Mathematics in the Modern World

MATHEMATICS IN OUR WORLD

CORE IDEA

Mathematics is a useful way to think about nature and our world.

LEARNING OUTCOMES

1. Identify patterns in nature and regularities in the world.
2. Articulate the importance of mathematics in one's life.
3. Argue about the nature of mathematics, what it is, how it is expressed, represented, and used.
4. Express appreciation for mathematics as a human endeavor.

METHODOLOGY

Video Watching, Pair/Group Sharing, Journal Writing, Class Discussion

RESOURCES

Textbooks, Pictures, Images, Illustrations, Online Videos

ASSESSMENT

Problem Set, Online Quizzes, Completion of Module Activities, Assignment, Recitation

WHAT IS MATHEMATICS?

- Mathematics is the study of pattern and structure. Mathematics is fundamental to the physical and biological sciences, engineering and information technology, to economics and increasingly to the social sciences.
- Mathematics is a useful way to think about nature and our world.
- Mathematics is a tool to quantify, organize and control our world, predict phenomena and make life easier for us.

WHERE IS MATHEMATICS?

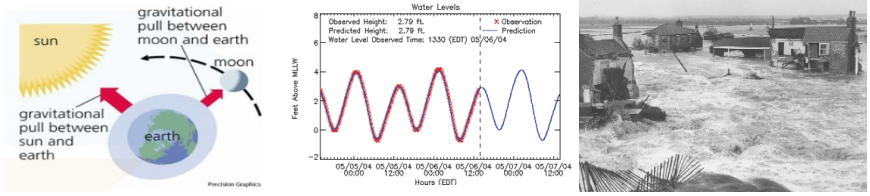
- Many patterns and occurrences exist in nature, in our world, in our life. Mathematics helps make sense of these patterns and occurrences.

WHAT ROLE DOES MATHEMATICS PLAY IN OUR WORLD?

- Mathematics helps organize patterns and regularities in our world.
- Mathematics helps predict the behavior of nature and phenomena in the world.
- Mathematics helps control nature and occurrences in the world for our own ends.
- Mathematics has numerous applications in the world making it indispensable.

Mathematics helps predict the behavior of nature and phenomena in the world.

- **Tides -** Tides have predictable patterns based on the positions of the sun and the moon.



- **Typhoon -** Computers analyze weather radar and satellite data patterns to predict the typhoon's paths.

Mathematics helps control nature and occurrences in the world for our own ends.

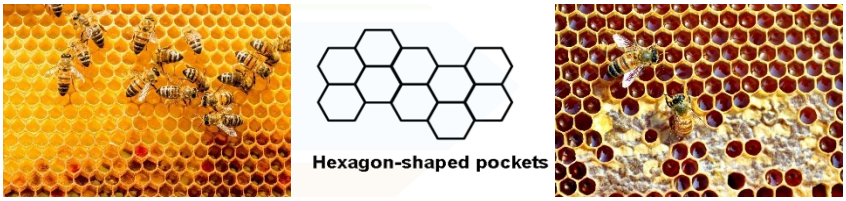
- **Mathematical Models of Nature -** Mathematical models of natural phenomena can help prepare people against disasters such as floods, drought, and earthquake.

- **Modeling Nature for Control -** Mathematics models are used to forecast some natural occurrences such as solar flares, volcanic eruptions, and typhoons, to help prevent disasters.

PATTERNS AND NUMBERS IN NATURE AND THE WORLD

Patterns in nature are visible regularities of form found in the natural world and can also be seen in the universe. Nature patterns which are not just to be admired, they are vital clues to the rules that govern natural processes.

- **Beehive Honeycomb** – is composed of hundreds of hexagon-shaped pockets.



- **Leaves of the Coconut Tree** – the leaves branch out in alternating pattern and the lengths taper to the tip.

- **Fruits and Leaves of the Mango Tree** – the mango fruits and leaves also have regular arrangements and patterns.

- **Sunflower** – the sunflower petals also have a nice pattern.

- **Snails** – patterns can also be found on snail's shell.

- **Zebra's Stripes & Peacock's Tail Feathers** – patterns can also be found on animals like zebra's stripes and peacock's tail feathers.

- **Tiger's Stripes & Hyena's Spots** – patterns can also be found on animals like tiger's stripes and hyena's spots.

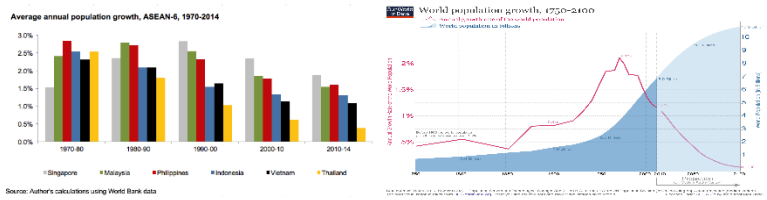
- **Fishes** - Patterns can also be seen in fishes. These attest to mathematical regularities in biological growth and form.

- **Cheetahs and Giraffes** - patterns can also be found on animals like cheetahs and giraffes who are covered with blotches.

- **Snow Flakes** – the snow flakes found in temperate countries during winter also have regular patterns which contains a sixfold symmetry which no two are exactly the same.

- **Weather Pattern – Seasons** - There are dry and wet seasons in the tropical countries like in the Philippines. There are four seasons in the temperate countries like Japan and the United States of America. These are summer, fall, winter and spring seasons.

- **Population** - the population profile and growth have a certain mathematical pattern.



- Natural patterns like the intricate waves across the oceans; sand dunes on deserts; formation of typhoon; water drop with ripple and others. These serves as clues to the rules that govern the flow of water, sand and air.

- Other patterns in nature can also be seen in the ball of mackerel, the v-formation of geese in the sky and the tornado formation of starlings.

THE FIBONACCI SEQUENCE

Leonardo Pisano Bigollo, Son of Bonacci, a.k.a Leonardo Fibonacci

Fibonacci was an Italian mathematician from the Republic of Pisa, considered to be "**the most talented Western mathematician of the Middle Ages**". (Smith, 1951)
Born: **1170, Pisa, Italy**
Full Name: **Leonardo Pisano Bigollo**
Nationality: **Italian**
Parents: **Alessandra Bonacci, Guglielmo Bonacci**
Siblings: **Bonaccinghus Bonacci**

The Fibonacci Sequence

- Fibonacci sequence is a series of numbers that follow a unique integer sequence.
- These numbers generate mathematical patterns that can be found in all aspects of life.
- The patterns can be seen in everything from the human body to the physiology of plants and animals.
- Starting from zero and one, the next number in the Fibonaccci number sequence is the sum of the preceding two numbers.

$$0,\ 1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,\ X_n,\ \ldots$$

$0 + 1 = 1$ $\quad$ $2 + 3 = 5$

$1 + 1 = 2$ $\quad$ $3 + 5 = 8$ $\quad$ $X_{n-2} + X_{n-1} = X_n$

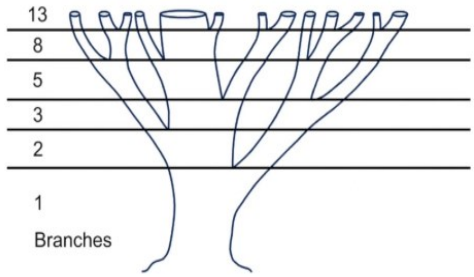
$1 + 2 = 3$ $\quad$ $5 + 8 = 13$

Fibonacci in Nature

- **Flower Petals** – Different flowers have different number of petals which exhibit the Fibonacci number.

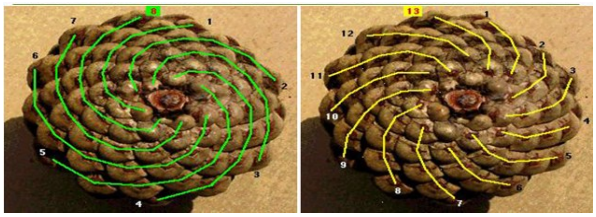
- **Sunflower seeds** - The pattern of two spirals goes in opposing directions (clockwise and counter-clockwise) conveys the Fibonacci sequence. The number of clockwise spirals and counter clockwise spirals are consecutive Fibonacci numbers and usually contains 34 and 55 seeds.

- **Tree branches** - The Fibonacci sequence can also be seen in the way tree branches form or split. A main trunk will grow until it produces a branch, which creates two growth points. Then, one of the new stem branches into two, while the other one lies dormant. This pattern of branching is repeated for each of the new stems. A good example is the sneezewort. Root systems and even algae exhibit this pattern.



- **Pineapples** – Pineapples have spirals formed by their hexagonal nubs. The nubs on many pineapples form 5 spirals and 8 spirals, or 8 spirals and 13 spirals that rotate diagonally upward to the right depending on the size of the pineapple. The numbers 5, 8 and 13 are Fibonacci numbers.

- **Pine cones** - Pine cones which contain spirals from the center have 5 arms and 8 arms, or 8 arms and 13 arms, depending on the size, which again two Fibonacci numbers.



Golden Ratio

Fibonacci discovery of Fibonacci sequence happened to approach the ratio asymptotically. He found the interesting and mysterious properties of the Fibonacci sequence that the series has a deep relationship with the golden ratio.

Extending the Fibonacci number sequence to bigger numbers and dividing the number in the sequence by its preceding number produces an interesting ratio. The ratio is called the Golden Ratio and is around 1.618 in decimal form.

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, . . .

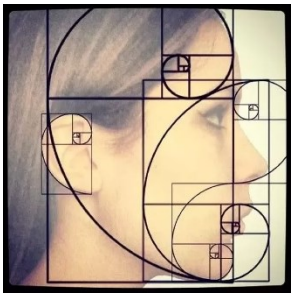
8 / 5	= 1.600000	144 / 89	= 1.617978	2584 / 1597	= 1.618034
13 / 8	= 1.625000	233 / 144	= 1.618056	4181 / 2584	= 1.618034
21 / 13	= 1.615385	377 / 233	= 1.618026	6765 / 4181	= 1.618034
34 / 21	= 1.619048	610 / 377	= 1.618037	10946 / 6765	= 1.618034
55 / 34	= 1.617647	987 / 610	= 1.618033	17711 / 10946	= 1.618034
89 / 55	= 1.618182	1597 / 987	= 1.618034	28657 / 17711	= 1.618034

Golden Ratio in Nature

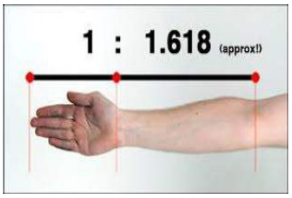
It is often said that math contains the answers to most of universe's questions. The Golden Ratio manifests itself in many places across the universe, including right here on Earth, it is part of Earth's nature and it is part of us.

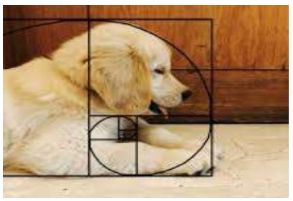
- **Flower petals -** number of petals in a flower is often one of the following numbers: 3, 5, 8, 13, 21, 34 or 55. For example, the lily has three petals, buttercups have five of them, the chicory has 21 of them, the daisy has often 34 or 55 petals, etc.

- **Faces -** Faces, both human and nonhuman, abound with examples of the Golden Ratio. The mouth and nose are each positioned at golden sections of the distance between the eyes and the bottom of the chin. Similar proportions can be seen from the side, and even the eye and ear itself.

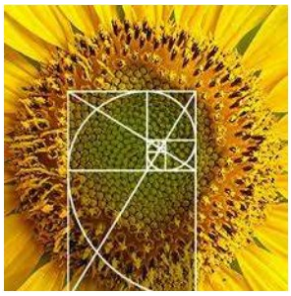


- **Body parts -** the Golden Section is manifested in the structure of the human body. The human body is based on Phi and the number 5. The number 5 appendages to the torso, in the arms, leg and head. 5 appendages on each of these, in the fingers and toes and 5 openings on the face. Animal bodies exhibit similar tendencies.

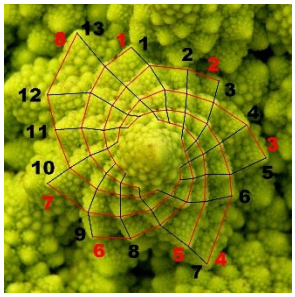




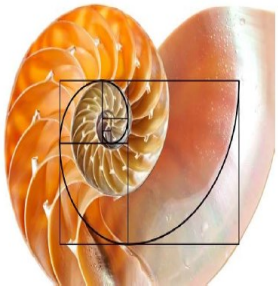
- **Seed heads -** Typically, seeds are produced at the center, and then migrate towards the outside to fill all the space. Sunflowers provide a great example of these spiraling patterns.



- **Fruits, Vegetables and Trees -** Spiraling patterns can be found on pineapples and cauliflower. Fibonacci numbers are seen in the branching of trees or the number of leaves on a floral stem; numbers like 4 are not. 3's and 5's, however, are abundant in nature.

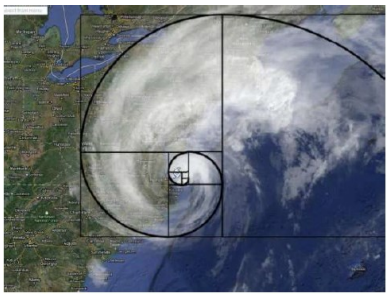


- **Shells -** Snail shells and nautilus shells follow the logarithmic spiral, as does the cochlea of the inner ear. It can also be seen in the horns of certain goats, and the shape of certain spider's webs.

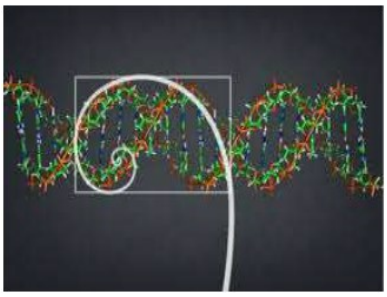


- **Spiral Galaxies -** Spiral galaxies are the most common galaxy shape. The Milky Way has several spiral arms, each of them a logarithmic spiral of about 12 degrees.

- **Hurricanes -** It's amazing how closely the powerful swirls of hurricane match the Fibonacci sequence.



- **DNA Molecules** – a DNA molecule measures 34 angstroms by 21 angstroms at each full cycle of the double helix spiral. In the Fibonacci series, 34 and 21 are successive numbers.



PATTERNS AND REGULARITIES

PATTERNS AND REGULARITIES

Mathematics is all around us. As we discover more about our environment, we can mathematically describe nature. The beauty of a flower, the majestic tree, even the rock formation exhibits nature's sense of symmetry.

TYPES OF PATTERNS

Behavior of nature can be observed around us. Natural regularities of nature include symmetry, fractals, spirals, trees, meanders, waves, foams, tessellations, cracks, stripes, and spots. Golden Ratio can be found in the beauty of nature, the growth patterns of many plants, insects, and the universe.

A. Symmetry is a sense of harmonious and beautiful proportion of balance or an object is invariant to any of various transformations (reflection, rotation or scaling).

- **Bilateral Symmetry -** is a symmetry in which the left and right sides of the organism can be divided into approximately mirror image of each other along the midline.

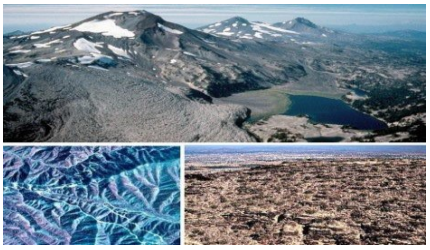
- **Radial Symmetry (Rotational Symmetry)** – is a type of symmetry around a fixed point known as the center and it can be classified as either cyclic or dihedral.

B. Fractal is curve or geometric figure, each part of which has the same statistical character as the whole. It is one of the newest and most exciting branches of mathematics. It is a class of highly irregular shapes that are related to continents, coastlines, and snowflakes.

• The fern is one of many flora that are fractal; it's an especially good example.

• Lightning's terrifying power is both awesome and beautiful. The fractals created by lightning are fascinatingly arbitrary and irregular.

• A special type of broccoli, this cruciferous and tasty cousin of the cabbage is a particularly symmetrical fractal. Cook it for your favorite mathematician.



• Both shorelines and mountain ranges are considered loosely fractal. These particular examples are beautiful.

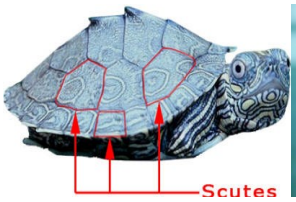
• From the macro view of a leaf to the span of a tree's branches, fractals turn up frequently.

C. Logarithmic Spiral (Growth Spiral) is a self-similar spiral curve which often appears in nature. It was first described by Rene Descartes and was later investigated by Jacob Bernoulli. Spirals are more evident in plants. We also see spirals in typhoon, whirlpool, galaxy, tail of chameleon, and shell among others.

D. Tessellations - patterns formed by repeating tiles all over a flat surface. Cells in the paper nests of social wasps, and the wax cells in honeycomb built by honey bees are well-known examples.

• Honeycombs of the bees show specific regular repeating hexagons. It uses the least amount of wax to store the honey giving a strong structure with no gaps.

• Turtles have growth rings called **"scutes"** which are hexagonal. Scutes estimates the age of the turtle. Smallest scute is in the center and is the oldest one, while the largest ones on the outside are the newer ones.


E. Meanders - The meander is one of a series of regular sinuous curves, bends, loops, turns, or windings in the channel of the body of water.

F. Waves - disturbances that carry energy as they move

G. Cracks - are linear openings that form in materials to relieve stress. Pattern of cracks indicates whether the material is elastic or not. Cracks can be found on the barks of trees which show some sort of weakness in the bark.

H. Bubbles, Foams - Foam bubbles formed by trapping pockets of gas in a liquid or solid. Foams of different materials occur in nature. At the scale of living cells, foam patterns are common.

I. Spots and Stripes - functions as camouflage, signaling and survival. these patterns have an evolutionary explanation.


J. Dunes - may form a range of patterns including crescents, very long straight lines, stars, domes, parabolas, and longitudinal or seif ('sword') shapes

APPLICATIONS OF MATHEMATICS IN THE WORLD

In our daily life, we use mathematics directly or indirectly in various fields.

1. **In forensic**, mathematics is applied specifically the differential and integral calculus to clarify the blurred image to clear image. Another application of calculus is optimization (maximize or minimize) surface areas, volumes, profit and cost analysis, projectile motion, etc.

2. **In medical field**, much of a function of a protein is determined by its shape and how the pieces move. Many drugs are designed to change the shape or motions of a protein by modeling using geometry and related areas. Mathematics is also being applied in the development of medicine to cure diseases.

3. **In fluid dynamics**, engineers use numerical analysis in phenomena involving heat, electricity and magnetism, relativistic mechanics, quantum mechanics and other theoretical constructs.

4. **In Information Technology**, modern computers are invented through the help of mathematics. An important area of applications of mathematics in the development of formal mathematical theories related to the development of computer science. Computer science development includes logic, relations, functions, basic set theory, counting techniques, graph theory, combinatorics, discrete probability, recursion, recurrence relations and number theory, computer-oriented numerical analysis and Operation Research techniques.

5. **Cryptography** is a combination of both mathematics and computer science and is affiliated closely with information theory, computer security and engineering. It is used in applications present in technologically advanced societies, examples include the security of ATM cards, computer passwords and electronic commerce.

6. **In archaeology**, archaeologists use a variety of mathematical and statistical techniques to present the data from archaeological surveys and try to find patterns to shed on past human behavior in carbon dating artifacts.

7. **In Social Sciences** such as economics, sociology, psychology and linguistics all now make extensive use of mathematical models, using the tools of calculus, probability, game theory, and network theory.

8. **In Economics**, mathematics such as matrices, probability and statistics are used. The models may be stochastic or deterministic, linear or non-linear, static or dynamic, continuous or discrete and all types of algebraic, differential, difference and integral equations arise for the solution of these models.

9. **In political Science**, political analysts study past election results to see changes in voting patterns and the influence of various factors on voting behavior or switching of votes among political parties and mathematical models for Conflict Resolution using Game Theory and Statistics.

10. **In music and arts,** the rhythm that we find in all music notes is the result of innumerable permutations and combinations. Music theorists understand musical structure and communicate new ways of hearing music by applying set theory, abstract algebra, and number theory.

References:

A. Aufmann, Richard et. al. (2013). *Mathematical Excursions 3rd Edition (International Edition).* BROOKS/COLE CENGAGE Learning.
B. Aufmann, Richard et. al. (2018). *Mathematics in the Modern World (Philippine Edition).* Rex Book Store, Inc.
C. Calubaquib, Medallon. (2018). Mathematics in the Modern World. Intramuros Manila, Philippines: Mindshapers Co., Inc.
D. Mathematics in the Modern World by PMSJR
E. Sirug, Winston S. (2018). Mathematics in the Modern World. Intramuros Manila, Philippines: Mindshapers Co., Inc.
F. Sobecki, Dave (2019). Math in Our World 4th Edition *(Philippine Edition)*. Mc Graw Hill Education
G. Ymas Jr., Sergio E. et. al. (2018). *Mathematics for the Modern World 2018*. Ymas Publishing House.