positions on a circle.

Bring the class together to discuss proof methods and share ideas. Then perhaps go onto the seven-pointed star. The animation in the problem shows one way of proving the result for a seven-pointed star. The total of the angles in the 7 triangles is the same as the sum of the interior angles of the heptagon and twice the sum of the angles at the points of the star.

Alternatively, some students may wish to consider the angle turned through as they mentally "walk" around the lines of the star.

Key questions

If you know the five interior angles of the pentagon, what other angles can you work out?

What do you know about the angle sum of a pentagon?

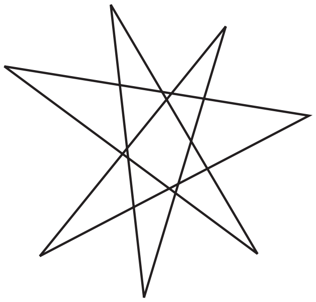
Could algebra help?

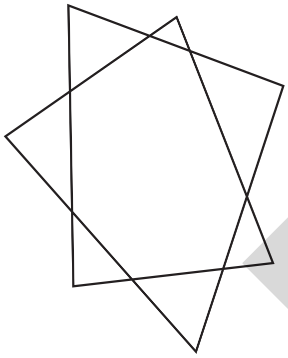
Possible support

Polygon Pictures offers students practice in calculating angles in regular polygons.

Possible extension

Students could investigate stars with more points, or different types of stars made by joining points around a circle differently. For example, the two stars below have been created by plotting seven points around a circle, and in the first image joining every third point, and in the second image joining every second point.





Student solutions

Go to the NRICH website to see some student solutions to this project.