Problems in

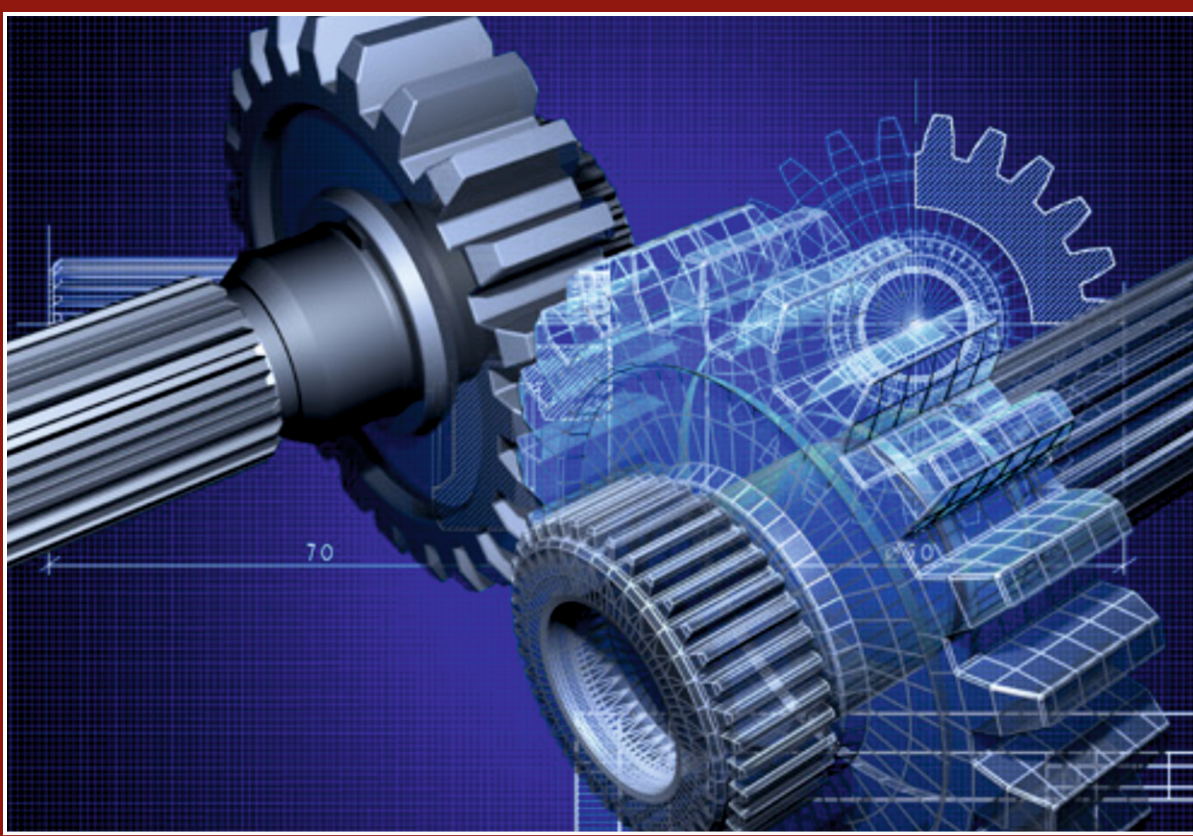
# PHYSICS for

# IIT-JEE ADVANCED

Fully Solved

Volume I

Mechanics | Waves




McGraw Hill Education

Shashi Bhushan Tiwari

# Problems in

# PHYSICS for

# IIT-JEE ADVANCED

# VOLUME I

**Shashi Bhushan Tiwari**

**McGraw Hill Education (India) Private Limited**
CHENNAI

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# PREFACE

In the past decade and a half, the entrance exam for IITs has seen many changes – in structure as well as in design of the question paper. No doubt, it has become more challenging. It requires high level of conceptual clarity and analytical skill, besides promptness and comprehension ability to excel in this exam. There are frequent surprises in terms of problems which require mathematical rigor or in depth understanding of physical conditions.

This book is being presented with a very simple objective – it will test you and nurture you on all parameters which are required to excel in JEE exam.

Every chapter in the book has been divided into three sections –

- LEVEL 1 – This section will test you on all basic fundamentals of the chapter. Problems are not very rigorous though they may be very conceptual.
- LEVEL 2 – This section will develop all necessary skills required to score a high rank in JEE exam. Few problems in this section may appear lengthy but they are the ones which test your confidence and patience. Don't be scared of them.
- LEVEL 3 – This section contains problems that may require exceptional reasoning skill or mathematical ability.

Since difficulty level is quite subjective and may vary from person to person — few problems may appear to you as misplaced in three sections described above. I have judged them to the best of my ability besides taking help from some very bright minds.

I have not tried to include every other problem that is available in this universe. Most of the books available in market have this issue – in the name of being exhaustive, they have become repetitive. Believe me, while solving problems from this book you will not feel like wasting your time in doing similar problems again and again..

Most of the solutions are quite descriptive so that a serious student can understand on his/her own. Diagrams have been included wherever possible to make things lucid.

JEE exam being objective, one may challenge the sanctity of a subjective book. Have no doubts in your mind — pattern of a question paper or type of question will never deter you if you have sound grasp of the subject and have developed right kind of temperament. Physics as a subject is notorious and can be learned only by subjecting yourself to the true rigor and complexity. While doing a subjective problem you cannot make a guess and bluff yourself!

This collection of problems will appear to you as fresh and challenging. Start and enjoy learning physics!

Suggestions are welcome.

# CONTENTS

# 01 BASIC MATHS

## Level 1

Q.1 In an experiment mileage of a car was measured to be 24 *kmpl* (Kilometer per liter of fuel consumed). After the experiment it was found that 4 % of the fuel used during the experiment was leaked through a small hole in the tank. Calculate the actual mileage of the car after the tank was repaired.

Q.2 A man is standing at a distance of 500m from a building. He notes that angle of elevation of the top of the building is 3.6°. Find the height of the building. Neglect the height of the man and take $\pi = 3.14$.

Q.3 A Smuggler in a hindi film is running with a bag 0.3 m × 0.2 m × 0.2 m in dimension. The bag is supposed to be completely filled with gold. Do you think than the director of the film made a technical mistake there? Density of gold is 19.6 g/cc.

Q.4 A particle moves along the curve $6y = x^3+2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

Q.5 The area of a regular octagon of side length *a* is A.

**(a)** Find the time rate of change of area of the octagon if its side length is being increased at a constant rate of $\beta$ *m/s*. Is the time rate of change of area of the octagon constant with time?

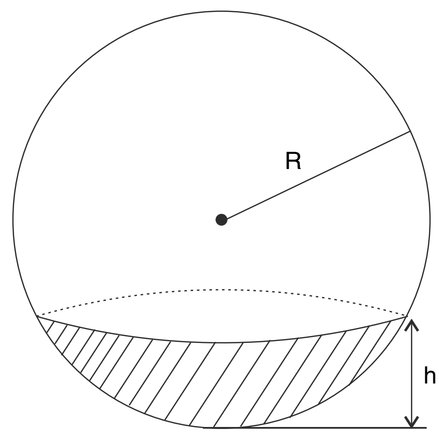
**(b)** Find the approximate change in area of the octagon as the side length is increased from 2.0 m to 2.001 m.

## Level 2

Q.6 Spirit in a bowl evaporates at a rate that is proportional to the surface area of the liquid. Initially, the height of liquid in the bowl is $H_0$. It becomes $\frac{H_0}{2}$ in time $t_0$. How much more time will be needed for the height of liquid to become $\frac{H_0}{4}$.

Q.7 Show that the volume of a segment of height h of a sphere of radius R is

$$V = \frac{1}{3}\pi h^2 \left(3R - h\right)$$




Q.8 The amount of energy a car expends against air resistance is approximately given by

$$E = K\,ADv^2$$

where E is measured in Joules. K is a constant, A is the cross-sectional area of the car viewed from the front (in $m^2$), D is the distance traveled (in m), and v is the speed of the car (in m/s). Julie wants to drive from Mumbai to Delhi and get good fuel mileage. For the following questions, assume that the energy loss is due solely to air resistance.

**(a)** Julie usually drives at a speed of 54 Km/hr. How much more energy will she use if she drives 20% faster?

**(b)** Harshit drives a very large SUV car, and Julie drives a small car. Every linear dimension of Harshit's car is double that of Julie's car. Find the ratio of energy spent by Harshit's car to

Julie's car when they cover same distance. Speed of Harshit was 10% faster compared to Julie's car.

**(c)** Write the dimensional formula for K. Will you believe that K depends on density of air?

Q.9 The volume flow rate $Q$ *(in $m^3$ $s^{-1}$)* of a liquid through pipe having diameter d is related to viscosity of water '$\eta$' (*unit Pascal. s*) and the pressure gradient along the pipe $\frac{dP}{dx}$ [pressure gradient $\frac{dP}{dx}$ is rate of change of pressure per unit length along the pipe], by a formula of the form

$$Q = k\eta^a d^b \left(\frac{dP}{dx}\right)^c$$

Where K is a dimensionless constant. Find a,b and c.

Q.10. The potential energy (U) of a particle can be expressed in certain case as $U = \frac{A^2}{2mr^2} - \frac{BMm}{r}$

Where m and M are mass and r is distance. Find the dimensional formulae for constants.

Q.11. In the following expression V and g are speed and acceleration respectively. Find the dimensional formulae of $a$ and $b$

$$\int \frac{VdV}{g - bV^2} = a$$

Q.12 The maximum height of a mountain on earth is limited by the rock flowing under the enormous weight above it. Studies show that maximum height depends on young's modulus (Y) of the rod, acceleration due to gravity (g) and the density of the rock (d).

(a) Write an equation showing the dependence of maximum height (h) of mountain on Y, g and d. It is given that unit of Y is $Nm^{-2}$.

(b) Take $d = 3 \times 10^3$ kg $m^{-3}$, $Y = 1 \times 10^{10}$ $Nm^{-2}$ and $g = 10$ $ms^{-2}$ and assume that maximum height of a mountain on the surface of earth is limited to 10 km [height of mount Everest is nearly 8 km]. Write the formula for h.

Q.13 A particle of mass m is given an initial speed $V_0$. It experiences a retarding force that is proportional to the speed of the particle ($F = aV$). $a$ is a constant.

(a) Write the dimensional formula of constant $a$.

(b) Using dimensional analysis, derive a formula for stopping time (t) of the particle. Does your formula tell you how 't' depends on initial speed $V_0$? What can you predict about the constant obtained in the formula?

Q.14 Assume that maximum mass $m_1$ of a boulder swept along by a river, depends on the speed $V$ of the river, the acceleration due to gravity g, and the density $d$ of the boulder. Calculate the percentage change in maximum mass of the boulder that can be swept by the river, when speed of the river increases by 1%.

Q.15 A massive object in space causes gravitational lensing. Light from a distant source gets deflected by a massive lensing object. This was first observed in 1919 and supported Einstein's general theory of relativity.

The angle $\theta$ by which light gets deflected due to a massive body depends on the mass (M) of the body, universal gravitational constant (G), speed of light (c) and the least distance (r) between the lensing object and the apparent path of light. Derive a formula for $\theta$ using method of dimensions. Make suitable assumptions.

Q.16 The Casimir effect describes the attraction between two unchanged conducting plates placed parallel to each other in vacuum. The astonishing force ( predicted in 1948 by Hendrik Casimir) per unit area of each plate depends on the planck's constant (h), speed of light (c) and separation between the plates (r).

(a) Using dimensional analysis prove that the formula for the Casimir force per unit area on the plates is given by

$F = k\frac{hc}{r^4}$ where k is a dimensionless constant

(b) If the force acting on 1x1 cm plates separated by $1\mu m$ is 0.013 dyne, calculate the value of constant k.

Q.17. Scattering of light is a process of absorption and prompt re-emission of light by atoms and molecules. Scattering involving particles smaller than wavelength ($\lambda$) of light is known as Rayleigh scattering. Let $a_i$ be amplitude of incident light on a scatterer of volume V. The scattered amplitude at a distance r from the scatterer is $a_s$. Assume and $a_s \,\alpha\, a_i$, $a_s \,\alpha\, \frac{1}{r}$ and $a_s \,\alpha V$.

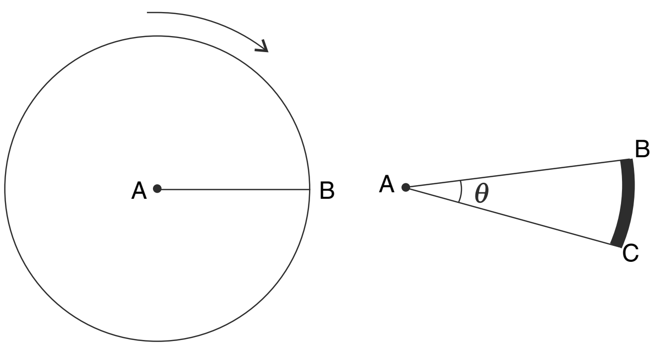
(i) Find the dimensions of the proportionality constant occurring in the expression of $a_s$

(ii) Assuming that this constant depends on $\lambda$, find the dependence of ratio $\frac{a_s}{a_i}$ on $\lambda$.

(iii) Knowing that intensify of light I $\alpha$ $a^2$ find the dependence of $\frac{I_s}{I_i}$ on $\lambda$.

Q.18 It is given that $\int \frac{dx}{1+x^2} = \tan^{-1} x + c$. Using methods of dimensions find $\int \frac{dx}{a^2 + x^2}$

Q.19




Two point sources of light are fixed at the centre (A) and circumference (point B) of a rotating turn table. A photograph of the rotating table is taken. On the photograph a point A and an arc BC appear. The angle $\theta$ was measured to be $\theta = 10.8° \pm 0.1°$ and the angular speed of the turntable was measured to be ω = (33.3 ± 0.1) revolution per minute. Calculate the exposure time of the camera.

Q.20 The speed (V) of wave on surface of water is given by

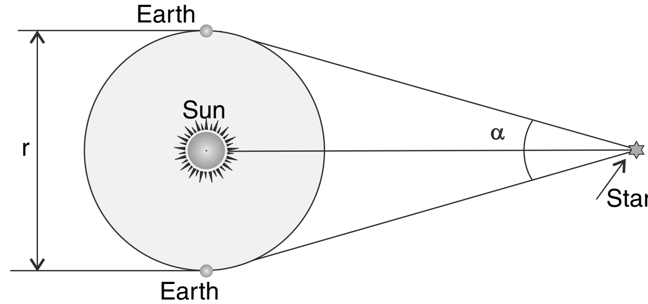
$$V = \sqrt{\frac{a\lambda}{2\pi} + \frac{2\pi b}{\rho\lambda}}$$

where $\lambda$ is the wavelength of the wave and $\rho$ is density of water. $a$ is a constant and b is a quantity that changes with liquid temperature.

(a) Find the dimensional formulae for $a$ and b.

(b) Surface wave of wavelength 30 mm have a speed of 0.240 $ms^{-1}$. If the temperature of water changes by 50°$C$, the speed of waves for same wavelength changes to 0.230 $ms^{-1}$. Assuming that the density of water remains constant at $1 \times 10^3$ kg $m^{-3}$, estimate the change in value of 'b' for temperature change of 50°$C$.

Q.21 The line of sight of the brightest star in the sky, Sirius has a maximum parallax angle of $\delta = 0.74 \pm 0.02$ arc second when observed at six month interval. The distance between two positions of earth (at six – month interval) is $r = 3.000 \times 10^{11} m$.




Calculate the distance of Sirius from the Sun with uncertainty, in unit of light year. Given 1 ly = $9.460 \times 10^{15}$ m. ; $\pi = 3.14$

## LEVEL 3

Q.22 You inhale about 0.5 liter of air in each breath and breath once in every five seconds. Air has about 1% argon. Mass of each air particle can be assumed to be nearly $5 \times 10^{-26}$ $kg$. Atmosphere can be assumed to be around 20 $km$ thick having a uniform density of 1.2 $kg\ m^{-3}$. Radius of the earth is $R = 6.4 \times 10^6$ m. Assume that when a person breathes, half of the argon atoms in each breath have never been in that person's lungs before. Argon atoms remain in atmosphere for long-long time without reacting with any other substance. Given : one year = $3.2 \times 10^7$s

(a) Estimate the number of argon atoms that passed through Newton's lungs in his 84 years of life.

(b) Estimate the total number of argon atoms in the Earth's atmosphere.

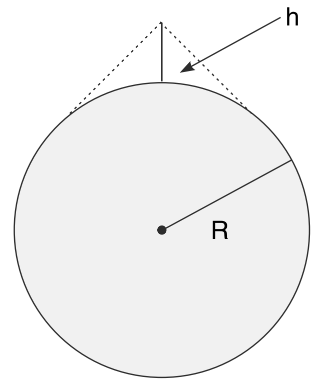
(c) Assume that the argon atoms breathed by Newton is now mixed uniformly through the atmosphere, estimate the number of argon atoms in each of your breath that were once in Newton's lungs.

Q.23 A rope is tightly wound along the equator of a large sphere of radius R. The length of the rope is increased by a small amount $\ell$ (<< R) and it is pulled away from the surface at a point to make it taut. To what height (h) from the surface will the point rise ?

If the radius of the earth is R=6400 km and $\ell$ =

*10 mm*, find the value of h. Does the value surprise you.

[For small $\theta$ take $\tan\theta = \theta + \frac{\theta^3}{3}$ and $\sec\theta = 1 + \frac{\theta^2}{2}$. Also take $(2.3)^{\frac{2}{3}} = 1.74$ ]

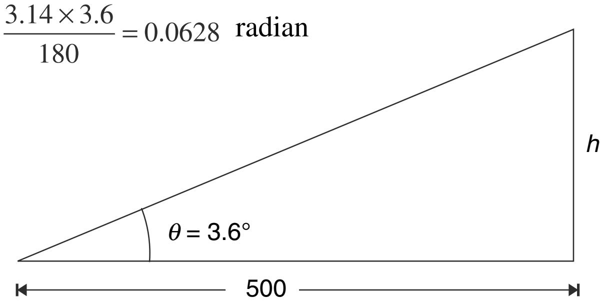



## ANSWERS

1. 25 kmpl
2. 31.40 m
3. Yes.
4. $(-4, -\frac{31}{3}), (4, 11)$
5. (a) $4(\sqrt{2}+1)\,a\,\beta$. No, it is not a constant
   (b) $0.0019\ m^2$
6. $\frac{t_0}{2}$
8. (a) 44% higher (b) 4.84 (c) $[ML^{-3}]$; Yes
9. $a = -1;\ b = 4;\ c = 1$
10. $[A] = [M^1L^2T^{-1}]$; $[B] = [M^{-1}L^3T^{-2}]$
11. $[a] = L$; $[b] = L^{-1}$
12. (a) $h = k\left(\frac{Y}{gd}\right)$; $k = a\ const$
    (b) $h = 0.03\left(\frac{Y}{gd}\right)$
13. (a) $[a] = [M^1T^{-1}]$ (b) $t = k\frac{m}{a}$; $t = \infty$
14. 6%
15. $\theta = k\frac{GM}{cr^2}$
16. (b) $k = 6.5 \times 10^{-3}$
17. (i) $[k] = [L^{-2}]$
    (ii) $\frac{a_s}{a_i} \propto \lambda^{-2}$
    (iii) $\frac{I_s}{I_i} \propto \frac{a_s^2}{a_i^2} \propto \lambda^{-4}$
18. $a\tan^{-1}\left(\frac{x}{a}\right) + c$
19. $(0.054 \pm 0.003)$s
20. (a) $[a] = [M^oL^1T^{-2}]$; $[b] = [M^1L^oT^{-2}]$
    (b) $\Delta b = -0.022\ kg\ s^{-2}$
21. $8.84 \pm 0.24\ ly$
22. (a) $3.2 \times 10^{28}$ (b) $2.5 \times 10^{42}$
    (c) $1.5 \times 10^6$
23. $5.6\ m$

## SOLUTIONS

**2.** $\theta = 3.6° = \frac{\pi}{180} \times 3.6\ rad = \frac{3.14 \times 3.6}{180} = 0.0628$ radian



# 02 KINEMATICS

## Level 1

**Q. 1.** A particle is travelling on a curved path. In an interval $\Delta t$ its speed changed from $v$ to $2v$. However, the change in magnitude of its velocity was found to be $\left|\overrightarrow{\Delta V}\right| = \sqrt{5}\, v$. What can you say about the direction of velocity at the beginning and at the end of the interval ($\Delta t$)?

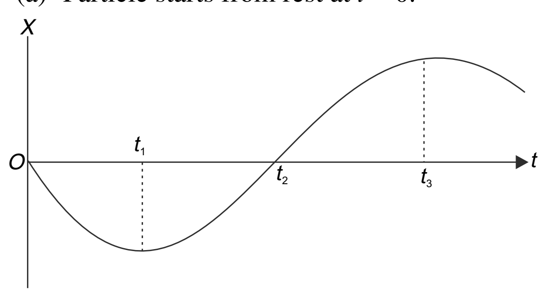
**Q. 2.** Two tourist $A$ and $B$ who are at a distance of 40 *km* from their camp must reach it together in the shortest possible time. They have one bicycle and they decide to use it in turn. '$A$' started walking at a speed of 5 $km\ hr^{-1}$ and $B$ moved on the bicycle at a speed of 15 $km\ hr^{-1}$. After moving certain distance $B$ left the bicycle and walked the remaining distance. $A$, on reaching near the bicycle, picks it up and covers the remaining distance riding it. Both reached the camp together.

(a) Find the average speed of each tourist.

(b) How long was the bicycle left unused?

**Q. 3.** The position time graph for a particle travelling along $x$ axis has been shown in the figure. State whether following statements are true of false.

(a) Particle starts from rest at $t = 0$.




(b) Particle is retarding in the interval 0 to $t_1$ and accelerating in the interval $t_1$ to $t_2$.

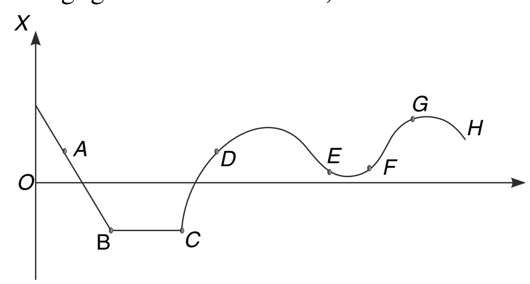
(c) The direction of acceleration has changed once during the interval 0 to $t_3$

**Q. 4.** The position time graph for a particle moving along $X$ axis has been shown in the fig. At which of the indicated points the particle has

(i) negative velocity but acceleration in positive $X$ direction.

(ii) positive velocity but acceleration in negative $X$ direction.

(iii) received a sharp blow (a large force for negligible interval of time)?




**Q. 5.** A particle is moving along positive $X$ direction and is retarding uniformly. The particle crosses the origin at time $t = 0$ and crosses the point $x = 4.0\ m$ at $t = 2\ s$.

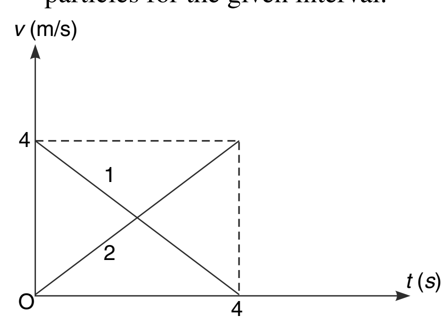
(a) Find the maximum speed that the particle can possess at $x = 0$.

(b) Find the maximum value of retardation that the particle can have.

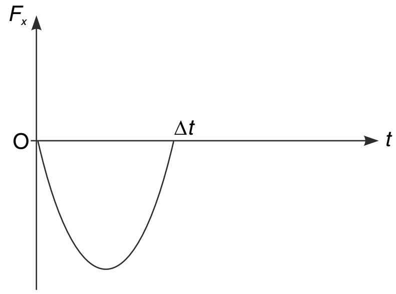
**Q. 6.** The velocity time graph for two particles (1 and 2) moving along $X$ axis is shown in fig. At time $t = 0$, both were at origin.

(a) During first 4 second of motion what is maximum separation between the particles? At what time the separation is maximum?

(b) Draw position ($x$) vs time ($t$) graph for the particles for the given interval.



Q. 7. A ball travelling in positive $X$ direction with speed $V_0$ hits a wall perpendicularly and rebounds with speed $V_0$. During the short interaction time ($\Delta t$) the force applied by the wall on the ball varies as shown in figure.



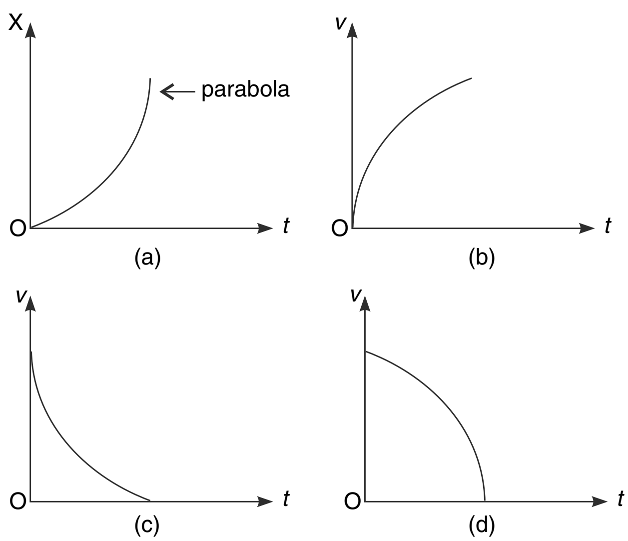

Draw the velocity-time graph for the ball during the interval 0 to $\Delta t$

Q. 8. For a particle moving along a straight line consider following graphs $A$, $B$, $C$ and $D$. Here $x$, $v$ and $t$ are position, velocity and time respectively.

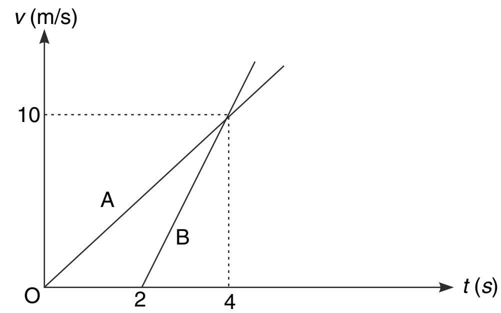
(i) In which of the graphs the magnitude of acceleration is decreasing with time?

(ii) In which of the graphs the magnitude of acceleration is increasing with time?

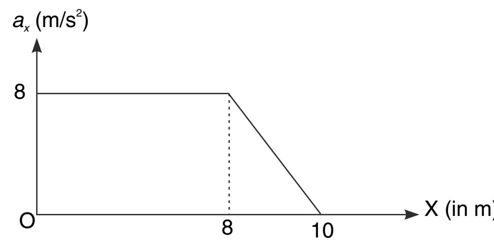
(iii) If the body is definitely going away from the starting point with time, which of the given graphs represent this condition.



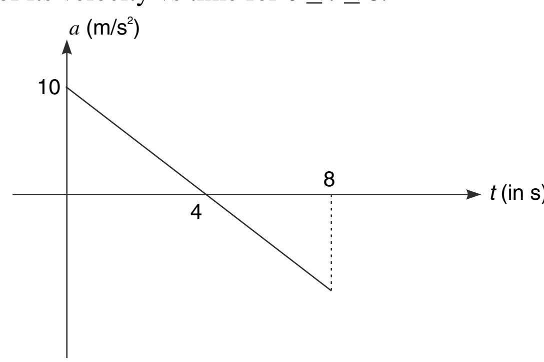

Q. 9. Two particles $A$ and $B$ start from same point and move along a straight line. Velocity-time graph for both of them has been shown in the fig. Find the maximum separation between the particles in the interval $0 < t < 5$ sec.




Q. 10. A particle starts from rest (at $x = 0$) when an acceleration is applied to it. The acceleration of the particle changes with its co-ordinate as shown in the fig. Find the speed of the particle at $x = 10m$.



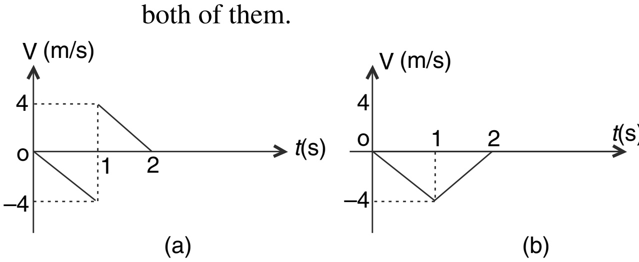

Q. 11. Acceleration vs time graph for a particle moving along a straight line is as shown. If the initial velocity of the particle is $u = 10$ $m/s$, draw a plot of its velocity vs time for $0 \leq t \leq 8$.




Q. 12. The velocity ($V$) – time ($t$) graphs for two particles $A$ and $B$ moving rectilinearly have been shown in the figure for an interval of 2 second.

(a) At $t = 1$ $s$, which of the two particles ($A$ or $B$) has received a severe blow?

(b) Draw displacement ($X$) – time ($t$) graph for both of them.



Q. 13. A particle starts moving rectilinearly at time $t = 0$ such that its velocity($v$) changes with time ($t$) as per equation –

$v = (t^2 - 2t)\ m/s$ for $0 \le t \le 2\ s$

$= (-t^2 + 6t - 8)\ m/s$ for $2 \le t \le 4\ s$

(a) Find the interval of time between $t = 0$ and $t = 4\ s$ when particle is retarding.

(b) Find the maximum speed of the particle in the interval $0 \le t \le 4\ s$.

Q. 14. Our universe is always expanding. The rate at which galaxies are receding from each other is given by Hubble's law (discovered in 1929 by *E.* Hubble). The law states that the rate of separation of two galaxies is directly proportional to their separation. It means relative speed of separation of two galaxies, presently at separation $r$ is given by $v = Hr$

$H$ is a constant known as Hubble's parameter. Currently accepted value of $H$ is $2.32 \times 10^{-18}\ s^{-1}$

(a) Express the value of $H$ in unit of

$$\frac{Km.\,s^{-1}}{\text{Mega light year}}$$

(b) Find time required for separation between two galaxies to change from $r$ to $2r$.

Q. 15. A stone is projected vertically up from a point on the ground, with a speed of 20 $m/s$. Plot the variation of followings with time during the entire course of flight –

(a) Velocity

(b) Speed

(c) Height above the ground

(d) distance travelled

Q. 16. A ball is dropped from a height $H$ above the ground. It hits the ground and bounces up vertically to a height $\frac{H}{2}$ where it is caught. Taking origin at the point from where the ball was dropped, plot the variation of its displacement *vs* velocity. Take vertically downward direction as positive.

Q. 17. A helicopter is rising vertically up with a velocity of 5 $ms^{-1}$. A ball is projected vertically up from the helicopter with a velocity $V$ (relative to the ground). The ball crosses the helicopter 3 second after its projection. Find $V$.

Q. 18. A chain of length $L$ supported at the upper end is hanging vertically. It is released. Determine the interval of time it takes the chain to pass a point $2L$ below the point of support, if all of the chain is a freely falling body.

Q. 19. Two nearly identical balls are released simultaneously from the top of a tower. One of the balls fall with a constant acceleration of $g_1 = 9.80\ ms^{-2}$ while the other falls with a constant acceleration that is 0.1% greater than $g_1$. [This difference may be attributed to variety of reasons. You may point out few of them]. What is the displacement of the first ball by the time the second one has fallen 1.0 $mm$ farther than the first ball?

Q. 20. Two projectiles are projected from same point on the ground in $x$-$y$ plane with $y$ direction as vertical. The initial velocity of projectiles are

$$\vec{V}_1 = V_{x_1}\hat{i} + V_{y1}\hat{j}$$
$$\vec{V}_2 = V_{x2}\hat{i} + V_{y2}\hat{j}$$

It is given that $V_{x1} > V_{x2}$ and $V_{y1} < V_{y2}$. Check whether all of the following statement/s are True.

(a) Time of flight of the second projectile is greater than that of the other.

(b) Range of first projectile may be equal to the range of the second.

(c) Range of the two projectiles are equal if $V_{x1}V_{y1} = V_{x2}V_{y2}$

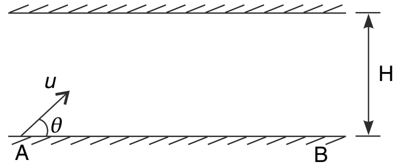
(d) The projectile having greater time of flight can have smaller range.

Q. 21. (a) A particle starts moving at $t = 0$ in $x$-$y$ plane such that its coordinates (in cm) with time (in sec) change as $x = 3t$ and $y = 4\sin(3t)$. Draw the path of the particle.

(b) If position vector of a particle is given by $\vec{r} = \left(4t^2 - 16t\right)\hat{i} + \left(3t^2 - 12t\right)\hat{j}$, then find distance travelled in first 4 sec.

Q. 22. Two particles projected at angles $\theta_1$ and $\theta_2$ ($<\theta_1$) to the horizontal attain same maximum height. Which of the two particles has larger range? Find the ratio of their range.

Q. 23. A ball is projected from the floor of a long hall having a roof height of $H = 10\ m$. The ball is projected with a velocity of $u = 25\ ms^{-1}$ making an angle of $\theta = 37°$ to the horizontal. On hitting the roof the ball loses its entire vertical component of velocity but there is no change in the horizontal component of its velocity. The ball was projected from point $A$ and it hits the floor at $B$. Find distance $AB$.




Q. 24. In a tennis match Maria Sharapova returns an incoming ball at an angle that is 4° below the horizontal at a speed of 15 $m/s$. The ball was hit at a height of 1.6 $m$ above the ground. The opponent, Sania Mirza, reacts 0.2 $s$ after the ball is hit and runs to the ball and manages to return it just before it hits the ground. Sania runs at a speed of 7.5 $m/s$ and she had to reach 0.8 $m$ forward, from where she stands, to hit the ball.

(a) At what distance Sania was standing from Maria at the time the ball was returned by Maria? Assume that Maria returned the ball directly towards Sania.

(b) With what speed did the ball hit the racket of Sania?

$[g = 9.8\ m/s^2]$

Q.25. A player initially at rest throws a ball with an initial speed $u = 19.5\ m/s$ at an angle $\theta = \sin^{-1}\left(\frac{12}{13}\right)$ to the horizontal. Immediately after throwing the ball he starts running to catch it. He runs with constant acceleration ($a$) for first 2 $s$ and thereafter runs with constant velocity. He just manages to catch the ball at exactly the same height at which he threw the ball. Find '$a$'. Take $g = 10\ m/s^2$. Do you think anybody can run at a speed at which the player ran?

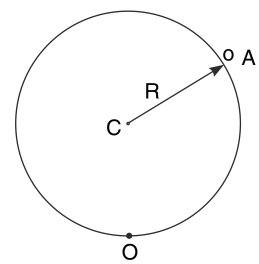
Q. 26. In a cricket match, a batsman hits the ball in air. A fielder, originally standing at a distance of 12 $m$ due east of the batsman, starts running 0.6 s after the ball is hits. He runs towards north at a constant speed of 5 $m/s$ and just manages to catch the ball 2.4 $s$ after he starts running.

Assume that the ball was hit and caught at the same height and take $g = 10\ m/s^2$ $g = 10\ m/s^2$

Find the speed at which the ball left the bat and the angle that its velocity made with the vertical.

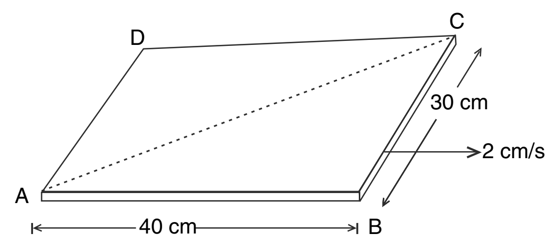
Q. 27. The time of flight, for a projectile, along two different paths to get a given range $R$, are in ratio 2 : 1. Find the ratio of this range $R$ to the maximum possible range for the projectile assuming the projection speed to be same in all cases.

Q. 28. A boy '$A$' is running on a circular track of radius $R$. His friend, standing at a point O on the circumference of the track is throwing balls at speed $u = \sqrt{gR}$. Balls are being thrown randomly in all possible directions. Find the length of the circumference of the circle on which the boy is completely safe from being hit by a ball.

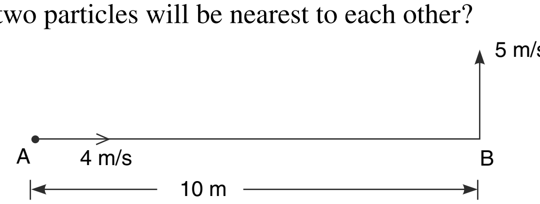



Q. 29. A rectangular cardboard $ABCD$ has dimensions of 40 $cm$ × 30 $cm$. It is moving in a direction perpendicular to its shorter side at a constant speed of 2 $cm/s$. A small insect starts at corner $A$ and moves to diagonally opposite corner $C$. On reaching $C$ it immediately turns back and moves to $A$. Throughout the motion the insect maintains a constant speed relative to the board. It takes 10 $s$ for the insect to reach $C$ starting from $A$.

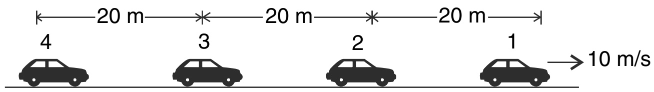
Find displacement and distance travelled by the insect in reference frame attached to the ground in the interval the insect starts from $A$ and comes back to $A$.




Q. 30. Two particles $A$ and $B$ separated by 10 $m$ at time $t = 0$ are moving uniformly. $A$ is moving along line $AB$ at a constant velocity of 4 $m/s$ and $B$ is moving perpendicular to the velocity of $A$ at a constant velocity of 5 $m/s$. After what time the two particles will be nearest to each other?



Q. 31. Four cars are moving along a straight road in the same direction. Velocity of car 1 is 10 *m/s*. It was found that distance between car 1 and 2 is decreasing at a rate of 2 *m/s*, whereas driver in car 4 observed that he was nearing car 2 at a speed of 8 *m/s*. The gap between car 2 and 3 is decreasing at a rate of 3 *m/s*.




(a) If cars were at equal separations of 20 *m* at time $t = 0$, after how much time $t_0$ will the driver of car 2 see for the first time, that another car overtakes him?

(b) Which car will be first to overtake car 1?

Q. 32. Acceleration of a particle as seen from two reference frames 1 and 2 has magnitude 3 $m/s^2$ and 4 $m/s^2$ respectively. What can be magnitude of acceleration of frame 2 with respect to frame 1?

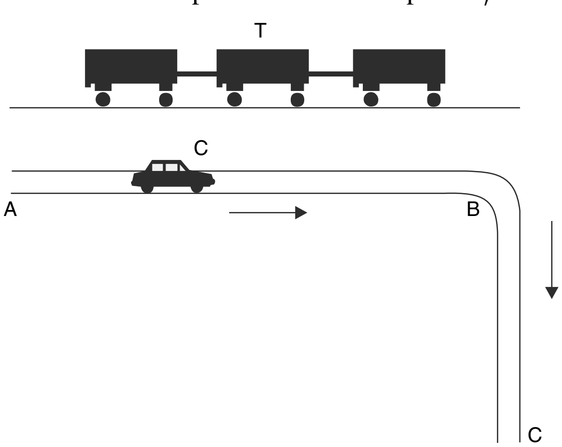
Q. 33. A physics professor was driving a Maruti car which has its rear wind screen inclined at $\theta = 37°$ to the horizontal. Suddenly it started raining with rain drops falling vertically. After some time the rain stopped and the professor found that the rear wind shield was absolutely dry. He knew that, during the period it was raining, his car was moving at a constant speed of $V_c = 20$ *km/hr*.

[tan 37° = 0.75]

(a) The professor calculated the maximum speed of vertically falling raindrops as $V_{max}$. What is value of $V_{max}$ that he obtained.

(b) Plot the minimum driving speed of the car vs. angle of rear wind screen with horizontal ($\theta$) so as to keep rain off the rear glass. Assume that rain drops fall at constant speed $V_r$

Q. 34.



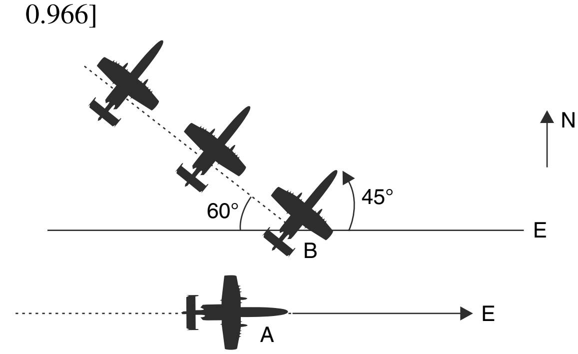

A train($T$) is running uniformly on a straight track. A car is travelling with constant speed along section AB of the road which is parallel to the rails. The driver of the car notices that the train is having a speed of 7 *m/s* with respect to him. The car maintains the speed but takes a right turn at $B$ and travels along $BC$. Now the driver of the car finds that the speed of train relative of him is 13 *m/s*. Find the possible speeds of the car.

**35.**

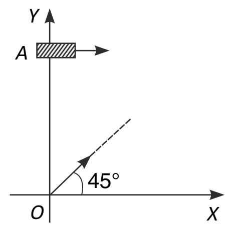

A police car $B$ is chasing a culprit's car $A$. Car $A$ and $B$ are moving at constant speed $V_1$ = 108 *km/hr* and $V_2$ = 90 *km/hr* respectively along a straight line. The police decides to open fire and a policeman starts firing with his machine gun directly aiming at car A. The bullets have a velocity $u$ = 305 *m/s* relative to the gun. The policeman keeps firing for an interval of $T_0 = 20$ *s*. The Culprit experiences that the time gap between the first and the last bullet hitting his car is $\Delta t$. Find $\Delta t$.

Q. 36. A chain of length $L$ is supported at one end and is hanging vertically when it is released. All of the chain falls freely with acceleration $g$. The moment, the chain is released a ball is projected up with speed u from a point 2 $L$ below the point of support. Find the interval of time in which the ball will cross through the entire chain.

Q. 37. Jet plane $A$ is moving towards east at a speed of 900 *km/hr*. Another plane $B$ has its nose pointed towards 45° $N$ of $E$ but appears to be moving in direction 60° $N$ of $W$ to the pilot in A. Find the true velocity of $B$. [sin 60° = 0.866 ; sin 75° = 0.966]



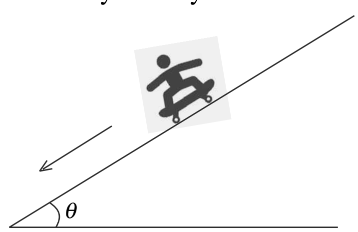

Q. 38. A small cart A starts moving on a horizontal surface, assumed to be $x$-$y$ plane along a straight line parallel to $x$-axis (see figure) with a constant acceleration of 4 $m/s^2$. Initially it is located on the positive $y$-axis at a distance 9 $m$ from origin. At the instant the cart starts moving, a ball is rolled along the surface from the origin in a direction making an angle 45° with the $x$-axis. The ball moves without friction at a constant velocity and hits the cart.



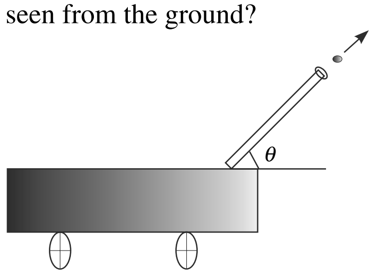

(a) Describe the path of the ball in a reference frame attached to the cart.

(b) Find the speed of the ball.

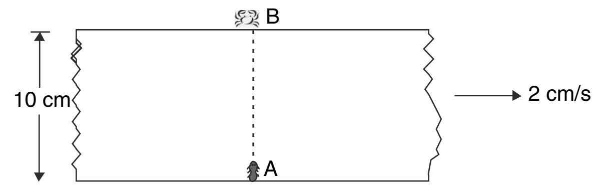
Q. 39. (a) A boy on a skateboard is sliding down on a smooth incline having inclination angle $\theta$. He throws a ball such that he catches it back after time $T$. With what velocity was the ball thrown by the boy relative to himself ?




(b) Barrel of an anti aircraft gun is rotating in vertical plane (it is rotating up from the horizontal position towards vertical orientation in the plane of the fig). The length of the barrel is $L = \sqrt{2}\ m$ and barrel is rotating with angular velocity $\omega = 2\ rad/s$. At the instant angle $\theta$ is 45° a shell is fired with a velocity $2\sqrt{2}\ m/s$ with respect to the exit point of the barrel. The tank recoils with speed 4 $m/s$. What is the launch speed of the shell as seen from the ground?




Q. 40. long piece of paper is10 $cm$ wide and is moving uniformly along its length with a velocity of 2 $cm/s$. An ant starts moving on the paper from point $A$ and moves uniformly with respect to the paper. A spider was located exactly opposite to the ant just outside the paper at point $B$ at the instant the ant started to move on the paper. The spider, without moving itself, was able to grab the ant 5 second after it (the ant) started to move. Find the speed of ant relative to the paper.



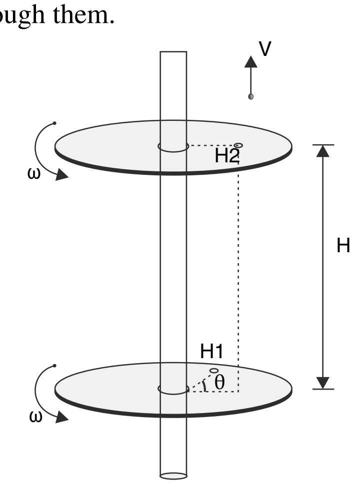

Q. 41. Two particles A and B are moving uniformly in a plane in two concentric circles. The time period of rotation is $T_A$ = 8 minute and $T_B$ = 11 minute respectively for the two particles. At time $t = 0$, the two particles are on a straight line passing through the centre of the circles. The particles are rotating in same sense. Find the minimum time when the two particles will again fall on a straight line passing through the centre.

Q. 42. A particle moves in $xy$ plane with its position vector changing with time ($t$) as

$\vec{r} = (\sin t)\hat{i} + (\cos t)\hat{j}$ (in meter)

Find the tangential acceleration of the particle as a function of time. Describe the path of the particle.

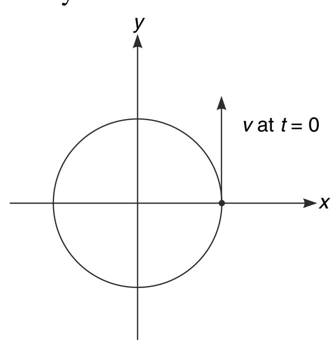
Q. 43. Two paper discs are mounted on a rotating vertical shaft. The shaft rotates with a constant angular speed $\omega$ and the separation between the discs is H. A bullet is fired vertically up so that it pierces through the two discs. It creates holes H1 and H2 in the lower and the upper discs. The angular separation between the two holes (measured with respect to the shaft axis) is $\theta$. Find the speed ($v$) of the bullet. Assume that the speed of the bullet does not change while travelling through distance H and that the discs do not complete even one revolution in the interval the bullet pierces through them.




Q. 44. (a) A car moves around a circular arc subtending an angle of 60° at the centre. The car moves at a constant speed $u_0$ and magnitude of its instantaneous acceleration is $a_0$. Find the average acceleration of the car over the 60° arc.

(b) The speed of an object undergoing uniform circular motion is 4 *m/s*. The magnitude of the change in the velocity during 0.5 sec is also 4 *m/s*. Find the minimum possible centripetal acceleration (in $m/s^2$) of the object.

Q. 45. A particle is fixed to the edge of a disk that is rotating uniformly in anticlockwise direction about its central axis. At time $t = 0$ the particle is on the $X$ axis at the position shown in figure and it has velocity $v$




(a) Draw a graph representing the variation of the $x$ component of the velocity of the particle as a function of time.

(b) Draw the $y$-component of the acceleration of the particle as a function of time.

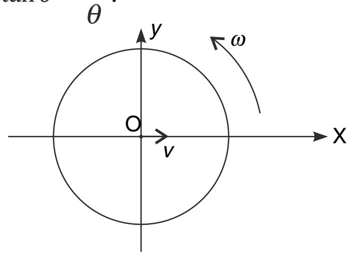
Q. 46. A disc is rotating with constant angular velocity $\omega$ in anticlockwise direction. An insect sitting at the centre (which is origin of our co-ordinate system) begins to crawl along a radius at time $t = 0$ with a constant speed $V$ relative to the disc. At time $t = 0$ the velocity of the insect is along the $X$ direction.

(a) Write the position vector $(\vec{r})$ of the insect at time '$t$'.

(b) Write the velocity vector $(\vec{v})$ of the insect at time '$t$'.

(c) Show that the $X$ component of the velocity of the insect become zero when the disc has rotated through an angle $\theta$ given by

$$\tan\theta = \frac{1}{\theta}.$$



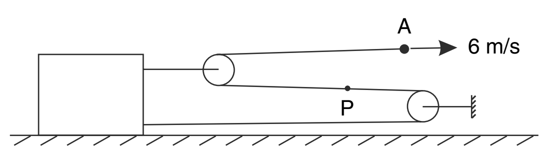

Q. 47. (a) A point moving in a circle of radius $R$ has a tangential component of acceleration that is always $n$ times the normal component of acceleration (radial acceleration). At a certain instant speed of particle is $v_0$. What is its speed after completing one revolution?

(b) The tangential acceleration of a particle moving in $xy$ plane is given by $a_t = a_0 \cos\theta$. Where $a_0$ is a positive constant and $\theta$ is the angle that the velocity vector makes with the positive direction of $X$ axis. Assuming the speed of the particle to be zero at $x = 0$, find the dependence of its speed on its $x$ co-ordinate.

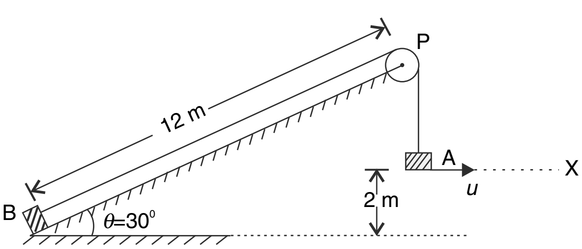
Q. 48. A particle is rotating in a circle. When it is at point $A$ its speed is $V$. The speed increases to 2 $V$ by the time the particle moves to $B$. Find the magnitude of change in velocity of the particle as it travels from $A$ to $B$. Also, find $\vec{V_A}.\vec{\Delta V}$ ; where $\vec{V_A}$ is its velocity at point A and $\vec{\Delta V}$ is change in velocity as it moves from $A$ to $B$.

Q. 49. A particle starts from rest moves on a circle with its speed increasing at a constant rate of . Find the angle through which it 0.8 $ms^{-2}$ would have turned by the time its acceleration becomes 1 $ms^2$.

Q. 50. In the arrangement shown in the fig, end $A$ of the string is being pulled with a constant horizontal velocity of 6 *m/s*. The block is free to slide on the horizontal surface and all string segments are horizontal. Find the velocity of point $P$ on the thread.




Q. 51. In the arrangement shown in the fig, block $A$ is pulled so that it moves horizontally along the line $AX$ with constant velocity $u$. Block $B$ moves along the incline. Find the time taken by $B$ to reach the pulley $P$ if $u$ = 1*m/s*. The string is inextensible.



## LEVEL 2

Q. 52. Two friends $A$ and $B$ are running on a circular track of perimeter equal to 40 *m*. At time $t = 0$ they are at same location running in the same direction. $A$ is running slowly at a uniform speed of 4.5 *km/hr* whereas $B$ is running swiftly at a speed of 18 *km/hr*.

(a) At what time $t_0$ the two friends will meet again?

(b) What is average velocity of $A$ and $B$ for the interval $t = 0$ to $t = t_0$?

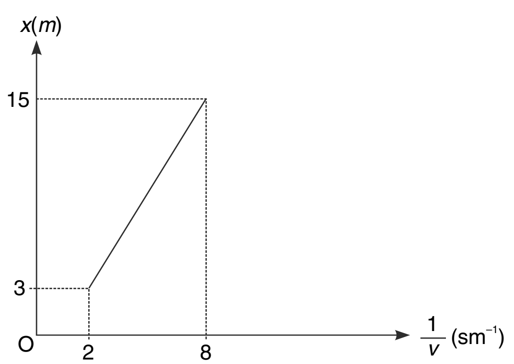
Q. 53. A particle is moving along $x$ axis. Its position as a function of time is given by $x = x(t)$. Say whether following statements are true or false.

(a) The particle is definitely slowing down if

$$\frac{d^2x}{dt^2} > 0 \text{ and } \frac{dx}{dt} < 0$$

(b) The particle is definitely moving towards the origin if $\frac{d\left(x^2\right)}{dt} < 0$

Q. 54. Graph of position ($x$) vs inverse of velocity $\left(\frac{1}{v}\right)$ for a particle moving on a straight line is as shown. Find the time taken by the particle to move from $x = 3$ *m* to $x = 15$ *m*.




Q. 55. Harshit and Akanksha both can run at speed $v$ and walk at speed $u (u < v)$. They together start on a journey to a place that is at a distance equal to $L$. Akanksha walks half of the distance and runs the second half. Harshit walks for half of his travel time and runs in the other half.

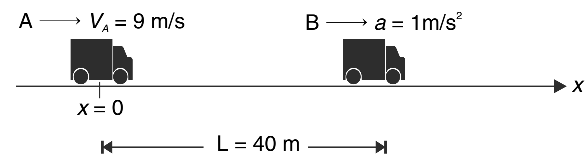
(a) Who wins?

(b) Draw a graph showing the positions of both Harshit and Akanksha versus time.

(c) Find Akanksha's average speed for covering distance $L$.

(d) How long does it take Harshit to cover the distance?

Q. 56. There are two cars on a straight road, marked as $x$ axis. Car $A$ is travelling at a constant speed of $V_A = 9$ *m/s*. Let the position of the Car $A$, at time $t = 0$, be the origin. Another car $B$ is $L = 40$ *m* ahead of car $A$ at $t = 0$ and starts moving at a constant acceleration of $a = 1$ *m/s*$^2$ (at $t = 0$). Consider the length of the two cars to be negligible and treat them as point objects.




(a) Plot the position–time ($x$–$t$) graph for the two cars on the same graph. The two graphs intersect at two points. Draw conclusion from this.

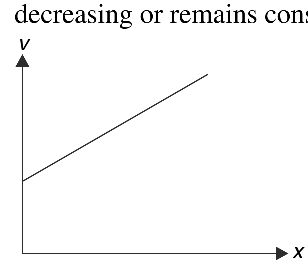
(b) Determine the maximum lead that car $A$ can have.

Q. 57. Particle $A$ is moving with a constant velocity of $V_A = 50\ ms^{-1}$ in positive $x$ direction. It crossed the origin at time $t = 10\ s$. Another particle $B$ started at $t = 0$ from the origin and moved with a uniform acceleration of $a_B = 2\ ms^{-2}$ in positive $x$ direction.

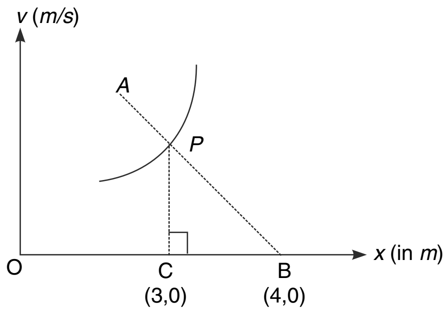
(a) For how long was $A$ ahead of $B$ during the subsequent journey?

(b) Draw the position ($x$) time ($t$) graph for the two particles and mark the interval for which $A$ was ahead of $B$.

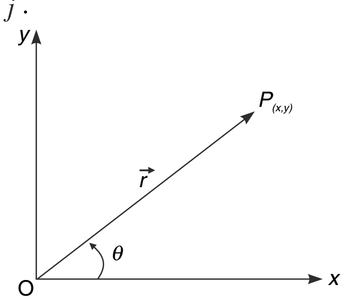
Q. 58. (a) A particle is moving along the $x$ axis and its velocity vs position graph is as shown. Is the acceleration of the particle increasing, decreasing or remains constant?



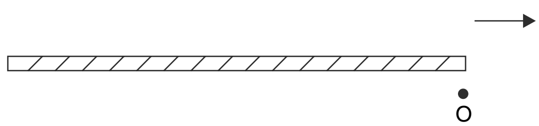

(b) A particle is moving along $x$ axis and its velocity ($v$) vs position ($x$) graph is a curve as shown in the figure. Line $APB$ is normal to the curve at point $P$. Find the instantaneous acceleration of the particle at $x = 3.0$ *m*.




Q. 59. A particle has co-ordinates $(x, y)$. Its position vector makes on angle $\theta$ with positive $x$ direction. In an infinitesimally small interval of time the particle moves such that length of its position vector does not change but angle $\theta$ increases by $d\theta$. Express the change in position vector of the particle in terms of $x$, $y$, $d\theta$ and unit vectors $\hat{i}$ and $\hat{j}$.



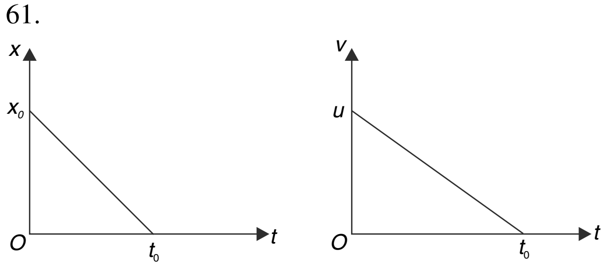

Q. 60. A rope is lying on a table with one of its end at point $O$ on the table. This end of the rope is pulled to the right with a constant acceleration starting from rest. It was observed that last 2 $m$ length of the rope took 5 $s$ in crossing the point $O$ and the last 1$m$ took 2 $s$ in crossing the point $O$.




(a) Find the time required by the complete rope to travel past point $O$.

(b) Find length of the rope.

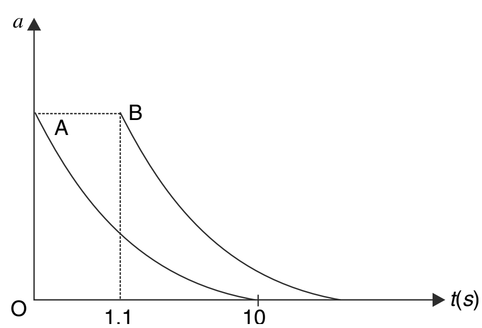
Q. 61.



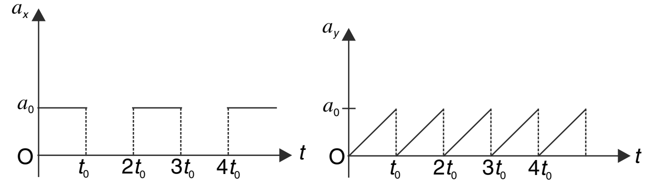

Two particles 1 and 2 move along the $x$ axis. The position $(x)$ - time $(t)$ graph for particle 1 and velocity $(v)$ - time $(t)$ graph for particle 2 has been shown in the figure. Find the time when the two particles collide. Also find the position $(x)$ where they collide. It is given that $x_0 = ut_0$, and that the particle 2 was at origin at $t = 0$.

Q. 62. Two stations $A$ and $B$ are 100$km$ apart. A passenger train crosses station $A$ travelling at a speed of 50 $km/hr$. The train maintains constant speed for 1 hour 48 minute and then the driven applies brakes to stop the train at station $B$ in next 6 minute. Another express train starts from station $B$ at the time the passenger train was crossing station $A$. The driver of the express train runs the train with uniform acceleration to attain a peak speed $v_0$. Immediately after the train attains the peak speed $v_0$, he applies breaks which cause the train to stop at station $A$ at the same time the passenger train stops at $B$. Brakes in both the trains cause uniform retardation of same magnitude. Find the travel time of two trains and $v_0$.

Q. 63. Particle $A$ starts from rest and moves along a straight line. Acceleration of the particle varies with time as shown in the graph. In 10 $s$ the velocity of the particle becomes 60 $m/s$ and the acceleration drops to zero. Another particle $B$ starts from the same location at time $t = 1.1$ $s$ and has acceleration – time relationship identical to $A$ with a delay of 1.1 $s$. Find distance between the particles at time $t = 15$ $s$.




Q. 64.




A particle is moving in $x$–$y$ plane. The $x$ and $y$ components of its acceleration change with time according to the graphs given in figure. At time $t = 0$, its velocity is $v_0$ directed along positive

$y$ direction. If $a_0 = \frac{v_0}{t_0}$, find the angle that the velocity of the particle makes with $x$ axis at time $t = 4t_0$.

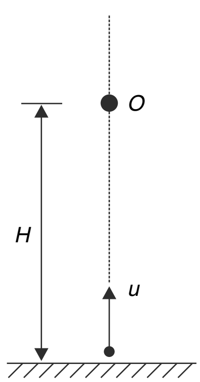
Q. 65. A particle is moving along positive $x$ direction and experiences a constant acceleration of $4\ m/s^2$ in negative $x$ direction. At time $t = 3$ second its velocity was observed to be $10\ m/s$ in positive $x$ direction.

(a) Find the distance travelled by the particle in the interval $t = 0$ to $t = 3\ s$. Also find distance travelled in the interval $t = 0$ to $t = 7.5\ s$..

(b) Plot the displacement – time graph for the interval $t = 0$ to $7.5\ s$.

Q. 66. A bead moves along a straight horizontal wire of length $L$, starting from the left end with velocity $v_0$. Its retardation is proportional to the distance that remains to the right end of the wire. Find the initial retardation (at left end of the wire) if the bead reaches the right end of the wire with a velocity $\frac{v_0}{2}$.

Q. 67. A ball is projected vertically up from the ground surface with an initial velocity of $u = 20\ m/s$. $O$ is a fixed point on the line of motion of the ball at a height of $H = 15\ m$ from the ground. Plot a graph showing variation of distance ($s$) of the ball from the fixed point $O$, with time ($t$). [Take $g = 10\ m/s^2$]. Plot the graph for the entire time of flight of the ball.




Q. 68. Two bodies 1 and 2 of different shapes are released on the surface of a deep pond. The mass of the two bodies are $m_1 = 1\ kg$ and $m_2 = 1.2\ kg$ respectively. While moving through water, the bodies experience resistive force given as $R = bv$, where $v$ is speed of the body and $b$ is a positive constant dependent on shape of the body. For bodies 1 and 2 value of $b$ is $2.5\ kg/s$ and $3.0\ kg/s$ respectively. Neglect all other forces apart from gravity and the resistive force, while answering following questions : [Hint : acceleration = force/mass]

(i) With what speed $v_{10}$ and $v_{20}$ will the two bodies hit the bed of the pond.

[Take $g = 10\ m/s^2$]

(ii) Which body will acquire speed equal to half the terminal speed in less time.

Q. 69. A prototype of a rocket is fired from the ground. The rocket rises vertically up with a uniform acceleration of $\frac{5}{4}\ m/s^2$. 8 second after the start a small nut gets detached from the rocket. Assume that the rocket keeps rising with the constant acceleration.

(a) What is the height of the rocket at the instant the nut lands on the ground

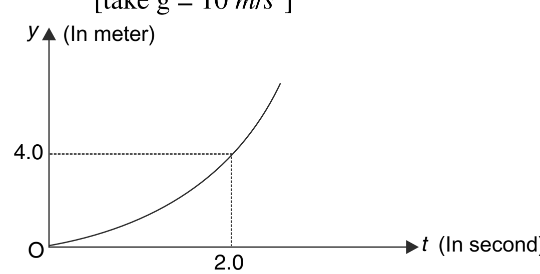
(b) Plot the velocity – time graph for the motion of the nut after it separates from the rocket till it hits the ground. Plot the same velocity–time graph in the reference frame of the rocket. Take vertically upward direction as positive and $g = 10\ m/s^2$

Q. 70. An elevator starts moving upward with constant acceleration. The position time graph for the floor of the elevator is as shown in the figure. The ceiling to floor distance of the elevator is $1.5\ m$. At $t = 2.0\ s$, a bolt breaks loose and drops from the ceiling.

(a) At what time $t_0$ does the bolt hit the floor?

(b) Draw the position time graph for the bolt starting from time $t = 0$.

[take g = $10\ m/s^2$]




Q. 71. At $t = 0$ a projectile is projected vertically up with a speed $u$ from the surface of a peculiar planet. The acceleration due to gravity on the planet changes linearly with time as per equation $g = \alpha t$ where $\alpha$ is a constant.

(a) Find the time required by the projectile to attain maximum height.

(b) Find maximum height attained.

(c) Find the total time of flight.

Q. 72. A wet ball is projected horizontally at a speed of $u = 10\ m/s$ from the top of a tower $h = 31.25\ m$ high. Water drops detach from the ball at regular intervals of $\Delta t = 1.0\ s$ after the throw.

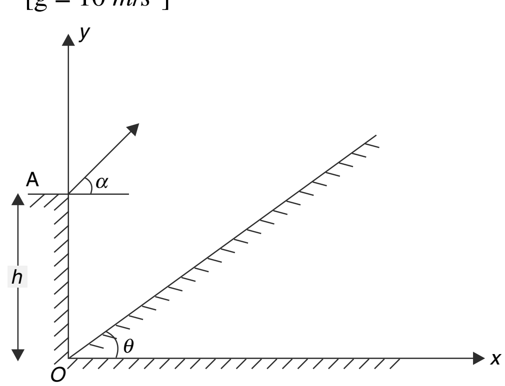
(a) How many drops will detach from the ball before it hits the ground.

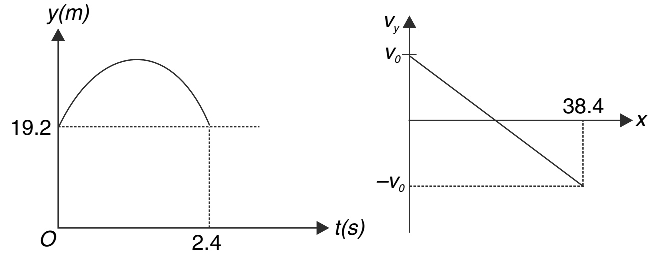
(b) How far away the drops strike the ground from the point where the ball hits the ground?

Q. 73. Two stones of mass $m$ and $M$ ($M > m$) are dropped $\Delta t$ time apart from the top of a tower. Take time $t = 0$ at the instant the second stone is released. Let $\Delta v$ and $\Delta s$ be the difference in their speed and their mutual separation respectively. Plot the variation of $\Delta v$ and $\Delta s$ with time for the interval both the stones are in flight. [$g = 10\ m/s^2$]

Q. 74. A particle is moving in the $xy$ plane on a sinusoidal course determined by $y = A \sin kx$, where $k$ and $A$ are constants. The $X$ component of the velocity of the particle is constant and is equal to $v_0$ and the particle was at origin at time $t = 0$. Find the magnitude of the acceleration of the particle when it is at point having $x$ co ordinate $x = \dfrac{\pi}{2k}$.

Q. 75. A ball is projected from a cliff of height $h = 19.2\ m$ at an angle $\alpha$ to the horizontal. It hits an incline passing through the foot of the cliff, inclined at an angle $\theta$ to the horizontal. Time of flight of the ball is $T = 2.4\ s$. Foot of the cliff is the origin of the co-ordinate system, horizontal is $x$ direction and vertical is $y$ direction (see figure). Plot of $y$ co-ordinate $vs$ time and $y$ component of velocity of the ball ($v_y$) $vs$ its $x$ co-ordinate ($x$) is as shown. $x$ and $y$ are in $m$ and time is in $s$ in the graph. [g = 10 $m/s^2$]




(a) Find the angle of projection $\alpha$

(b) Find the inclination ($\theta$) of the incline.

(c) If the ball is projected with same speed but at an angle $\theta$ (= inclination of incline) to the horizontal, will it hit the incline above or below the point where it struck the incline earlier?

Q. 76. (i) A canon can fire shells at speed $u$. Inclination of its barrel to the horizontal can be changed in steps of $\Delta\theta = 1°$ ranging from $\theta_1 = 15°$ to $\theta_2 = 85°$. Let $R_n$ be the horizontal range for projection angle $\theta = n°$.

$$\Delta R_n = |R_n - R_{n+1}|$$

For what value of $n$ the value of $\Delta R_n$ is maximum? Neglect air resistance.

(ii) A small water sprinkler is in the shape of a hemisphere with large number of uniformly spread holes on its surface. It is placed on ground and water comes out of each hole with speed u. Assume that we mentally divide the ground into many small identical patches – each having area $\Delta S$. What is the distance of a patch from the sprinkler which receives maximum amount of water ?

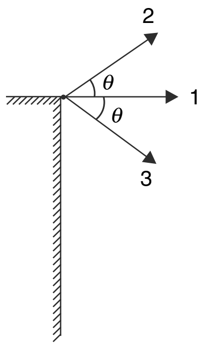
Q. 77. A gun fires a large number of bullets upward. Due to shaking of hands some bullets deviate as much as 1° from the vertical. The muzzle speed of the gun is 150 $m/s$ and the height of gun above the ground is negligible. The radius of the head of the person firing the gun is 10$cm$. You can assume that acceleration due to gravity is nearly constant for heights involved and its value is g = 10 $m/s^2$. The gun fires 1000 bullets and they fall uniformly over a circle of radius $r$. Neglect air resistance.

You can use the fact $\sin\theta \simeq \theta$ when $\theta$ is small.

(a) Find the approximate value of $r$.

(b) What is the probability that a bullet will fall on the person's head who is firing?

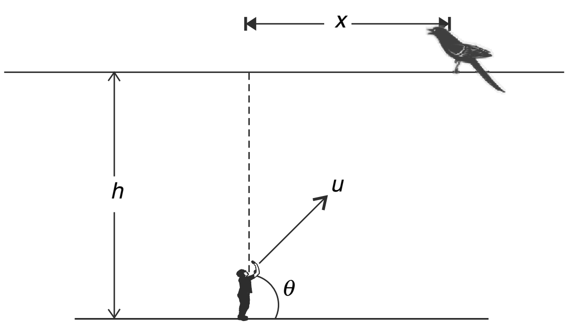
**Q. 78.** Three stones are projected simultaneously with same speed $u$ from the top of a tower. Stone 1 is projected horizontally and stone 2 and stone 3 are projected making an angle $\theta$ with the horizontal as shown in fig. Before stone 3 hits the ground, the distance between 1 and 2 was found to increase at a constant rate $u$.




(a) Find $\theta$

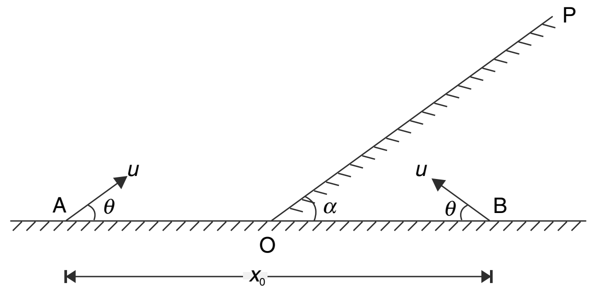
(b) Find the rate at which the distance between 2 and 3 increases.

Q. 79. A horizontal electric wire is stretched at a height $h = 10\ m$ above the ground. A boy standing on the ground can throw a stone at a speed $u = 20\ ms^{-1}$. Find the maximum horizontal distance $x$ at which a bird sitting on the wire can be hit by the stone.



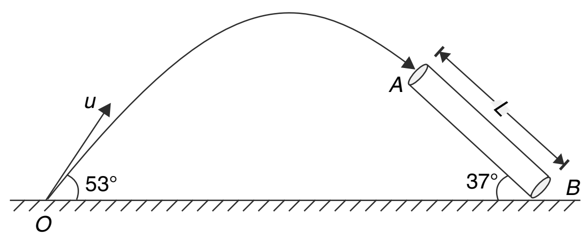

Q. 80. A wall $OP$ is inclined to the horizontal ground at an angle $\alpha$. Two particles are projected from points $A$ and $B$ on the ground with same speed ($u$) in directions making an angle $\theta$ to the horizontal (see figure). Distance between points $A$ and $B$ is $x_0 = 24\ m$. Both particles hit the wall elastically and fall back on the ground. Time of flight (time required to hit the wall and then fall back on to the ground) for particles projected from $A$ and $B$ are 4 $s$ and 2 $s$ respectively. Both the particles strike the wall perpendicularly and at the same location. [In elastic collision, the velocity component of the particle that is perpendicular to the wall gets reversed without change in magnitude]

(a) Calculate maximum height attained by the particle projected from $A$.




(b) Calculate the inclination of the wall to the horizontal ($\alpha$) [$g = 10\ m/s^2$]

Q. 81. $AB$ is a pipe fixed to the ground at an inclination of 37°. A ball is projected from point $O$ at a speed of $u = 20 m/s$ at an angle of 53° to the horizontal and it smoothly enters into the pipe with its velocity parallel to the axis of the pipe. [Take $g = 10\ ms^{-2}$]

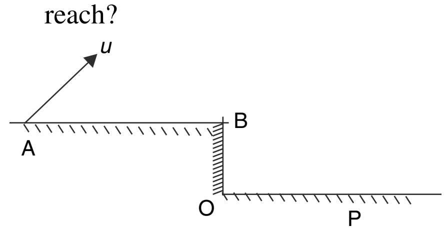



(a) Find the length $L$ of the pipe

(b) Find the distance of end $B$ of the pipe from point $O$.

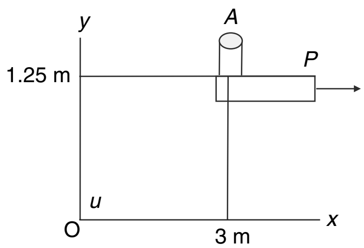
Q. 82. (a) A boy throws several balls out of the window of his house at different angles to the horizontal. All balls are thrown at speed $u = 10\ m/s$ and it was found that all of them hit the ground making an angle of 45° or larger than that with the horizontal. Find the height of the window above the ground [take $g = 10\ m/s^2$]

(b) A gun is mounted on an elevated platform $AB$. The distance of the gun at A from the edge $B$ is $AB = 960\ m$. Height of platform is $OB = 960\ m$. The gun can fire shells with a velocity of $u = 100\ m/s$ at any angle. What is the minimum distance ($OP$) from the foot of the platform where the shell of gun can reach?

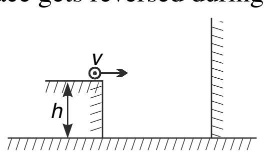



Q. 83 An object A is kept fixed at the point $x = 3\ m$

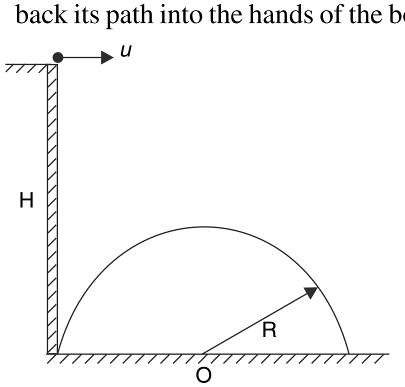
and $y = 1.25\ m$ on a plank $P$ raised above the ground. At time $t = 0$ the plank starts moving along the $+\ x$ direction with an acceleration $1.5\ m/s^2$. At the same instant a stone is projected from the origin with a velocity $u$ as shown. A stationary person on the ground observes the stone hitting the object during its downwards motion at an angle of 45° to the horizontal. All the motions are in $x$-$y$ plane. Find $u$ and the time after which the stone hits the object. Take $g = 10\ m/s^2$




Q. 84. (a) A particle is thrown from a height $h$ horizontally towards a vertical wall with a speed $v$ as shown in the figure. If the particle returns to the point of projection after suffering two elastic collisions, one with the wall and another with the ground, find the total time of flight. [Elastic collision means the velocity component perpendicular to the surface gets reversed during collision.]




(b) Touching a hemispherical dome of radius $R$ there is a vertical tower of height $H = 4\ R$. A boy projects a ball horizontally at speed $u$ from the top of the tower. The ball strikes the dome at a height $\frac{R}{2}$ from ground and rebounds. After rebounding the ball retraces back its path into the hands of the boy. Find $u$.




Q. 85. A city bus has a horizontal rectangular roof and a rectangular vertical windscreen. One day it was raining steadily and there was no wind.

(a) Will the quantity of water falling on the roof in unit time be different for the two cases (i) the bus is still (ii) the bus is moving with speed $v$ on a horizontal road ?

(b) Draw a graph showing the variation of quantity of water striking the windscreen in unit time with speed of the bus ($v$).

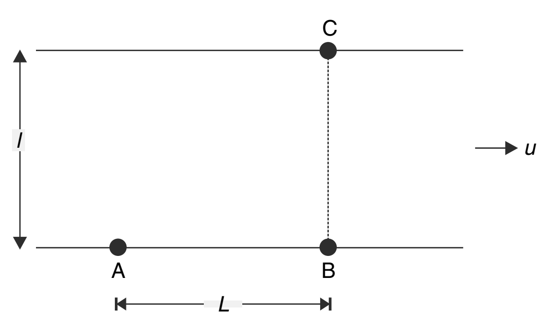
Q. 86. A truck is travelling due north descending a hill of slope angle $\theta = \tan^{-1}(0.1)$ at a constant speed of 90 $km/hr$. At the base of the hill there is a gentle curve and beyond that the road is level and heads 30° east of north. A south bound police car is travelling at 80 $km/hr$ along the level road at the base of the hill approaching the truck. Find the velocity of the truck relative to police car in terms of unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$. Take $x$ axis towards east, $y$ axis towards north and $z$ axis vertically upwards.

Q. 87. Two persons $A$ and $B$ travelling at 60 $km/hr^{-1}$ in their cars moving in opposite directions on a straight road observe an airplane. To the person $A$, the airplane appears to be moving perpendicular to the road while to the observe $B$ the plane appears to cross the road making an angle of 45°.

(a) At what angle does the plane actually cross the road (relative to the ground).

(b) Find the speed of the plane relative to the ground.

Q. 88.

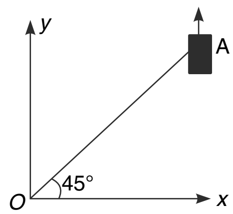



Two friends $A$ and $B$ are standing on a river bank $L$ distance apart. They have decided to meet at a point $C$ on the other bank exactly opposite to $B$. Both of them start rowing simultaneously on boats which can travel with velocity $V = 5\ km/hr$ in still water. It was found that both reached at $C$ at the same time. Assume that path of both the boats are straight lines. Width of the river is $l = 3.0$ *km* and water is flowing at a uniform speed of $u = 3.0$ *km/hr*.

(a) In how much time the two friends crossed the river.

(b) Find $L$.

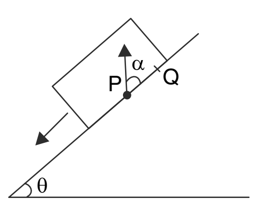
**Q. 89.** On a frictionless horizontal surface, assumed to be the *x-y* plane, a small trolley A is moving along a straight line parallel to the y-axis (see figure) with a constant velocity of $(\sqrt{3} - 1)$ *m/s*. At a particular instant, when the line *OA* makes an angle of 45° with the *x*-axis, a ball is thrown along the surface from the origin *O*. Its velocity makes an angle $\phi$ with the *x*-axis and it hits the trolley.




(a) The motion of the ball is observed from the frame of the trolley. Calculate the angle $\theta$ made by the velocity vector of the ball with the *x*-axis in this frame.

(b) Find the speed of the ball with respect to the surface, if $\phi = \frac{4\theta}{3}$ .

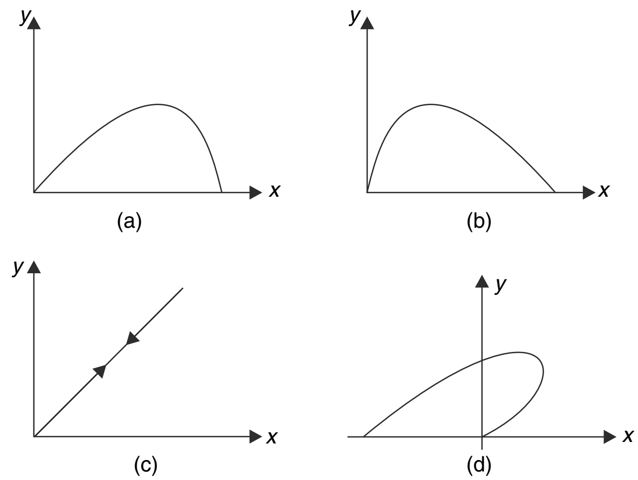
**Q. 90.** A large heavy box is sliding without friction down a smooth plane having inclination angle $\theta$. From a point *P* at the bottom of a box, a particle is projected inside the box. The initial speed of the particle with respect to box is *u* and the direction of projection makes an angle $\alpha$ with the bottom as shown in figure



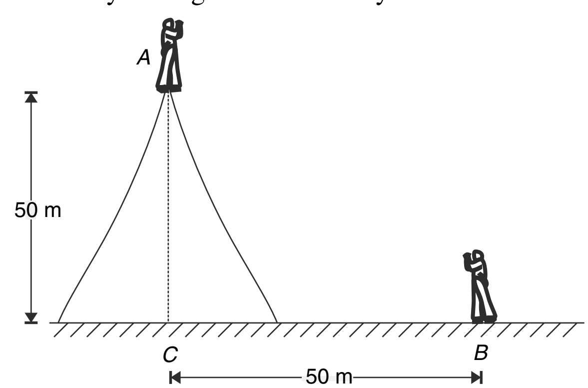

(a) Find the distance along the bottom of the box between the point of projection *P* and the point *Q* where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance)

(b) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when the particle was projected.

Q. 91. A ball is projected in vertical *x–y* plane from a car moving along horizontal *x* direction. The car is speeding up with constant acceleration. Which one of the following trajectory of the ball is not possible in the reference frame attached to the car? Give reason for your answer. Explain the condition in which other trajectories are possible. Consider origin at the point of projection.




Q. 92. A boy standing on a cliff 50 *m* high throws a ball with speed 40 *m/s* directly aiming towards a man standing on ground at *B*. At the same time the man at *B* throws a stone with a speed of 10 *m/s* directly aiming towards the boy.

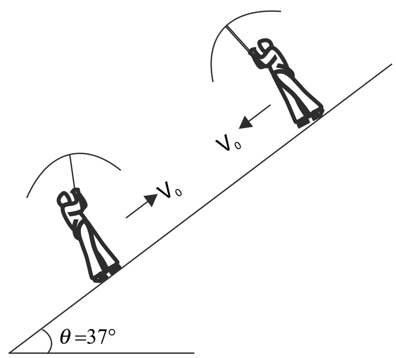



(a) Will the ball and the stone collide? If yes, at what time after projection?

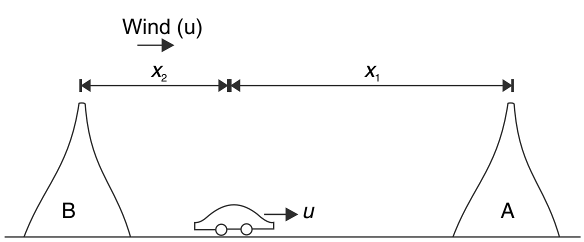
(b) At what height above the ground the two objects collide?

(c) Draw the path of ball in the reference frame of the stone.

Q. 93. A man walking downhill with velocity $V_0$ finds that his umbrella gives him maximum protection from rain when he holds it such that the stick is perpendicular to the hill surface. When the man turns back and climbs the hill with velocity $V_0$, he finds that it is most appropriate the hold the umbrella stick vertical. Find the actual speed of raindrops in terms of $V_0$. The inclination of the hill is $\theta = 37°$.

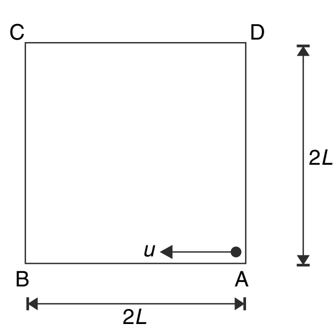



Q. 94. There are two hills $A$ and $B$ and a car is travelling towards hill $A$ along the line joining the two hills. Car is travelling at a constant speed $u$. There is a wind blowing at speed $u$ in the direction of motion of the car (i.e., from hill $B$ to $A$). When the car is at a distance $x_1$ from $A$ and $x_2$ from $B$ it sounds horn (for very short interval). Driver hears the echo of horn from both the hills at the same time.




Find the ratio $\frac{x_1}{x_2}$ taking speed of sound in still air to be $V$.

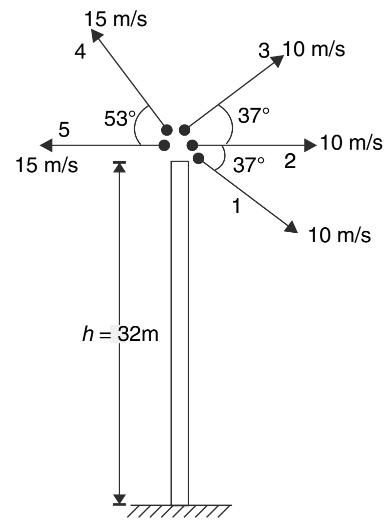
Q. 95. The figure shows a square train wagon $ABCD$ which has a smooth floor and side length of 2 $L$. The train is moving with uniform acceleration ($a$) in a direction parallel to $DA$. A 'ball is rolled along the floor with a velocity $u$, parallel to $AB$, with respect to the wagon. The ball passes through the centre of the wagon floor. At the instant it is at the centre, brakes are applied and the train begins to retard at a uniform rate that is equal to its previous acceleration ($a$)




(a) Will the ball hit the wall $BC$ or wall $CD$ or the corner $C$?

(b) What is speed of the ball, relative to the wagon at the instant it hits a wall ?

Q. 96. Five particles are projected simultaneously from the top of a tower that is $h = 32\ m$ high. The initial velocities of projection are as shown in figure. Velocity of 2 and 5 are horizontal.




(a) Which particle will hit the ground first?

(b) Separation between which two particles is maximum at the instant the first particle hits the ground?

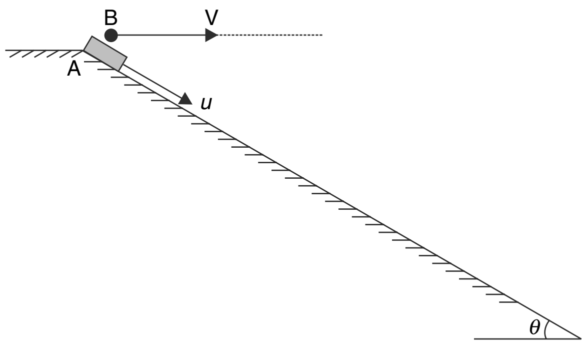
(c) Which two particles are last and last but one to hit the ground? Calculate the distance between these two particles (still in air), at a time 0.3$s$ after the third particle lands on ground.

[$g = 10\ m/s^2$, $\tan 37° = \frac{3}{4}$]

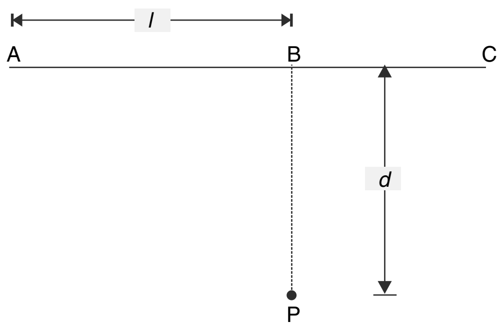
Q. 97. From the top of a long smooth incline a small body $A$ is projected along the surface with speed $u$. Simultaneously, another small object $B$ is thrown horizontally with velocity $v = 10\ m/s$, from the same point. The two bodies travel in the same vertical plane and body $B$ hits body $A$ on the incline. If the inclination angle of the incline is $\theta = \cos^{-1}\left(\frac{4}{5}\right)$ find

(a) the speed $u$ with which $A$ was projected.

(b) the distance from the point of projection, where the two bodies collide.




Q. 98. A man is on straight road $AC$, standing at $A$. He wants to get to a point $P$ which is in field at a distance '$d$' off the road (see figure). Distance $AB$ is $l = 50$. The man can run on the road at a speed $v_1 = 5\ m/s$ and his speed in the field is $v_2 = 3\ m/s$.




(a) Find the minimum value of '$d$' for which man can reach point $P$ in least possible time by travelling only in the field along the straight line $AP$.

(b) If value of '$d$' is half the value found in (a), what length the man must run on the road before entering the field, in order to reach '$P$' in least possible time.

Q. 99. Two particles, $A$ and $B$ are moving in concentric circles in anticlockwise sense in the same plane with radii of the circles being $\gamma_A = 1.0\ m$ and $\gamma_B = 2.0\ m$ respectively. The particles move with same angular speed of $\omega = 4\ rad/s$.

Find the angular velocity of $B$ as observed by $A$ if

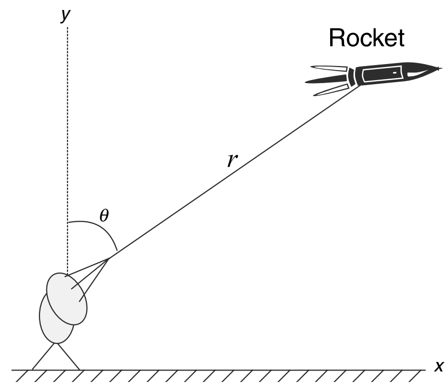
(a) Particles lie on a line passing through the centre of the circle.

(b) Particles lie on two perpendicular lines passing through the centre.

Q. 100. (a) An unpowered rocket is in flight in air. At a moment the tracking radar gives following data regarding the rocket.

$r$ = distance of the rocket from the radar = $4000\ m$, $\frac{dr}{dt} = 0, \frac{d\theta}{dt}$ = 1.8 deg/sec;

where $\theta$ is the angle made by position vector of the rocket with respect to the vertical.




(a) Neglect atmospheric resistance and take $g = 9.8\ m/s^2$ at the concerned height. Neglect height of radar. Calculate the height of the rocket above the ground.

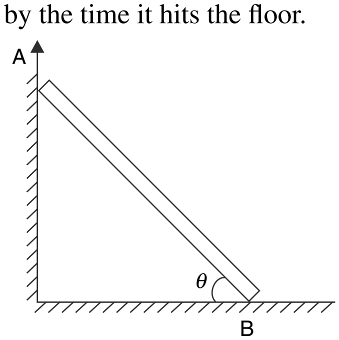
(b) Two points A and B are moving in $X$ - $Y$ plane with constant velocity of $V_A = \left(6\hat{i} - 9\hat{j}\right)\ m/s$ and $V_B = \left(\hat{i} + \hat{j}\right)\ m/s$ respectively. At time $t = 0$ they are 15 $m$ apart and both of them lie on $y$ axis with $A$ lying away on positive $Y$ axis with respect to $B$. What is the angular velocity of $A$ with respect to $B$ at $t = 1\ s$?

Q. 101. A stone is projected horizontally with speed $u$ from the top of a tower of height $h$.

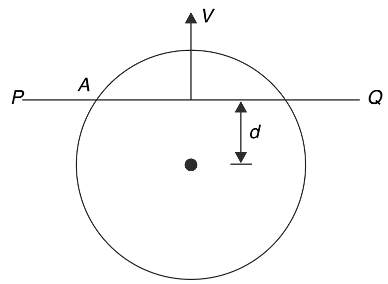
(a) Calculate the radius of curvature of the path of the stone at the point where its tangential and radial accelerations are equal.

(b) What shall be the height ($h$) of the tower so that radius of curvature of the path is always less than the value obtained in ($a$) above.

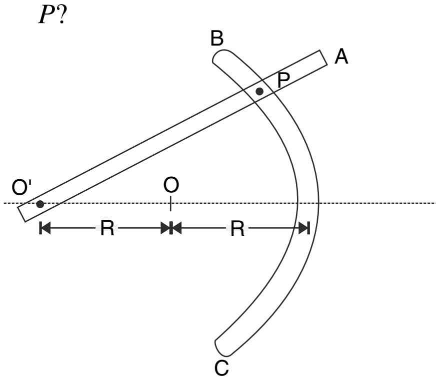
Q. 102. A stick of length $L = 2.0\ m$ is leaned against a wall as shown. It is released from a position when $\theta = 60°$. The end $A$ of the stick remains in contact with the wall and its other end $B$ remains in contact with the floor as the stick slides down. Find the distance travelled by the centre of the stick by the time it hits the floor.



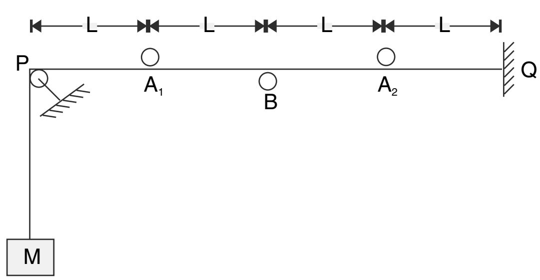
Q. 103. (a) A line $PQ$ is moving on a fixed circle of radius $R$. The line has a constant velocity $v$ perpendicular to itself. Find the speed of point of intersection ($A$) of the line with the circle at the moment the line is at a distance $d = R/2$ from the centre of the circle.



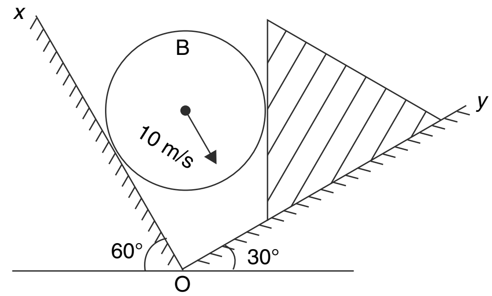

(b) In the figure shown a pin $P$ is confined to move in a fixed circular slot of radius $R$. The pin is also constrained to remains inside the slot in a straight arm $O'A$. The arm moves with a constant angular speed $\omega$ about the hinge $O'$. What is the acceleration of point $P$?



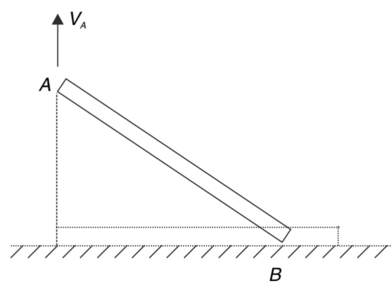

Q. 104. A flexible inextensible cord supports a mass $M$ as shown in figure. $A_1$, $A_2$ and $B$ are small pulleys in contact with the cord. At time $t = 0$ cord $PQ$ is horizontal and $A_1$, $A_2$ start moving vertically down at a constant speed of $v_1$, whereas $B$ moves up at a constant speed of $v_2$. Find the velocity of mass $M$ as a function of time.




Q. 105. In the arrangement shown in the figure $A$ is an equilateral wedge and the ball $B$ is rolling down the incline $XO$. Find the velocity of the wedge (of course, along $OY$) at the moment velocity of the ball is 10 $m/s$ parallel to the incline $XO$.



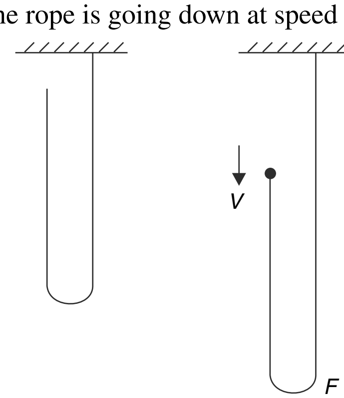

Q. 106. A meter stick $AB$ is lying on a horizontal table. Its end $A$ is pulled up so as to move it with a constant velocity $V_A = 4ms^{-1}$ along a vertical line. End $B$ slides along the floor.



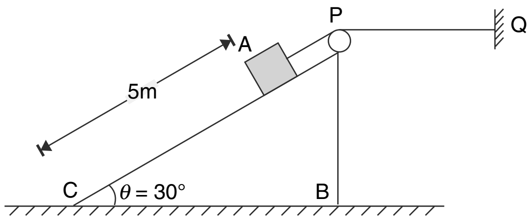

(a) After how much time ($t_0$) speed ($V_B$) of end B becomes equal to the speed ($V_A$) of end A ?

(b) Find distance travelled by the end $B$ in time $t_0$.

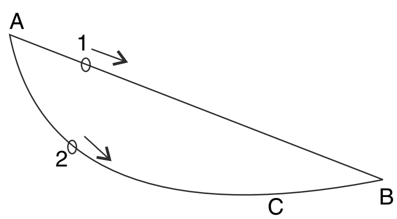
Q. 107. One end of a rope is fixed at a point on the ceiling the other end is held close to the first end so that the rope is folded. The second end is released from this position. Find the speed at which the fold at $F$ is descending at the instant the free end of the rope is going down at speed $V$.




Q. 108. Block $A$ rests on inclined surface of wedge $B$ which rests on a horizontal surface. The block $A$ is connected to a string, which passes over a pulley $P$ (fixed rigidly to the wedge $B$) and its other end is securely fixed to a wall at $Q$. Segment $PQ$ of the string is horizontal and $Q$ is at a large distance from $P$. The system is let go from rest and the wedge slides to right as $A$ moves on its inclined face. Find the distance travelled by $A$ by the time it reaches the bottom of the inclined surface.

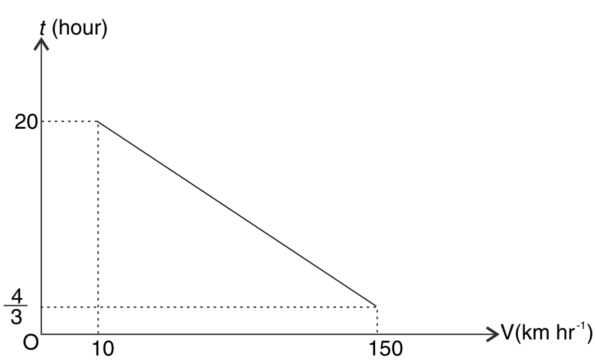



Q. 109. Two frictionless ropes connect points $A$ & $B$ in vertical plane. Bead 1 is allowed to slide along the straight rope $AB$ and bead 2 slides along the curved rope $ACB$. Which bead will reach $B$ in less time?




## LEVEL 3

Q. 110. A car manufacturer usually tells a optimum speed ($V_0$) at which the car should be driven to get maximum mileage. In order to find the optimum speed for a new model, an engineer of the car company experimented a lot and finally plotted a graph between the extreme time $t$ (defined as number of hours a tank full of petrol lasts) vs the constant speed $V$ at which car was run.




(a) Calculate the optimum speed $V_0$ for this new model.

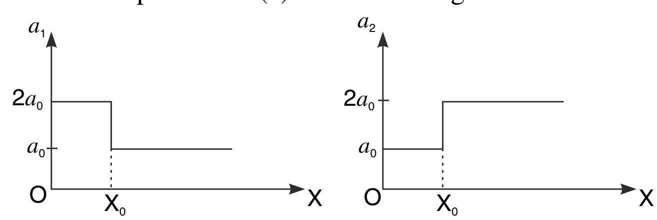
(b) If the fuel tank capacity of this car is 50 litre, what maximum mileage can be obtained from this car?

Q. 111. While starting from a station, a train driver was instructed to stop his train after time $T$ and to cover maximum possible distance in that time.

(a) If the maximum acceleration and retardation for the train are both equal to '$a$', find the maximum distance it can cover.

(b) Will the train travel more distance if maximum acceleration is '$a$' but the maximum retardation caused by the brakes is '$2a$'? Find this distance.

Q. 112. Two particles 1 and 2 start simultaneously from origin and move along the positive $X$ direction. Initial velocity of both particles is zero. The acceleration of the two particles depends on their displacement ($x$) as shown in fig.




(a) Particles 1 and 2 take $t_1$ and $t_2$ time respectively for their displacement to become $x_0$. Find $\frac{t_2}{t_1}$.

(b) Which particle will cover $2x_0$ distance in least time? Which particle will cross the point $x = 2x_0$ with greater speed?

(c) The two particles have same speed at a certain time after the start. Calculate this common speed in terms of $a_0$ and $x_0$.

Q. 113. A cat is following a rat. The rat is running with a constant velocity $u$. The cat moves with constant speed $v$ with her velocity always directed towards the rat. Consider time to be $t = 0$ at an instant when both are moving perpendicular to each other and separation between them is $L$.

(a) Find acceleration of the cat at $t = 0$.

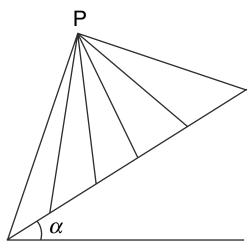
(b) Find the time $t_0$ when the rat is caught.

(c) Find the acceleration of the cat immediately before it catches the rat.

(d) Draw the path of the rat as seen by the cat.

Q. 114.(a) Prove that bodies starting at the same time $t = 0$ from the same point, and following frictionless slopes in different directions in the same vertical plane, all lie in a circle at any subsequent time.

(b) Using the above result do the following problem. $A$ point $P$ lies above an inclined plane of inclination angle $\alpha$. $P$ is joined to the plane at number of points by smooth wires, running in all possible directions. Small bodies (in shape of beads) are released from $P$ along all the wires simultaneously. Which body will take least time to reach the plane.



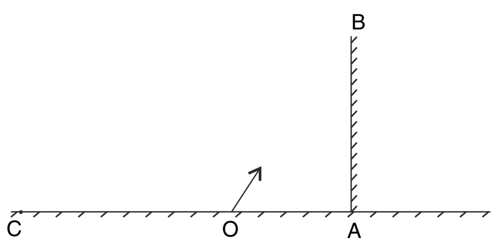

Q. 115. The acceleration due to gravity near the surface of the earth is $\vec{g}$. A ball is projected with velocity $\vec{u}$ from the ground.

(a) Express the time of flight of the ball.

(b) Write the expression of average velocity of the ball for its entire duration of flight.

Express both answers in terms of $\vec{u}$ and $\vec{g}$.

Q. 116. A ball is projected from point $O$ on the ground. It hits a smooth vertical wall $AB$ at a height $h$ and rebounds elastically. The ball finally lands at a point $C$ on the ground. During the course of motion, the maximum height attained by the ball is $H$.




(a) Find the ratio $\frac{h}{H}$ if $\frac{OA}{OC} = \frac{1}{3}$

(b) Find the magnitude of average acceleration of the projectile for its entire course of flight if it was projected at an angle of 45° to the horizontal.

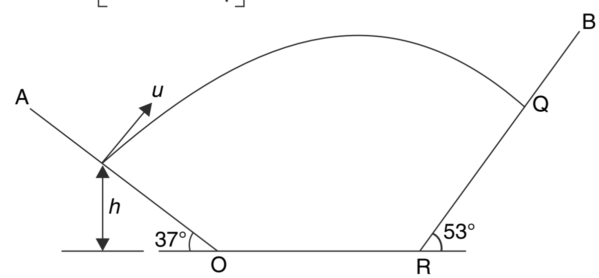
Q. 117. A boy can throw a ball up to a speed of $u = 30\ m/s$. He throws the ball many a times, ensuring that maximum height attained by the ball in each throw is $h = 20\ m$. Calculate the maximum horizontal distance at which a ball might have landed from the point of projection. Neglect the height of the boy. [$g = 10\ m/s^2$]

Q. 118. A valley has two walls inclined at 37° and 53° to the horizontal. A particle is projected from point $P$ with a velocity of $u = 20\ m/s$ along a direction perpendicular to the incline wall $OA$. The Particle hits the incline surface RB perpendicularly at Q. Take $g = 10\ m/s^2$ and find:

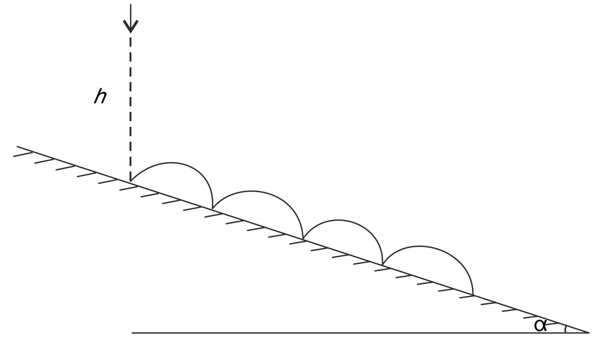
(a) The time of flight of the particle.

(b) Vertical height h of the point $P$ from horizontal surface $OR$.

$$\left[\tan 37° = \frac{3}{4}\right]$$




**Q. 119.**




A ball is released in air above an incline plane inclined at an angle $\alpha$ to the horizontal. After falling vertically through a distance $h$ it hits the incline and rebounds. The ball flies in air and then again makes an impact with the incline. This way the ball rebounds multiple times. Assume that collisions are elastic, i.e., the ball rebound without any loss in speed and in accordance to the law of reflection.

(a) Distance between the points on the incline where the ball makes first and second impact is $l_1$ and distance between points where the ball makes second and third impact is $l_2$. Which is large $l_1$ or $l_2$?

(b) Calculate the distance between the points on the incline where the ball makes second and fifth impact.

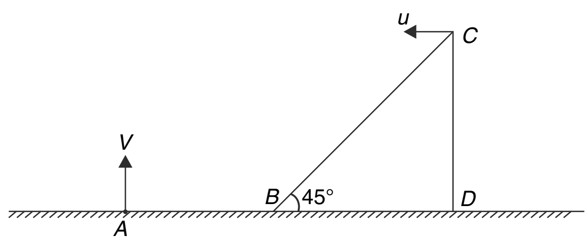
Q. 120. A terrorist '$A$' is walking at a constant speed of 7.5 $km/hr$ due West. At time $t = 0$, he was exactly

South of an army camp at a distance of 1 km. At this instant a large number of army men scattered in every possible direction from their camp in search of the terrorist. Each army person walked in a straight line at a constant speed of 6 *km/hr*.

(a) What will be the closest distance of an army person from the terrorist in this search operation?

(b) At what time will the terrorist get nearest to an army person?

Q. 121. A large wedge *BCD*, having its inclined surface at an angle $\theta = 45°$ to the horizontal, is travelling horizontally leftwards with uniform velocity $u = 10\ m/s$




At some instant a particle is projected vertically up with speed $V = 20\ m/s$ from point A on ground lying at some distance right to the lower edge B of the wedge. The particle strikes the incline BC normally, while it was falling. [$g = 10\ m/s^2$]

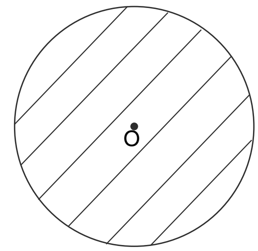
(a) Find the distance *AB* at the instant the particle was projected from *A*.

(b) Find the distance of lower edge *B* of the wedge from point *A* at the instant the particle strikes the incline.

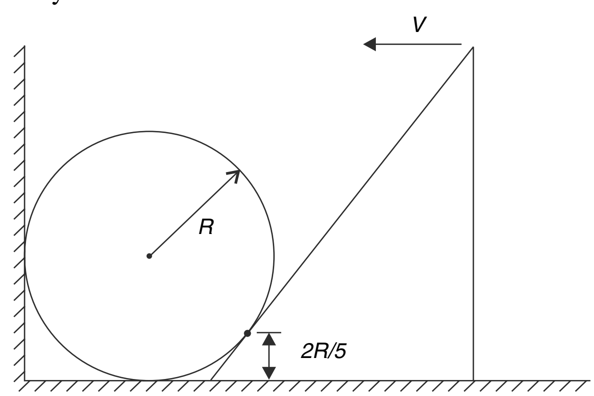
(c) Trace the path of the particle in the reference frame attached to the wedge.

Q. 122. The speed of river current close to banks is nearly zero. The current speed increases linearly from the banks to become maximum ($= V_0$) in the middle of the river. A boat has speed '*u*' in still water. It starts from one bank and crosses the river. Its velocity relative to water is always kept perpendicular to the current. Find the distance through which the boat will get carried away by the current (along the direction of flow) while it crosses the river. Width of the river is *l*.

Q. 123. A water sprinkler is positioned at *O* on horizontal ground. It issues water drops in every possible direction with fixed speed *u*. This way the sprinkler is able to completely wet a circular area of the ground (see fig). A horizontal wind starts blowing at a speed of $\frac{u}{2\sqrt{2}}$. Mark the area on the ground that the sprinkler will now be able to wet.




Q. 124. A cylinder of radius R has been placed in a corner as shown in the fig. A wedge is pressed against the cylinder such that its inclined surfaces touches the cylinder at a height of $\frac{2R}{5}$ from the ground. Now the wedge is pushed to the left at a constant speed $V = 15\ m/s$. With what speed will the cylinder move?

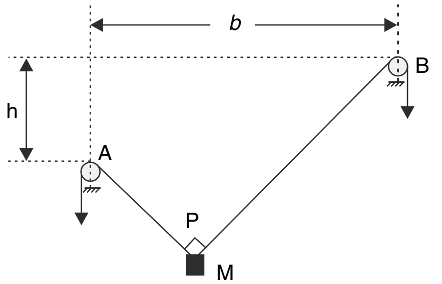



Q. 125. The entrance to a harbour consists of 50 *m* gap between two points *A* and *B* such that *B* is due east of *A*. Outside the harbour there is a 8 km/hr current flowing due east. A motor boat is located 300 m due south of A. Neglect size of the boat for answering following questions-

(a) Calculate the least speed ($V_{min}$) that the motor boat must maintain to enter the harbour.

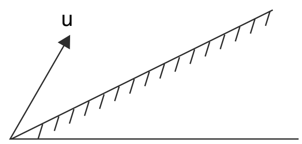
(b) Show that the course it must steer when moving at $V_{min}$ does not depend on the speed of the current.

Q. 126. Two small pegs (*A* and *B*) are at horizontal and vertical separation *b* and *h* respectively. A small block of mass *M* is suspended with the help of two light strings passing over *A* and *B* as shown in fig. The two string are always kept at right angles (i.e., $\angle APB = 90°$). Find the minimum possible gravitation potential energy of the mass assuming the reference level at location of peg *A*. [Hint: the potential energy is minimum when the block is at its lowest position]



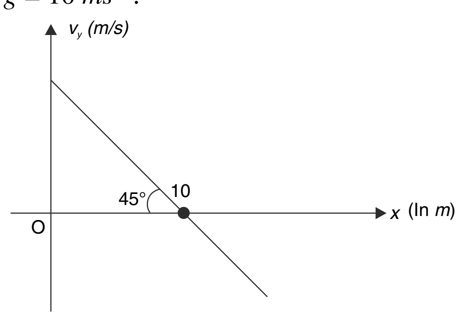

Q. 127. (a) A canon fires a shell up on an inclined plane. Prove that in order to maximize the range along the incline the shell should be fired in a direction bisecting the angle between the incline and the vertical. Assume that the shell fires at same speed all the time.

(b) A canon is used to hit a target a distance $R$ up an inclined plane. Assume that the energy used to fire the projectile is proportional to square of its projection speed. Prove that the angle at which the shell shall be fired to hit the target but use the least amount of energy is same as the angle found in part (a)

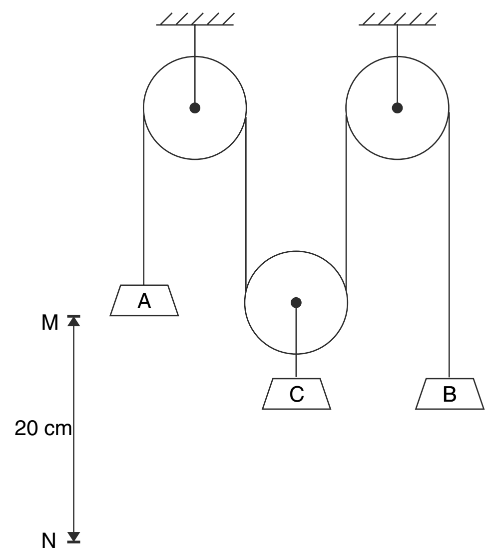



Q. 128. A ball of mass $m$ is projected from ground making an angle $\theta$ to the horizontal. There is a horizontal wind blowing in the direction of motion of the ball. Due to wind the ball experiences a constant horizontal force of $\frac{mg}{\sqrt{3}}$ in direction of its motion. Find $\theta$ for which the horizontal range of the ball will be maximum.

Q. 129. A projectile is projected from a level ground making an angle $\theta$ with the horizontal ($x$ direction). The vertical ($y$) component of its velocity changes with its $x$ co-ordinate according to the graph shown in figure. Calculate $\theta$. Take $g = 10\ ms^{-2}$.




Q. 130. In the arrangement shown in the figure, the block $C$ begins to move down at a constant speed of 7.5 $cm/s$ at time $t = 0$. At the same instant block $A$ is made to start moving down at constant acceleration. It starts at $M$ and its speed is 30 $cm/s$ when it reaches $N$ ($MN = 20\ cm$). Assuming that $B$ started from rest, find its position, velocity and acceleration when block $A$ reaches $N$.




Q. 131. A rocket prototype is fired from ground at time $t = 0$ and it goes straight up. Take the launch point as origin and vertically upward direction as positive $x$ direction. The acceleration of the rocket is given by

$$a = \frac{g}{2} - kt^2;\ \ 0 < t \le t_0$$
$$= -g; \qquad t > t_0$$

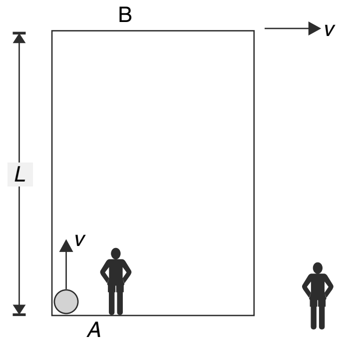
Where $t_0 = \sqrt{\frac{3g}{2k}}$

(a) Find maximum velocity of the rocket during the up journey.

(b) Find maximum height attained by the rocket.

(c) Find total time of flight.

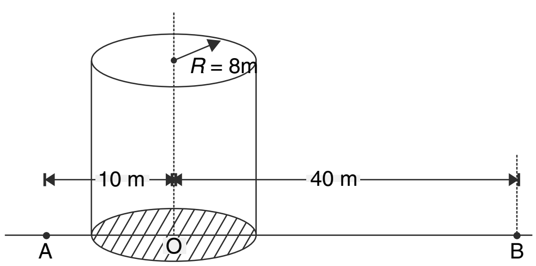
Q. 132. A man standing inside a room of length $L$ rolls a ball along the floor at time $t = 0$. The ball travels at constant speed $v$ relative to the floor, hits the front wall ($B$) and rebounds back with same speed $v$. The man catches the ball back at the wall $A$ at time $t_0$. The ball travelled along a straight line relative to the man inside the room. Another observer standing outside the room found that the entire room was travelling horizontally at constant velocity $v$ in a direction parallel to the two walls $A$ and $B$.




(a) Find the average speed of the ball in the time interval $t = 0$ to $t = t_0$ as observed by the observer outside the room.

(b) If the room has acceleration in the direction of its velocity draw a sketch of the path of the ball as observed by the observer standing outside. Assume that velocity of room was $v$ at the instant the ball was released.

Q. 133. There is a tall cylindrical building standing in a field. Radius of the cylinder is $R = 8\ m$. A boy standing at $A$ (at a distance of 10 $m$ from the centre of the cylindrical base of the building) knows that his friend is standing at $B$ behind the building. The line joining $A$ and $B$ passes through the centre of the base of the building. Distance between $A$ and $B$ is 50 $m$. A wants to throw a ball to $B$ but he realizes that the building is too tall and he cannot throw the ball over it. He throws the ball at a speed of 20 $m/s$ such that his friend at $B$ has to move minimum distance to catch it.



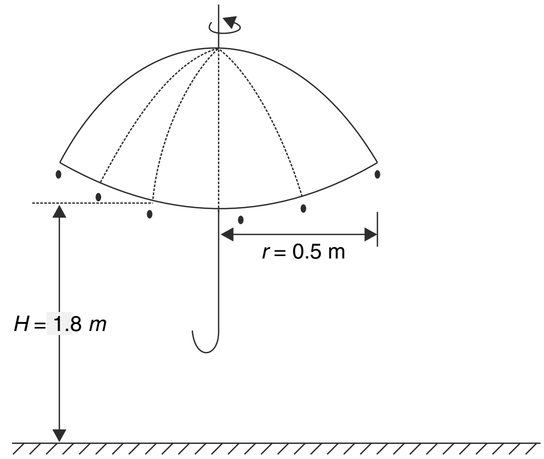

(a) What is the minimum distance that boy at $B$ will have to move to catch the ball?

(b) At what angle to the horizontal does the boy at $A$ throws the ball?

Assume that the ball is released and caught at same height above the ground.

[Take $g = 10\ m/s^2$ and $\sin^{-1}(0.75) \simeq 48.6°$

Q. 134. A wet umbrella is held upright (see figure). The man holding it is rotating it about its vertical shaft at an angular speed of $\omega = 5\ rad\ s^{-1}$. The rim of the umbrella has a radius of $r = 0.5\ m$ and it is at a height of $H = 1.8\ m$ from the floor. The man holding the umbrella gradually increases the angular speed to make it $2\ \omega$. Calculate the area of the floor that will get wet due to water drops spun off the rim and hitting the floor. [$g = 10\ m/s^2$]

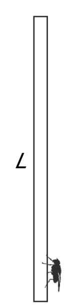



Q. 135. A ball is projected vertically up from ground. Boy $A$ standing at the window of first floor of a nearby building observes that the time interval between the ball crossing him while going up and the ball crossing him while going down is $t_1$. Another boy $B$ standing on the second floor notices that time interval between the ball passing him twice (during up motion and down motion) is $t_2$.

(a) Calculate the height difference ($h$) between the boy $B$ and $A$.

(b) Assume that the height of boy $A$ from the point of projection of the ball is also equal to $h$ and calculate the speed with which the ball was projected.

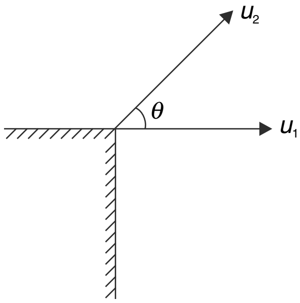
Q. 136. A stick of length $L$ is dropped from a high tower. An ant sitting at the lower end of the stick begins to crawl up at the instant the stick is released. Velocity of the ant relative to the stick remains constant and is equal to $u$. Assume that the stick remains vertical during its fall, and length of the stick is sufficiently long.



(a) Calculate the maximum height attained by the ant measured from its initial position.

(b) What time after the start the ant will be at the same height from where it started?

Q. 137. Two balls are projected simultaneously from the top of a tall building. The first ball is projected horizontally at speed $u_1 = 10\ m/s$ and the other one is projected at an angle $\theta = \tan^{-1}\left(\frac{4}{3}\right)$ to the horizontal with a velocity $u_2$. [$g = 10\ m/s^2$]



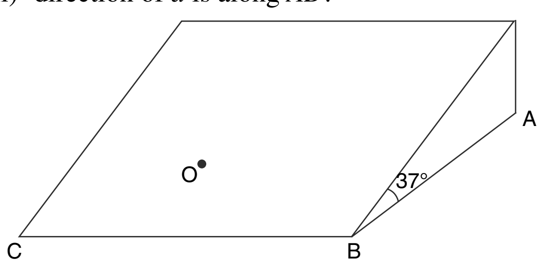

(a) Find minimum value of $u_2$ (= $u_0$) so that the velocity vector of the two balls can get perpendicular to each other at some point of time during their course of flight.

(b) Find the time after which velocities of the two balls become perpendicular if the second one was projected with speed $u_0$.

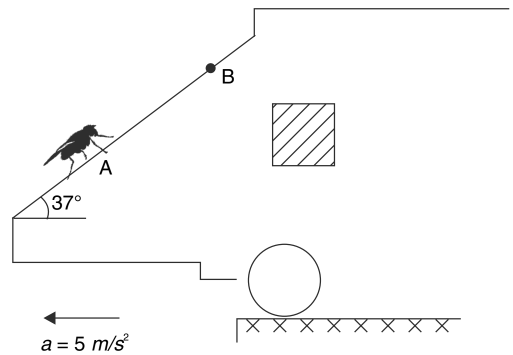
Q. 138. There is a large wedge placed on a horizontal surface with its incline face making an angle of 37° to the horizontal. A particle is projected in vertically upward direction with a velocity of $u = 6.5\ m/s$ from a point $O$ on the inclined surface. At the instant the particle is projected, the wedge begins to move horizontally with a constant acceleration of $a = 4\ m/s^2$. At what distance from point $O$ will the particle hit the incline surface if

(i) direction of $a$ is along $BC$?

(ii) direction of $a$ is along $AB$?



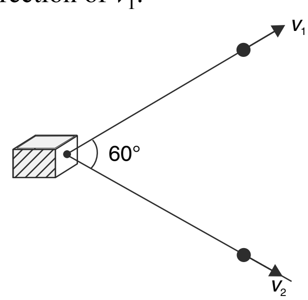

Q. 139. The windshield of a truck is inclined at 37° to the horizontal. The truck is moving horizontally with a constant acceleration of $a = 5\ m/s^2$. At the instant the velocity of the truck is $v_0 = 0.77\ m/s$, an insect jumps from point $A$ on the windshield, with a velocity $u = 2.64\ m/s$ (relative to ground) in vertically upward direction. It falls back at point $B$ on the windshield. Calculate distance $AB$. Assume that the insect moves freely under gravity and $g = 10\ m/s^2$.



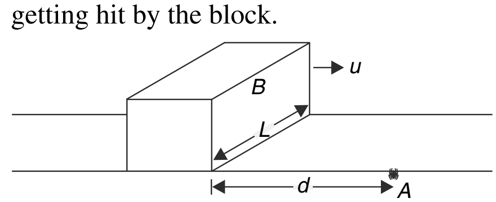

Q. 140. Two persons are pulling a heavy block with the help of horizontal inextensible strings. At the instant shown, the velocities of the two persons are $v_1$ and $v_2$ directed along the respective strings with the strings making an angle of 60° between them.

(a) Find the speed of the block at the instant shown.

(b) For what ratio of $v_1$ and $v_2$ the instantaneous velocity of the block will be along the direction of $v_1$.




Q. 141. A heavy block '$B$' is sliding with constant velocity $u$ on a horizontal table. The width of the block is $L$. There is an insect $A$ at a distance $d$ from the block as shown in the figure. The insect wants to cross to the opposite side of the table. It begins to crawl at a constant velocity $v$ at the instant shown in the figure. Find the least value of $v$ for which the insect can cross to the other side without getting hit by the block.



Q. 142. A projectile is thrown from ground at a speed $v_0$ at an angle $\alpha$ to the horizontal. Consider point of projection as origin, horizontal direction as $X$ axis and vertically upward as $Y$ axis. Let $t$ be the time when the velocity vector of the projectile becomes perpendicular to its position vector.

(a) Write a quadratic equation in $t$.

(b) What is the maximum angle $\alpha$ for which the distance of projectile from the point of projection always keeps on increasing?

[Hint: Start from the equation you obtained in part (a)]

Q. 143. A projectile is thrown from a point on ground, with initial velocity $u$ at some angle to the horizontal. Show that it can clear a pole of height $h$ at a distance $d$ from the point of projection if

$u^2 \geq g[h + \sqrt{h^2 + d^2}]$

Q. 144. A particle rotates in a circle with angular speed $\omega_0$. A retarding force decelerates it such that angular deceleration is always proportional to square root of angular velocity. Find the mean angular velocity of the particle averaged over the whole time of rotation.

# ANSWERS

**1.** The two velocities are perpendicular.

**2.** (a) 7.5 $km/hr^{-1}$

(b) 2 $hr$ 40 min

**3.** (a) $F$

(b) $T$

(c) $T$

**4.** (a) $E$,

(b) $D$,$G$

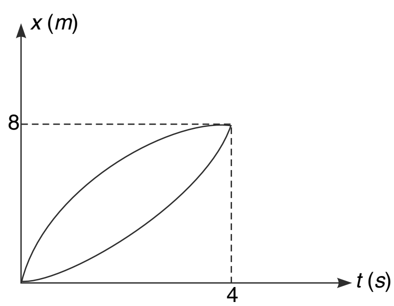
(c) $B$,$C$

**5.** (a) 4 $m/s$

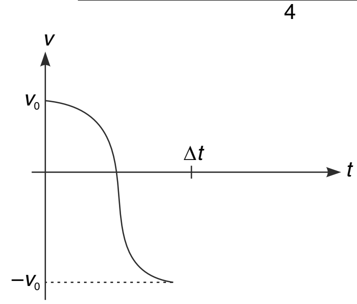
(b) 2 $m/s^2$

**6.** (a) $X_{max} = 4\ m$ ; $t = 2\ s$

(b)




**7.**




**8.** (i) $B$ and $C$

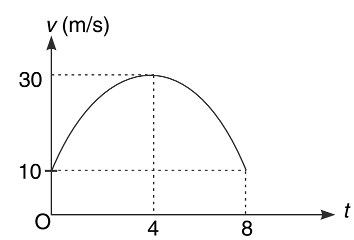
(ii) $D$

(iii) $A$, $B$, $C$, $D$

**9.** 10 $m$

**10.** $v = 12\ m/s$

**11.**




**12.** (a) particle $A$

(b) see solution for graph

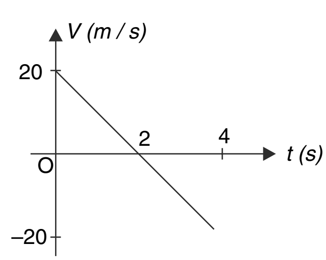
**13.** (a) $1 < t < 2\ s$ and $3 < t < 4\ s$

(b) 1 $m/s$

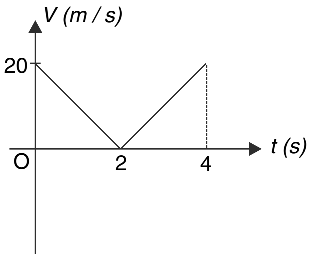
**14.** (a) 22 $(Km)$ $(s^{-1})$ $(MLy^{-1})$

(b) $\dfrac{\ln(2)}{H}$

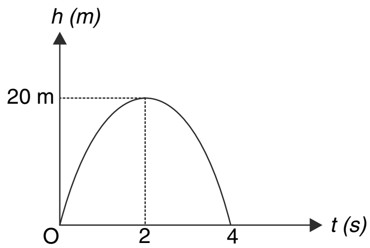
**15.** (a)



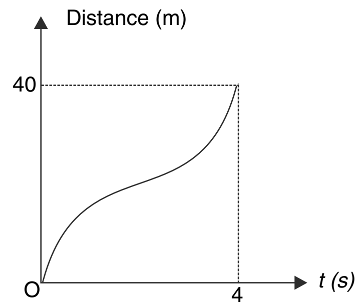
(b)



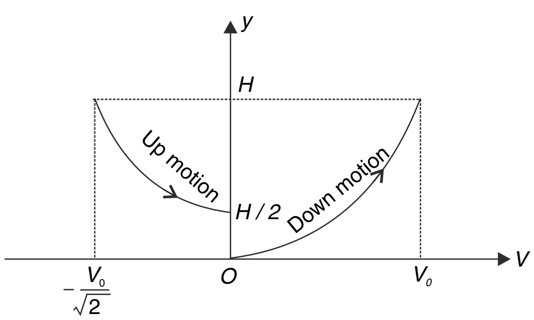

(c)




(d)




**16.**



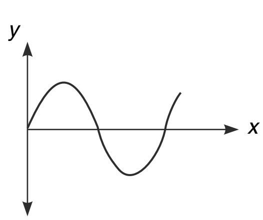

**17.** $V = 20\ ms^{-1}$

**18.** $\Delta t = \sqrt{\frac{2L}{g}}\ \left[\sqrt{2}-1\right]$

**19.** 1 *m*

**20.** All statements are true

**21.** (a)




(b) 40 *m*

**22.** The one that is projected at $\theta_2$

$$\frac{R_1}{R_2} = \frac{\tan\theta_2}{\tan\theta_1}$$

**23.** $20(1+\sqrt{2})\ m$

**24.** (a) 12.13 *m*

(b) 16 *m/s*

**25.** $a = 5.19\ m/s^2$

**26.** $u = 16\ m/s;\ \theta = \tan^{-1}\left(\frac{4\sqrt{2}}{15}\right)$

**27.** $\frac{4}{5}$

**28.** $\frac{4}{3}\pi R$

**29.** Displacement = 40 *cm*

Distance = $(30\sqrt{5} + 10\sqrt{13})\ cm$

**30.** $\frac{40}{41} s$

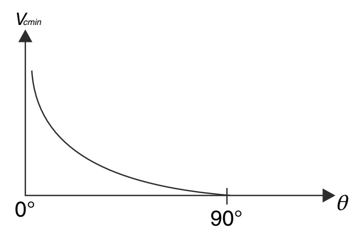
**31.** (a) $t_0 = 5\ s$

(b) car 4

**32.** $1\ m/s^2$ to $7\ m/s^2$

**33.** (a) $V_{max} = 12\ km/hr$

(b)




**34.** 5 *m/s*, 12 *m/s*

**35.** $\Delta t = 23.33\ s$

**36.** $\frac{L}{u}$

**37.** 807 *kph*

**38.** (a) Parabolic path

(b) 6 *m/s*

**39.** (a) $\frac{1}{2}Tg\cos\theta$ Perpendicular to the incline

(b) $4\sqrt{2}\ ms^{-1}$

**40.** $2\sqrt{2}\ cms^{-1}$

**41.** $\frac{88}{3}$ min

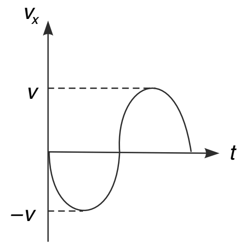
**42.** $a_t = 0$; path is circular

**43.** $v = \frac{\omega H}{\theta}$

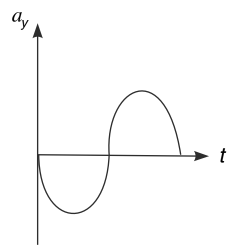
**44.** (a) $\langle a \rangle = \frac{3}{\pi} a_0$

(b) 8.37 m/s$^2$

**45.** (a)




(b)




**46.** (a) $\vec{r} = vt\,[\cos(\omega t)\,\hat{i} + \sin(\omega t)]\,\hat{j}$

(b) $\vec{V_p} = V\,[\cos(\omega t) - \omega t \sin(\omega t)]$
$\hat{i} + V\,[\sin(\omega t) + \omega t \cos(\omega t)]\,\hat{j}$

**47.** (a) $v_0 e^{2\pi n}$

(b) $V = \sqrt{2a_0 x}$

**48.** $\sqrt{3}\,\upsilon$, zero

**49.** $\frac{3}{8}$ *rad*

**50.** 2 *m/s*

**51.** 1.59 *s*

**52.** (a) $t_0 = \frac{32}{3} s$;

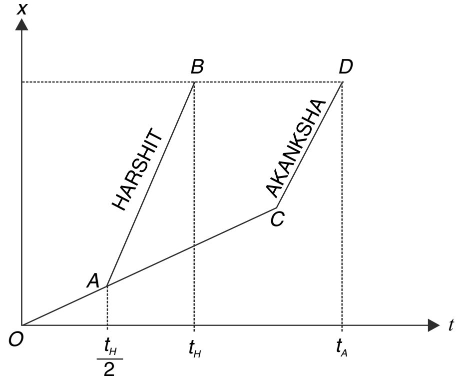
(b) $< V_A > = < V_B > = \frac{15\sqrt{3}}{8\pi} m/s$

**53.** Both are true

**54.** 60 *s*

**55.** (a) Harshit

(b)



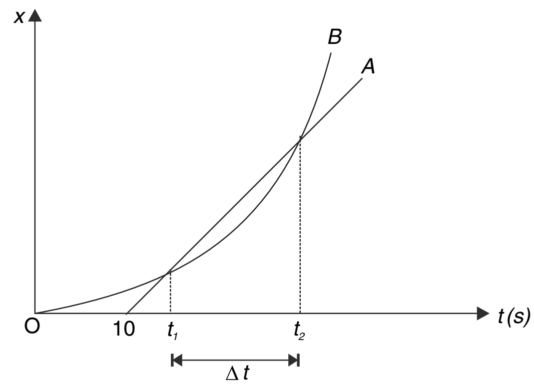

(c) $\frac{2uv}{u+v}$

(d) $\frac{2L}{u+v}$

**56.** (b) 0.5 *m*

**57.** (a) $10\sqrt{5}\,s$

(b)




**58.** (a) Acceleration is increasing

(b) 1 $m/s^2$

**59.** $\vec{\Delta r} = \left(-y\hat{i} + x\hat{j}\right)d\theta$

**60.** (a) 8.5 *s*

(b) 2.41 *m*

**61.** $t = (2 - \sqrt{2})t_0$ ; $x = (\sqrt{2} - 1)x_0$

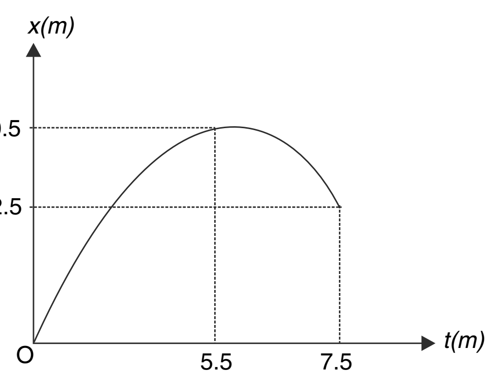
**62.** 2.2 *hr* ; 90.9 *km/hr*

**63.** 66 *m*

**64.** $\theta = \tan^{-1}\left(\frac{3}{2}\right)$

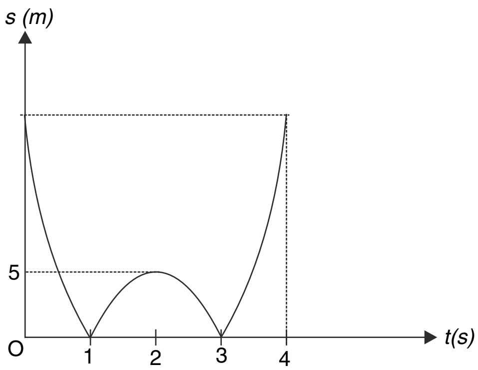
**65.** (a) 48 *m*, 68.5 *m*

(b)




**66.** $\frac{3v_0^2}{4L}$

**67.**

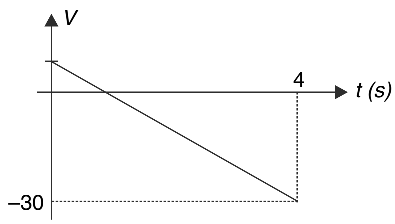



**68.** (i) $v_{10} = v_{20} = 4 m/s$

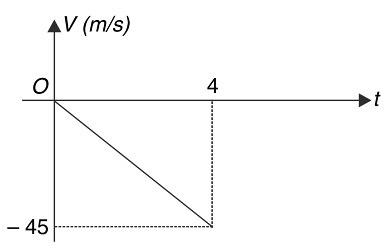
(ii) Both will take same time

**69.** (a) 90 $m$

(b)

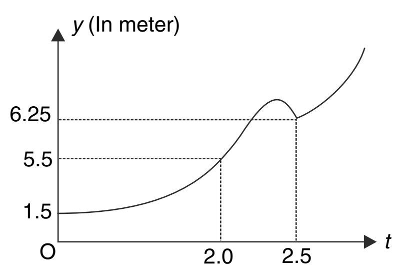



(c)




**70.** (a) 2.5 $s$

(b)




**71.** (a) $t_0 = \sqrt{\dfrac{2u}{\alpha}}$

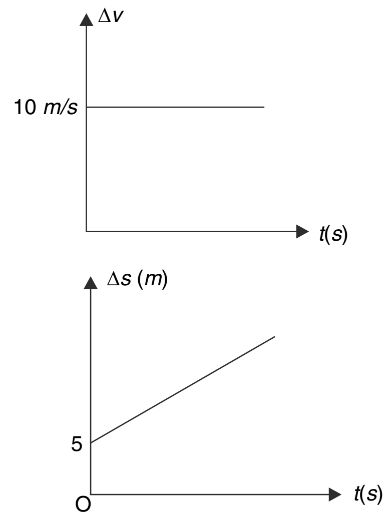
(b) $\dfrac{(2u)^{3/2}}{3\alpha^{1/2}}$

(c) $\sqrt{\dfrac{6u}{\alpha}}$

**72.** (a) 2

(b) zero

**73.**




**74.** $Ak_0 v_0^{\,2}$

**75.** (a) $\alpha = \tan^{-1}\left(\dfrac{3}{4}\right)$

(b) $\theta = \tan^{-1}\dfrac{1}{2}$

(c) The ball will hit at a point lower than the earlier spot.

**76.** (i) $n = 84°$

(ii) $\dfrac{u^2}{g}$

**77.** (a) 80 $m$

(b) $1.6 \times 10^{-3}$

**78.** (a) $\theta = 60°$

(b) $\sqrt{3}\,u$

**79.** $20\sqrt{2}\ m$

**80.** (a) 11.25 $m$

(b) $\tan^{-1}\left(\dfrac{8}{5}\right)$

**81.** (a) $L = 14.58\ m$

(b) $OB = 41.66\ m$

**82.** (a) 5 $m$

(b) 480 $m$

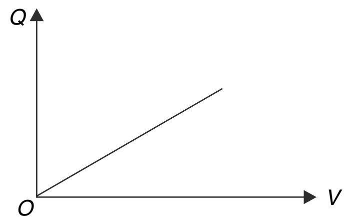
**83.** $u = 7.29\ m/s$, $t = 1\ s$.

**84.** (a) $2\sqrt{\dfrac{2h}{g}}$

(b) $u = \sqrt{21gR}$

**85.** (a) No

(b)




**86.** $\left(40\hat{i}+158.9\hat{j}-8.9\hat{k}\right)$ $km\ hr^{-1}$

**87.** (a) $\theta = \tan^{-1}(2)$

(b) $60\sqrt{5}\ kmhr^{-1}$

**88.** (a) $\frac{3}{4}hr$

(b) 4.5 *km*

**89.** (a) 45°

(b) 2 *m/s*

**90.** (a) $\frac{u^2 \sin 2\alpha}{g\cos\theta}$

(b) $\frac{u\cos(\alpha+\theta)}{\cos\theta}$

**91.** (b)

**92.** (a) yes, $\sqrt{2}\ s$

(b) zero

(c) straight line

**93.** $\frac{\sqrt{73}}{3}V_0$

**94.** $\frac{x_1}{x_2}=\frac{v+u}{v-u}$

**95.** (a) Corner C

(b) *u*

**96.** (a) particle 1

(b) Particle 2 and 5

(c) particle 3 and 4 ; 50.94 *m*

**97.** (a) $u = 8\ m/s$,

(b) 18.75 *m*

**98.** (a) $d_{\min}=\frac{200}{3}$

(b) 25 *m*

**99.** (a) $\omega = 4\ rad/s$

(b) $\omega = 4\ rad/s$

**100.** (a) 1600 *m*

(b) $\frac{3}{2}rad/\sec$

**101.** (a) $R=\frac{2\sqrt{2}u^2}{g}$

(b) $h<\frac{u^2}{2g}$

**102.** $\frac{\pi}{3}m$

**103.** (a) $\frac{2v}{\sqrt{3}}$

(b) $4\omega^2R$

**104.** $v=\frac{dy}{dt}=\frac{2v_1^2t}{\sqrt{L^2+v_1^2t^2}}+\frac{2(v_1+v_2)^2\,t}{\sqrt{L^2+(v_1+v_2)^2\,t^2}}$

**105.** $\frac{10}{\sqrt{3}}$ *m/s*

**106.** (a) $t_0=\frac{1}{4\sqrt{2}}\ s$

(b) $\left(1-\frac{1}{\sqrt{2}}\right)m$

**107.** *V*/2

**108.** 10 sin 15°

**109.** Bead 2

**110.** (a) $80\ kmhr^{-1}$

(b) $17\ kml^{-1}$

**111.** (a) $\frac{1}{4}aT^2$

(b) yes, $\frac{1}{3}aT^2$

**112.** (a) $\sqrt{2}$

(b) particle 1 will cover $2x_0$ in lesser time. Both will cross $2x_0$ with same speed.

(c) $v = (2+\sqrt{2})\sqrt{a_0x_0}$

**113.** (a) $\frac{uv}{L}$

(b) $t_0=\frac{vL}{v^2-u^2}$

(c) Zero

(d) The path will be like a spiral

**114.** (b) Body travelling along a line making an angle $\frac{\alpha}{2}$

with vertical

**115.** (a) $t = -\dfrac{2\vec{u}\cdot\vec{g}}{|\vec{g}|^2}$

(b) $\overrightarrow{V_{av}} = \vec{u} - \dfrac{\vec{g}\,(\vec{u}\cdot\vec{g})}{|\vec{g}|^2}$

**116.** (a) $\dfrac{16}{25}$

(b) $\sqrt{2}\,g$

**117.** $40\sqrt{5}\ m$

**118.** (a) 2.5 *s*

(b) 4.05 *m*

**119.** (a) $l_2 > l_1$

(b) $72\,h \sin\alpha$

**120.** (a) $\dfrac{3}{5} km$

(b) 8 min

**121.** (a) 15 *m*

(b) 15 *m*

(c) parabolic

**122.** $\dfrac{V_0 l}{2u}$

**123.** A circle of same size shifted from the original circle by $\Delta X = \dfrac{u^2}{2g}$ in the direction of wind.

**124.** 20 *m/s*

**125.** (a) $\sqrt{\dfrac{48}{37}}\ km/hr$

**126.** $U_{\min} = -\dfrac{1}{2} Mg \left[\sqrt{h^2+b^2} - h\right]$

**128.** $\theta = 60°$

**129.** $\theta = 45°$

**130.** Position: 40 *cm* up from starting position

$V_B = 45\ cm/s\ (\uparrow)$

$a_B = 22.5\ cm/s^2\ (\uparrow)$

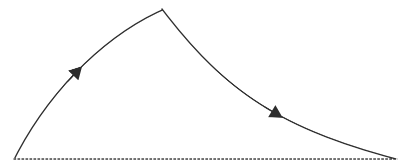
**131.** (a) $V_{\max} = \sqrt{\dfrac{g^3}{18k}}$

(b) $X_0 = \dfrac{3g^2}{16k}$

(c) $T = \dfrac{3}{2}\sqrt{\dfrac{3g}{2k}}$

**132.** (a) $\sqrt{2}\,v$

(b) path is as shown



**133.** (a) 40 *m*

(b) 24.3° or 65.7°

**134.** $21.2\ m^2$

**135.** (a) $h = \dfrac{g\,(t_1^2 - t_2^2)}{8}$

(b) $u = \dfrac{g}{2}\sqrt{2t_1^2 - t_2^2}$

**136.** (a) $H_{\max} = \dfrac{u^2}{2g}$

(b) $\dfrac{2u}{g}$

**137.** (a) $u_0 = 37.5\ m/s$

(b) $t = 1.5\ m/s$

**138.** (i) 3.38 *m*

(ii) 2.5 *m*

**139.** $AB = 0.57\ m$

**140.** (a) $\dfrac{2}{\sqrt{3}}\sqrt{v_1^2 + v_2^2 - v_1 v_2}$

(b) $\dfrac{v_1}{v_2} = 2$

**141.** $v_{\min} = \dfrac{uL}{\sqrt{d^2 + L^2}}$

**142.** (a) $t^2 - \dfrac{3v_0 \sin\alpha}{g} t + \dfrac{2v_0^2}{g^2} = 0$

(b) $\sin^{-1}\dfrac{8}{9}$

**144.** $\dfrac{\omega_0}{3}$

# 03 NEWTON'S LAWS

## LEVEL 1

Q. 1. Let $\vec{u}$ be the initial velocity of a particle and $\vec{F}$ be the resultant force acting on it. Describe the path that the particle can take if

(a) $\vec{u} \times \vec{F} = 0$ and $\vec{F}$ = constant

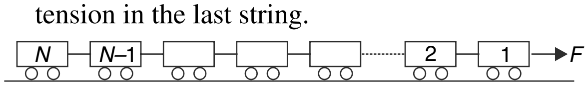
(b) $\vec{u}.\vec{F} = 0$ and $\vec{F} =$ constant

In which case can the particle retrace its path.

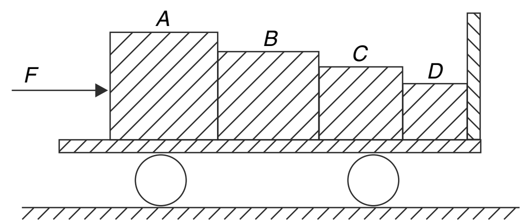
Q. 2. A ball is projected vertically up from the floor of a room. The ball experiences air resistance that is proportional to speed of the ball. Just before hitting the ceiling the speed of the ball is 10 *m/s* and its retardation is $2g$. The ball rebounds from the ceiling without any loss of speed and falls on the floor $2s$ after making impact with the ceiling. How high is the ceiling? Take $g = 10\ m/s^2$.

Q. 3. A small body of super dense material, whose mass is half the mass of the earth (but whose size is very small compared to the size of the earth), starts from rest at a height $H$ above the earth's surface, and reaches the earth's surface in time $t$. Calculate time $t$ assuming that $H$ is very small compared to the radius of the earth. Acceleration due to gravity near the surface of the earth is $g$.

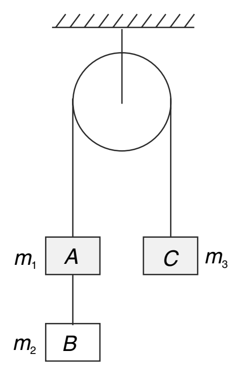
Q. 4. $N$ identical carts are connected to each other using strings of negligible mass. A pulling force $F$ is applied on the first cart and the system moves without friction along the horizontal ground. The tension in the string connecting $4^{th}$ and $5^{th}$ cart is twice the tension in the string connecting $8^{th}$ and $9^{th}$ cart. Find the total number of carts ($N$) and tension in the last string.




Q. 5. A toy cart has mass of 4 *kg* and is kept on a smooth horizontal surface. Four blocks $A$, $B$, $C$ and $D$ of masses 2 *kg*, 2 *kg*, 1 *kg* and 1 *kg* respectively have been placed on the cart. A horizontal force of $F = 40\ N$ is applied to the block $A$ (see figure). Find the contact force between block $D$ and the front vertical wall of the cart.




Q. 6. (i) Three blocks $A$, $B$ and $C$ are placed in an ideal Atwood machine as shown in the figure. When the system is allowed to move freely it was found that tension in the string connecting $A$ to $C$ was more than thrice the tension in the string connecting $A$ and $B$. The masses of the three blocks $A$, $B$ and $C$ are $m_1$, $m_2$ and $m_3$, respectively. State whether the following statements are true or false [All masses have finite non zero values and the system has a non zero acceleration].



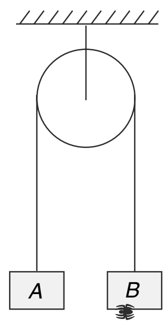

(a) $m_3$ can have any finite value

(b) $m_1 > 2m_2$

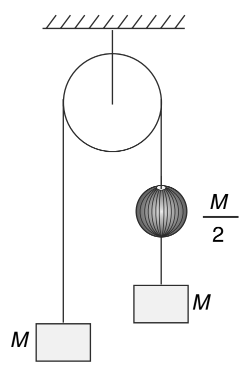
(ii) In an Atwood machine the sum of two masses is a constant. If the string can sustain a tension equal to $\left(\frac{24}{30}\right)$ of the weight of the sum of two masses, find the least acceleration of the masses. The string and pulley are light.

(iii) A load of $w$ newton is to be raised vertically through a height $h$ using a light rope. The greatest tension that the rope can bear is $\eta w$ ($\eta > 1$). Calculate the least time of ascent if it is required that the load starts from rest and must come to rest when it reaches a height $h$.

Q. 7. In the arrangement shown in the figure the system is in equilibrium. Mass of the block $A$ is $M$ and that of the insect clinging to block $B$ is $m$. Pulley and string are light. The insect loses contact with the block $B$ and begins to fall. After how much time the insect and the block B will have a separation $L$ between them.



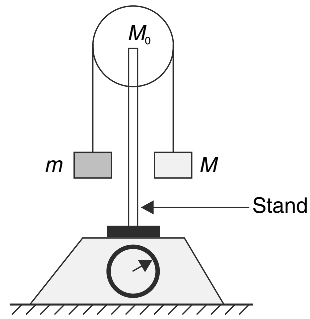

Q. 8. Two blocks of equal mass, $M$ each, are connected to two ends of a massless string passing over a massless pulley. On one side of the string there is a bead of mass $\frac{M}{2}$.



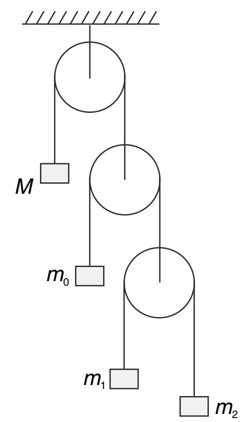

(a) When the system is released from rest the bead continues to remain at rest while the two blocks accelerate. Find the acceleration of the blocks.

(b) Find the acceleration of the two blocks if it was observed that the bead was sliding down with a constant velocity relative to the string.

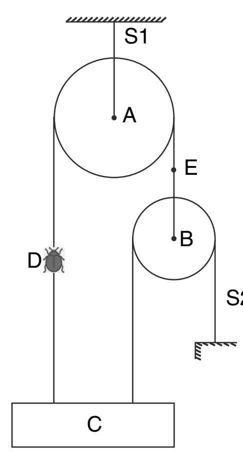
Q. 9. A pulley is mounted on a stand which is placed over a weighing scale. The combined mass of the stand and the pulley is $M_0$. A light string passes over the smooth pulley and two masses $m$ and $M$ ($> m$) are connected to its ends (see figure). Find the reading of the scale when the two masses are left free to move.




Q. 10. In the given arrangement, all strings and pulleys are light. When the system was released it was observed that $M$ and $m_0$ do not move. Find the masses $M$ and $m_0$ in terms of $m_1$ and $m_2$. Find the acceleration of all the masses if string is cut just above $m_2$.




Q.11 The system shown in the fig. is in equilibrium. Pulleys $A$ and $B$ have mass $M$ each and the block C has mass $2M$. The strings are light. There is an insect (D) of mass $M/2$ sitting at the middle or the right string. Insect does not move.



(a) Just by inspection, say if the tension in the string S1 is equal to, more than or less than 9/2 $Mg$.

(b) Find tension in the string $S2$, and $S1$.

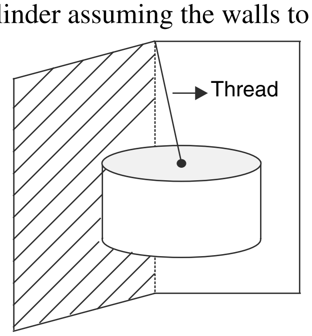
(c) Find tension in $S2$ if the insect flies and sits at point E on the string.

Q. 12. A block slides down a frictionless plane inclined at an angle $\theta$. For what value of angle $\theta$ the horizontal component of acceleration of the block is maximum? Find this maximum horizontal acceleration.

Q. 13. A tall elevator is going up with an acceleration of $a = 4\ m/s^2$. A 4 $kg$ snake is climbing up the vertical wall of the elevator with an acceleration of $a$. A 50 g insect is riding on the back of the snake and it is moving up relative to the snake at an acceleration of $a$. Find the friction force between the elevator wall and the snake. Assume that the snake remains straight.

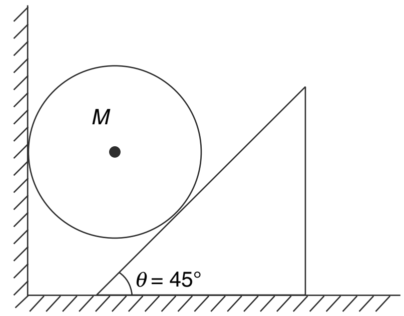
Q. 14. Due to air drag the falling bodies usually acquire a constant speed when the drag force becomes equal to weight. Two bodies, of identical shape, experience air drag force proportional to square of their speed ($F_{drag} = kv^2$, $k$ is a constant). The mass ratio of two bodies is 1 : 4. Both are simultaneously released from a large height and very quickly acquire their terminal speeds. If the lighter body reaches the ground in 25 $s$, find the approximate time taken by the other body to reach the ground.

Q. 15. A cylinder of mass $M$ and radius $r$ is suspended at the corner of a room. Length of the thread is twice the radius of the cylinder. Find the tension in the thread and normal force applied by each wall on the cylinder assuming the walls to be smooth.




Q. 16. A rod of mass $M$ and length $L$ lies on an incline having inclination of $\theta = 37°$. The coefficient of friction between the rod and the incline surface is $\mu = 0.90$. Find the tension at the mid point of the rod.

Q. 17. A ball of mass $M$ is in equilibrium between a vertical wall and the inclined surface of a wedge. The inclination of the wedge is $\theta = 45°$ and its mass is very small compared to that of the ball. The coefficient of friction between the wedge and the floor is $\mu$ and there is no friction elsewhere. Find minimum value of $\mu$ for which this equilibrium is possible.



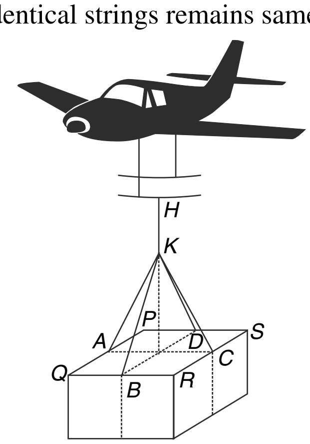

Q.18 A helicopter of mass $M = 15000\ kg$ is lifting a cubical box of mass $m = 2000\ kg$. The helicopter is going up with an acceleration of $a = 1.2\ m/s^2$. The four strings are tied at mid points of the sides of the square face $PQRS$ of the box. The strings are identical and form a knot at $K$. Another string $KH$ connects the knot to the helicopter. Neglect mass of all strings and take g = 10 $m/s^2$. Length of each string $AK$, $BK$, $CK$ and $DK$ is equal to side length of the cube.

(a) Find tension $T$ in string $AK$.

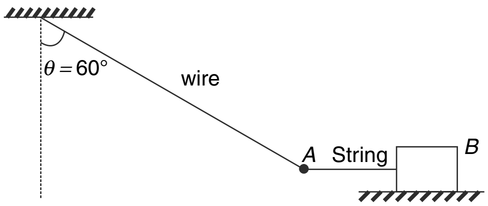
(b) Find tension $T_0$ in string $KH$.

(c) Find the force ($F$) applied by the atmosphere on the helicopter. Assume that the atmosphere exerts a negligible force on the box.

(d) If the four strings are tied at $P, Q, R$ and $S$ instead of $A, B, C$ & $D$, how will the quantities $T$, $T_0$ and $F$ change? Will they increase or decrease? Assume that length of the four identical strings remains same.



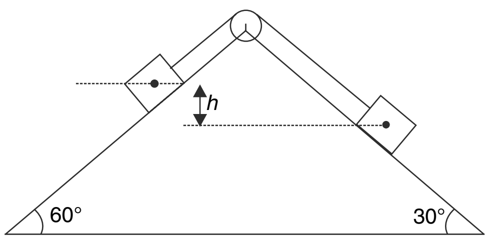
Q. 19. A pendulum has a bob connected to a light wire. Bob '*A*' is in equilibrium in the position shown. The string is horizontal and is connected to a block *B* resting on a rough surface. The block *B* is on verge of sliding when $\theta = 60°$.



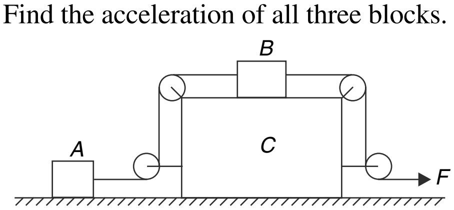

(a) Is equilibrium possible if $\theta$ were 70°?

(b) With $\theta$ = 60°, calculate the ratio of tension in the pendulum wire immediately after the string is cut to the tension in the wire before the string is cut.

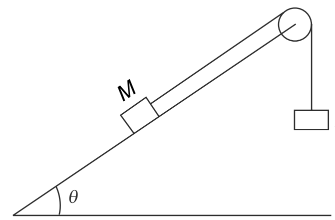
Q. 20. Two blocks of equal mass have been placed on two faces of a fixed wedge as shown in figure. The blocks are released from position where centre of one block is at a height *h* above the centre of the other block. Find the time after which the centre of the two blocks will be at same horizontal level. There is no friction anywhere.



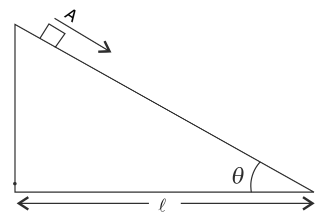

Q. 21. In the system shown in the figure, all surfaces are smooth. Block *A* and *B* have mass *m* each and mass of block *C* is 2 *m*. All pulleys are massless and fixed to block *C*. Strings are light and the force *F* applied at the free end of the string is horizontal. Find the acceleration of all three blocks.



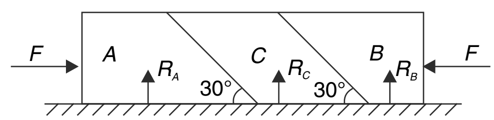

Q. 22. A particle of mass *M* rests on a rough inclined plane at an angle $\theta$ to the horizontal $\left(\sin\theta = \frac{4}{5}\right)$. It is connected to another mass *m* as shown in fig. The pulley and string are light. The largest value of *m* for which equilibrium is possible is *M*. Find the smallest value of *m* for which equilibrium is possible.



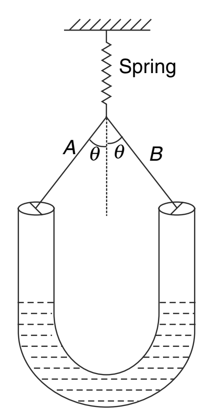

Q. 23. A small body *A* starts sliding down from the top of a wedge (see fig) whose base is equal to $\ell$. The coefficient of friction between the body and wedge surface is $\mu = 1.0$. At what value of angle $\theta$ will the time of sliding be least?




Q. 24. Three blocks *A*, *B* and *C* each of mass *m* are placed on a smooth horizontal table. There is no friction between the contact surfaces of the blocks as well. Horizontal force *F* is applied on each of *A* and *B* as shown. Find the ratio of normal force applied by the table on the three blocks (i.e., $R_A : R_B : R_C$). Take $F = \frac{mg}{2\sqrt{3}}$




Q. 25. A *U* shaped container has uniform cross sectional area *S*. It is suspended vertically with the help of a spring and two strings *A* and *B* as shown in the figure. The spring and strings are light. When water (density = *d* ) is poured slowly into the container it was observed that the level of water remained unchanged with respect to the ground. Find the force constant of the spring.

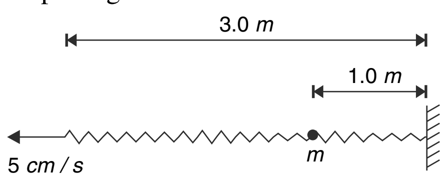


Q. 26. A uniform light spring has unstretched length of 3.0 *m*. One of its end is fixed to a wall. A particle of mass $m = 20\ g$ is glued to the spring at a point 1.0 m away from its fixed end. The free end of the spring is pulled away from the wall at a constant speed of 5 *cm*/*s*.

Assume that the spring remains horizontal (i.e., neglect gravity). Force constant of spring = 0.6 N/*cm*.

(a) With what speed does the particle of mass *m* move?

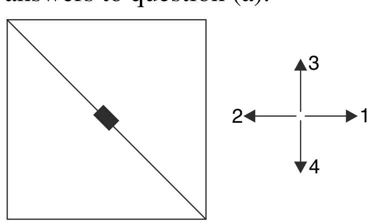
(b) Find the force applied by the external agent pulling the spring at time 2.0 s after he started pulling.




Q. 27. It was observed that a small block of mass *m* remains in equilibrium at the centre of a vertical square frame, which was accelerated. The block is held by two identical light strings as shown. [Both strings are along the diagonal]

(a) Which of 1, 2, 3 & 4 is/are possible direction/s of acceleration of the frame for block to remain in equilibrium inside it?

(b) Find the acceleration of the frame for your answers to question (a).



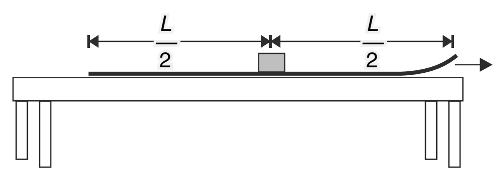

Q.28 In an emergency situation while driving one has tendency to jam the brakes, trying to stop in shortest distance. With wheels locked, the car slides and steering get useless. In ABS system the electronic sensors keep varying the brake pressure so as to keep the wheels rolling (without slipping) while ensuring that the friction remains limiting.

Your friend has an old car with good brakes. He boasts saying that all the four wheels of his car get firmly locked and stop rotation immediately after the brakes are applied. You know that your new car which has a computerized anti lock braking system (*ABS*) is much safer. How will you convince your friend?

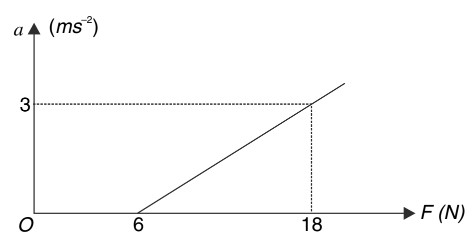
In a typical situation, car without *ABS* needs 20 *m* as minimum stopping distance. Under identical conditions, what minimum distance a car with *ABS* would need to stop? Coefficient of kinetic friction between tyre and road is 25% less than the coefficient of static friction.

Q. 29. Starting from rest a car takes at least '*t*' second to travel through a distance *s* on a flat concrete road. Find the minimum time that will be needed for it to climb through a distance '*s*' on an inclined concrete road. Assume that the car starts from rest and inclination of road is $\theta = 5°$ with horizontal. Coefficient of friction between tyres and the concrete road is $\mu = 1$.

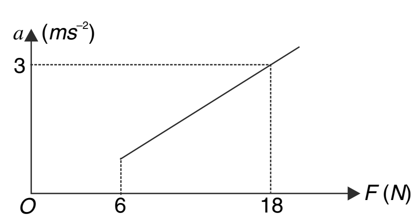
Q. 30. A table cloth of length *L* is lying on a table with one of its end at the edge of the table. A block is kept at the centre of the table cloth. A man pulls the end of the table cloth horizontally so as to take it off the table. The cloth is pulled at a constant speed $V_0$. What can you say about the coefficient of friction between the block and the cloth if the block remains on the table (i.e., it does not fall off the edge) as the cloth is pulled out.



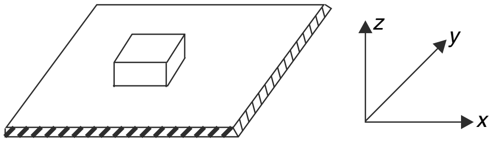

Q. 31. A block rests on a horizontal surface. A horizontal force *F* is applied to the block. The acceleration (*a*) produced in the block as a function of applied force (*F*) has been plotted in a graph (see figure). Find the mass of the block.




Q. 32. Repeat the last problem if the graph is as shown below.

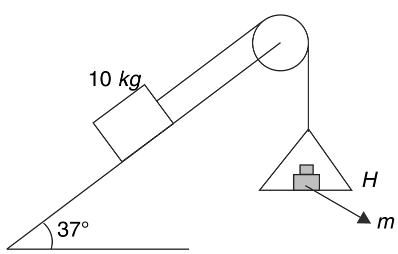


Q. 33. A solid block of mass $m$ = 1 $kg$ is resting on a horizontal platform as shown in figure. The $z$ direction is vertically up. Coefficient of friction between the block and the platform is $\mu = 0.2$. The platform is moved with a time dependent velocity given by $\vec{V} = \left(2t\hat{i} + t\hat{j} + 3t\hat{k}\right)\text{m/s}$. Calculate the magnitude of the force exerted by the block on the platform. Take g = 10 $m/s^2$

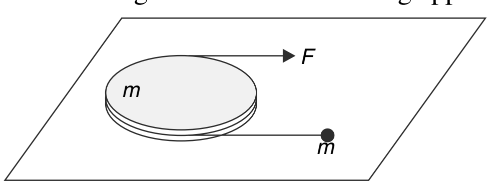



Q. 34. In the system shown in the figure, the string is light and coefficient of friction between the 10 $kg$ block and the incline surface is $\mu = 0.5$. Mass of the hanger, $H$ is 0.5 $kg$. A boy places a block of mass $m$ on the hanger and finds that the system does not move. What could be values of mass $m$?

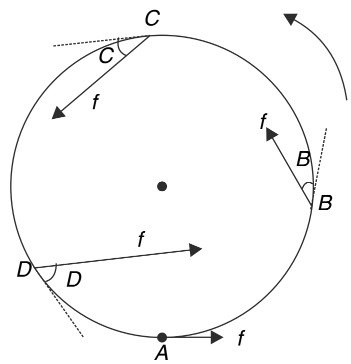
$\tan 37° = \frac{3}{4}$ *and* $g = 10\ \text{m/s}^2$



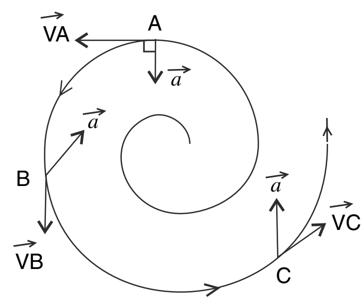

Q. 35. A disc of mass $m$ lies flat on a smooth horizontal table. A light string runs halfway around it as shown in figure. One end of the string is attached to a particle of mass $m$ and the other end is being pulled with a force $F$. There is no friction between the disc and the string. Find acceleration of the end of the string to which force is being applied.




Q. 36. (a) A car starts moving (at point $A$) on a horizontal circular track and moves in anticlockwise sense. The speed of the car is made to increase uniformly. The car slips just after point $D$. The figure shows the friction force ($f$) acting on the car at points $A$, $B$, $C$ and $D$. The length of the arrow indicates the magnitude of the friction and it is given that $\angle D > \angle B > \angle C$. At which point ($A$, $B$, $C$ or $D$) the friction forces represented is certainly wrong ?

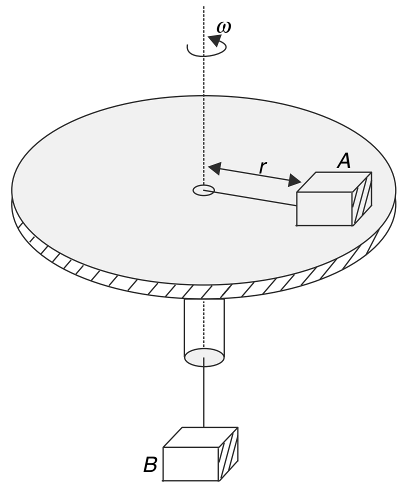



(b) A particle is moving along an expanding spiral (shown in fig) such that the normal force on the particle [i.e., component of force perpendicular to the path of the particle] remains constant in magnitude. The possible direction of acceleration $\left(\vec{a}\right)$ of the particle has been shown at three points A, B and C on its path. At which of these points the direction of acceleration has been represented correctly.




(c) A particle is moving in XY plane with a velocity. $\vec{v} = 4\hat{i} + 2t\hat{j}\ \text{ms}^{-1}$. Calculate its rate of change of speed and normal acceleration at $t$ = 2 $s$.

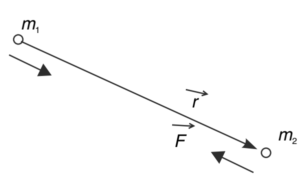
Q. 37. (i) A spinning disk has a hole at its centre. The surface of the disk is horizontal and a small block $A$ of mass $m$ = 1 $kg$ is placed on it.



Block $A$ is tied to a light inextensible string, other end of which passes through the hole and supports another block $B$ of mass $M = 2\ kg$. The coefficient of friction between $A$ and the disk surface is 0.5. It was observed that the disk is spinning with block $A$ remaining at rest relative to the disk. Block $B$ was found to be stationary. It was estimated that length of horizontal segment of the string ($r$) was anywhere between 1.0 $m$ to 1.5 $m$. With this data what estimate can be made about the angular speed ($\omega$) of the disk. [$g = 10\ m/s^2$]

(ii) A spring has force constant equal to $k = 100\ Nm^{-1}$. Ends of the spring are joined to give it a circular shape of radius $R = 20\ cm$. Now the spring is rotated about its symmetry axis (perpendicular to its plane) such that the circumference of the circle increases by 1%. Find the angular speed ($\omega$). Mass of one meter length of the spring is $\lambda = 0.126\ gm^{-1}$.

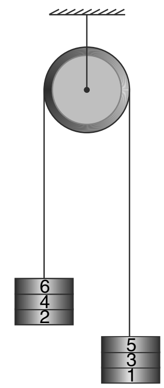
Q. 38. Two particles of mass $m_1$ and $m_2$ are in space at separation $\vec{r}$ [vector from $m_1$ to $m_2$]. The only force that the two particles experience is the mutual gravitational pull. The force applied by $m_1$ on $m_2$ is $\vec{F}$. Prove that $\mu \frac{d^2\vec{r}}{dt^2} = \vec{F}$ Where $\mu \frac{m_1 m_2}{m_1 + m_2}$ is known as reduced mass for the two particle system.




Q. 39. Six identical blocks – numbered 1 to 6 – have been glued in two groups of three each and have been suspended over a pulley as shown in fig. The pulley and string are massless and the system is in equilibrium. The block 1, 2, 3, and 4 get detached from the system in sequence starting with block 1. The time gap between separation of two consecutive block (i.e., time gap between separation of 1 and 2 or gap between separation of 2 and 3) is $t_0$. Finally, blocks 5 and 6 remain connected to the string.

(a) Find the final speed of blocks 5 and 6.

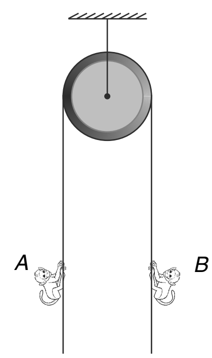
(b) Plot the graph of variation of speed of block 5 with respect to time. Take $t = 0$ when block 1 gets detached.




Q. 40. Two monkeys $A$ and $B$ are holding on the two sides of a light string passing over a smooth pulley. Mass of the two monkeys are $m_A = 8\ kg$ and $m_B = 10\ kg$ respectively [$g = 10\ m/s^2$]

(a) Monkey $A$ holds the string tightly and $B$ goes down with an acceleration $a_r = 2\ m/s^2$ relative to the string. Find the weight that $A$ feels of his own body.

(b) What is the weight experienced by two monkeys if $A$ holds the string tightly and $B$ goes down with an acceleration $a_r = 4\ m/s^2$ relative to the string.




## LEVEL 2

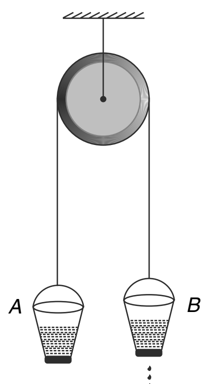
Q. 41. Two strange particles $A$ and $B$ in space, exert no force on each other when they are at a separation greater than $x_0 = 1.0\ m$. When they are at a distance less than $x_0$, they repel one another along the line joining them. The repulsion force is constant and does not depend on the distance between the particles. This repulsive force produces an acceleration of 6 $ms^{-2}$ in $A$ and 2 $ms^{-2}$ in $B$ when the particles are at separation less than $x_0$. In one experiment particle $B$ is projected towards $A$ with a velocity of 2 $ms^{-1}$ from a large distance so as to hit $A$ head on. The particle $A$ is originally at rest and the system of two particles do not experience any external force.

(a) Find the ratio of mass of $A$ to that of $B$.

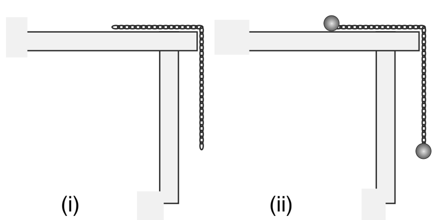
(b) Find the minimum distance between the particles during subsequent motion.

(c) Find the final velocity of the two particles.

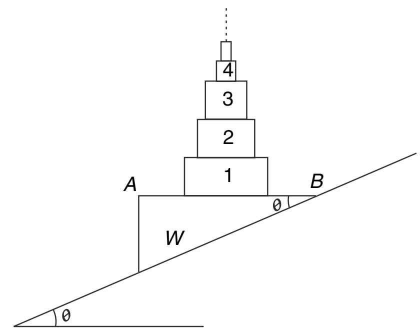
Q. 42. A light string passing over a smooth pulley holds two identical buckets at its ends. Mass of each empty bucket is $M$ and each of them holds $M$ mass of sand. The system was in equilibrium when a small leak developed in bucket $B$ (take this time as $t = 0$). The sand leaves the bucket at a constant rate of $\mu$ $kg$/s. Assume that the leaving sand particles have no relative speed with respect to the bucket (it means that there is no impulsive force on the bucket like leaving exhaust gases exert on a rocket). Find the speed ($V_0$) of the two bucket when $B$ is just empty.



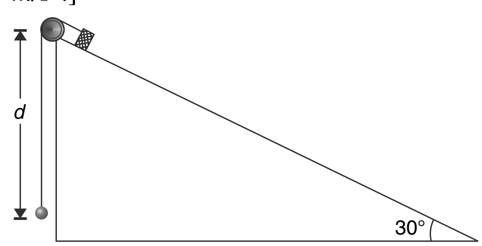

Q. 43. A chain is lying on a smooth table with half its length hanging over the edge of the table [fig(i)]. If the chain is released it slips off the table in time $t_1$. Now, two identical small balls are attached to the two ends of the chain and the system is released [fig(ii)]. This time the chain took $t_2$ time to slip off the table. Which time is larger, $t_1$ or $t_2$?




Q. 44. A triangular wedge $W$ having mass $M$ is placed on an incline plane with its face $AB$ horizontal. Inclination of the incline is $\theta$. On the flat horizontal surface of the wedge there lies an infinite tower of rectangular blocks. Blocks 1, 2, 3, 4 .......... have masses $M, \frac{M}{2}, \frac{M}{4}, \frac{M}{8}$ ..................respectively. All surfaces are smooth. Find the contact force between the block 1 and 2 after the system is released from rest. Also find the acceleration of the wedge.




Q. 45. In the system shown in fig, mass of the block is $m_1 = 4$ $kg$ and that of the hanging particle is $m_2 = 1$ $kg$. The incline is fixed and surface is smooth. Block is initially held at the top of the incline and the particle hangs a distance $d = 2.0$ m below it. [Assume that the block and the particle are on same vertical line in this position]. System is released from this position. After what time will the distance between the block and the particle be minimum ? Find this minimum distance. [$g = 10$ $m/s^2$.]



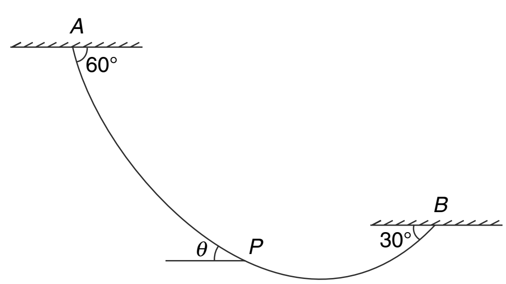

Q. 46. A uniform chain of mass $M = 4.8$ $kg$ hangs in vertical plane as shown in the fig.

(a) Show that horizontal component of tension is same throughout the chain.

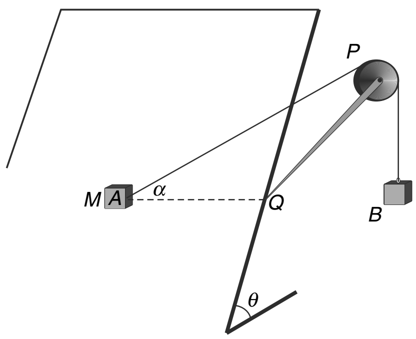
(b) Find tension in the chain at point $P$ where the chain makes an angle $\theta = 15°$ with horizontal.

(c) Find mass of segment $AP$ of the chain.

[Take $g = 10$ $m/s^2$; cos 15° = 0.96, sin 15° = 0.25]

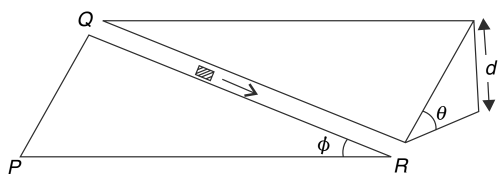


Q. 47. Block $A$ of mass $M$ is placed on an incline plane, connected to a string, passing over a pulley as shown in the fig. The other end of the string also carries a block $B$ of mass $M$. The system is held in the position shown such that triangle $APQ$ lies in a vertical plane with horizontal line $AQ$ in the plane of the incline surface.

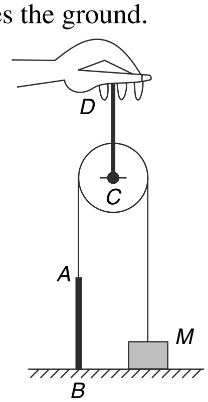



Find the minimum coefficient of friction between the incline surface and block $A$ such that the system remains at rest after it is released. Take $\theta = \alpha = 45°$.

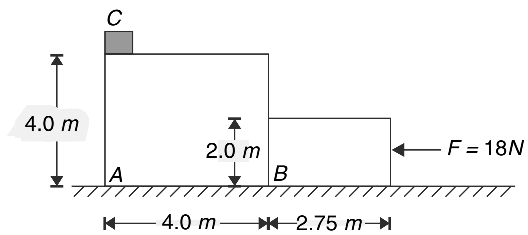
Q. 48. Figure shown a fixed surface inclined at an angle $\theta$ to the horizontal. A smooth groove is cut on the incline along $QR$ forming an angle $\phi$ with $PR$. A small block is released at point $Q$ and it slides down to $R$ in time $t$. Find $t$.




Q. 49. In the system shown in the figure $AB$ and $CD$ are identical elastic cords having force constant $K$. The string connected to the block of mass $M$ is inextensible and massless. The pulley is also massless. Initially, the cords are just taut. The end $D$ of the cord $CD$ is gradually moved up. Find the vertical displacement of the end $D$ by the time the block leaves the ground.



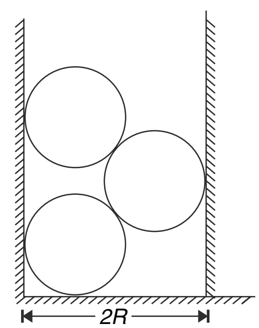

Q. 50. Blocks $A$ and $B$ have dimensions as shown in the fig. and their masses are 8 $kg$ and 1 $kg$ respectively. $A$ small block $C$ of mass 0.5 $kg$ is placed on the top left corner of block $A$. All surfaces are smooth. A horizontal force $F$ = 18 $N$ is applied to the block $B$ at time $t$ = 0. At what time will the block $C$ hit the ground surface? Take $g = 10\ m/s^2$.



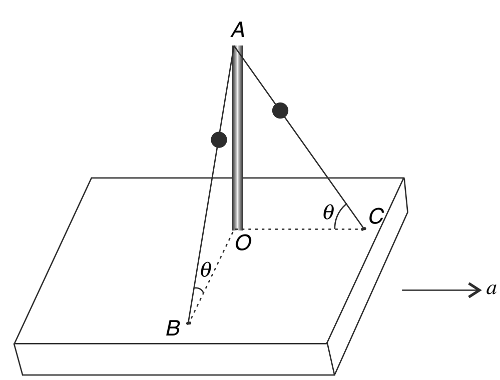

Q. 51. Three identical smooth balls are placed between two vertical walls as shown in fig. Mass of each ball is $m$ and radius is $r = \frac{5R}{9}$ where $2R$ is separation between the walls.

(a) Force between which two contact surface is maximum? Find its value.

(b) Force between which two contact surface is minimum and what is its value?




Q. 52. A horizontal wooden block has a fixed rod $OA$ standing on it. From top point $A$ of the rod, two wires have been fixed to points $B$ and C on the block. The plane of triangle $OAB$ is perpendicular to the plane of the triangle $OAC$. There are two identical beads on the two wires. One of the wires

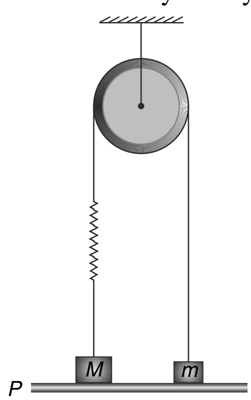


is perfectly smooth while the other is rough. The wooden block is moved with a horizontal acceleration ($a$) that is perpendicular to the line $OB$ and it is observed that both the beads do not slide on the wire. Find the minimum coefficient of friction between the rough wire and the bead.

Q. 53. In the arrangement shown in the fig. the pulley, the spring and the thread are ideal. The spring is stretched and the two blocks are in contact with a horizontal platform $P$. When the platform is gradually moved up by 2 *cm* the tension in the string becomes zero. If the platform is gradually moved down by 2 *cm* from its original position one of the blocks lose contact with the platform. Given $M = 4\ kg$; $m = 2\ kg$.

(a) Find the force constant ($k$) of the spring

(b) If the platform continues to move down after one of the blocks loses contact, will the other block also lose contact? Assume that that the platform moves very slowly.

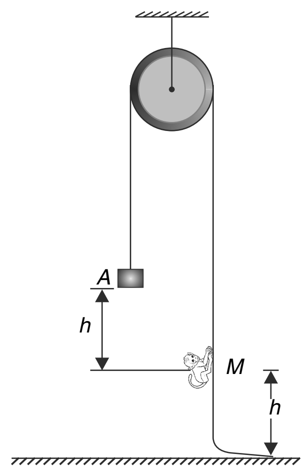



Q. 54. In the arrangement shown in the fig. a monkey of mass $M$ keeps itself as well as block $A$ at rest by firmly holding the rope. Rope is massless and the pulley is ideal. Height of the monkey and block $A$ from the floor is $h$ and $2h$ respectively [$h = 2.5\ m$]

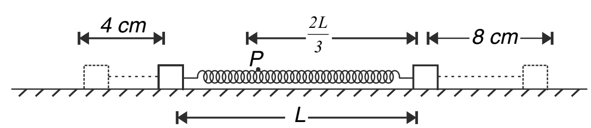
(a) The monkey loosens its grip on the rope and slides down to the floor. At what height from the ground is block A at the instant the money hits the ground?

(b) Another block of mass equal to that of $A$ is stuck to the block $A$ and the system is released. The monkey decides to keep itself at height $h$ above the ground and it allows the rope to slide through its hand. With what speed will the block strike the ground?

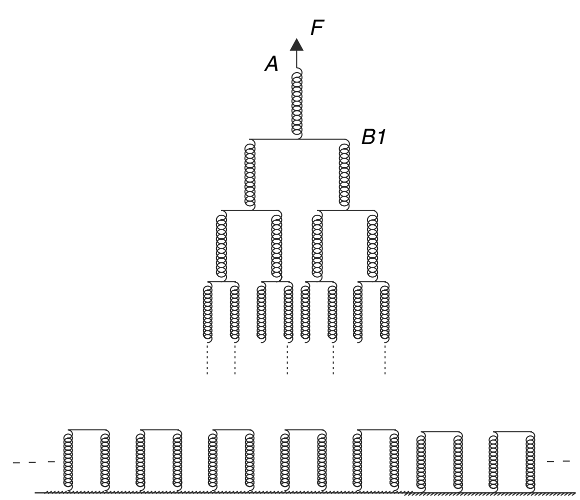
(c) In the situation described in (b), the monkey decides to prevent the block from striking the floor. The monkey remains at height $h$ till the block crosses it. At the instant the block is crossing the monkey it begins climbing up the rope. Find the minimum acceleration of the monkey relative to the rope, so that the block is not able to hit the floor. Do you think that a monkey can climb with such an acceleration? ($g = 10\ ms^{-2}$)




Q. 55. An ideal spring is in its natural length ($L$) with two objects $A$ and $B$ connected to its ends. A point $P$ on the unstretched spring is at a distance $\frac{2L}{3}$ from $B$. Now the objects $A$ and $B$ are moved by 4 *cm* to the left and 8 *cm* to the right respectively. Find the displacement of point $P$.




Q. 56. The fig. shows an infinite tower of identical springs each having force constant $k$. The connecting

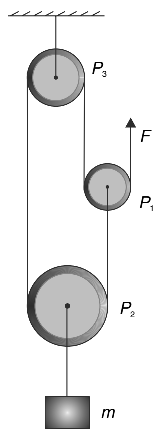


bars and all springs are massless. All springs are relaxed and the bottom row of springs is fixed to horizontal ground. The free end of the top spring is pulled up with a constant force $F$. In equilibrium, find

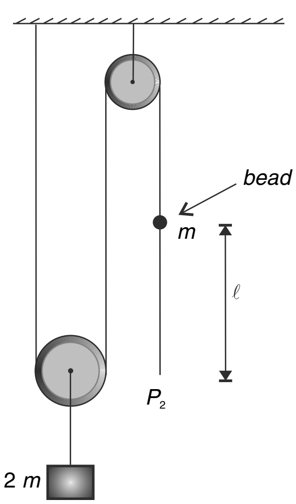
(a) The displacement of free end $A$ of the top spring from relaxed position.

(b) The displacement of the top bar $B1$ from the initial relaxed position.

Q. 57. In the system shown in the fig. there is no friction and string is light. Mass of movable pulley $P_2$ is $M_2$. If pulley $P_1$ is massless, what should be value of applied force $F$ to keep the system in equilibrium?



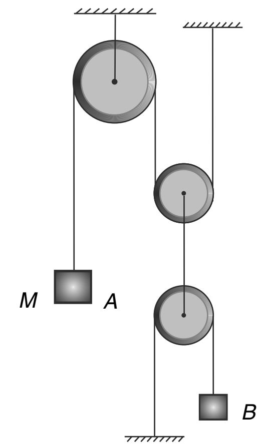

Q. 58. In the system shown in the fig., the bead of mass $m$ can slide on the string. There is friction between the bead and the string. Block has mass equal to twice that of the bead. The system is released from rest with length $l$ of the string hanging below the bead. Calculate the distance moved by the block before the bead slips out of the thread. Assume the string and pulley to be massless.




Q. 59. In the arrangement shown in the fig. all pulleys are mass less and the strings are inextensible and light. Block $A$ has mass $M$.

(a) If the system stays at rest after it is released, find the mass of the block $B$.

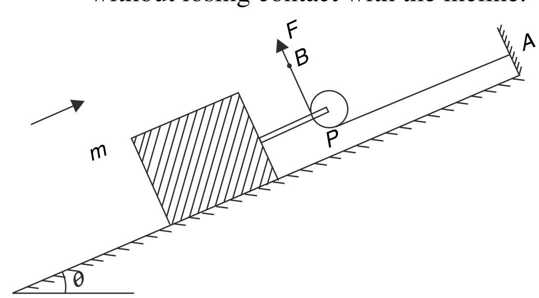
(b) If mass of the block $B$ is twice the value found in part (a) of the problem, calculate the acceleration of block $A$.




Q. 60. In the fig. shown, the pulley and string are mass less and the incline is frictionless. The segment $AP$ of the string is parallel to the incline and the segment $PB$ is perpendicular to the incline. End of the string is pulled with a constant force $F$.

(a) If the block is moving up the incline with acceleration while being in contact with the incline, then angle $\theta$ must be less then $\theta_0$. Find $\theta_0$

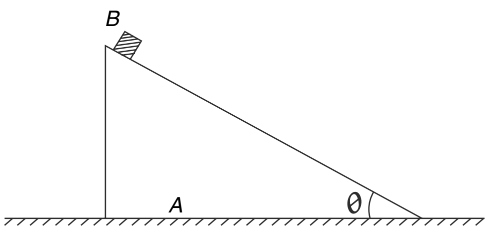
(b) If $\theta = \frac{\theta_0}{2}$ find the maximum acceleration with which the block can move up the plane without losing contact with the incline.



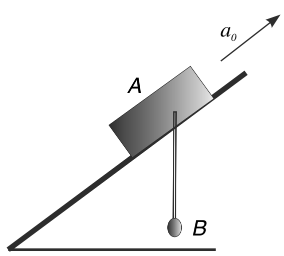

Q.61. A triangular wedge $A$ is held fixed and a block $B$ is released on its inclined surface, from the top. Block $B$ reaches the horizontal ground in time $t$. In another experiment, the wedge $A$ was free to slide on the horizontal surface and it took $t'$ time for the block $B$ to reach the ground surface after it was released from the top. Neglect friction and assume that $B$ remains in contact with $A$.

(a) Which time is larger $t$ or $t'$? Tell by simple observation.

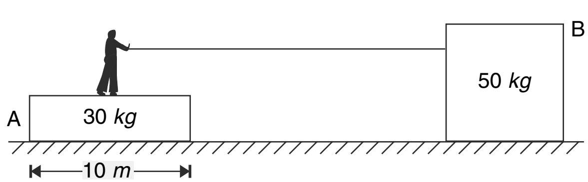
(b) When wedge $A$ was free to move, it was observed that it moved leftward with an acceleration $\frac{g}{4}$ and one of the two measured times ($t$ & $t'$) was twice the other. Find the inclination $\theta$ of the inclined surface of the wedge.




Q. 62. A block $A$ is made to move up an inclined plane of inclination $\theta$ with constant acceleration $a_0$ as shown in figure. Bob $B$, hanging from block $A$ by a light inextensible string, is held vertical and is moving along with the block. Calculate the magnitude of acceleration of block $A$ relative to the bob immediately after bob is released.




Q. 63. A 50 *kg* man is standing at the centre of a 30 *kg* platform $A$. Length of the platform is 10 m and coefficient of friction between the platform and the horizontal ground is 0.2. Man is holding one end of a light rope which is connected to a 50 *kg* box $B$. The coefficient of friction between the box and the ground is 0.5. The man pulls the rope so as to slowly move the box and ensuring that he himself does not move relative to the ground. If the shoes of the man does not slip on the platform, calculate how much time it will take for the man to fall off the platform. Assume that rope remains horizontal, and coefficient of friction between shoes and the platform is 0.6.

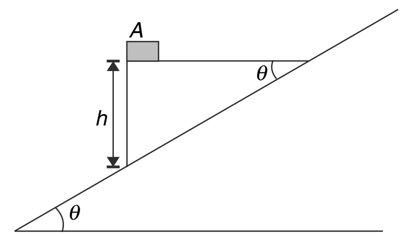



Q. 64. A wedge is placed on the smooth surface of a fixed incline having inclination $\theta$ with the horizontal. The vertical wall of the wedge has height $h$ and there is a small block $A$ on the edge of the horizontal surface of the wedge. Mass of the wedge and the small block are $M$ and $m$ respectively.

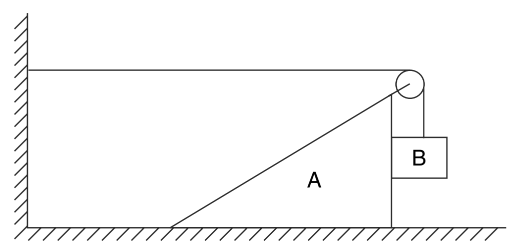
(a) Find the acceleration of the wedge if friction between block $A$ and the wedge is large enough to prevent slipping between the two.

(b) Find friction force between the block and the wedge in the above case. Also find the normal force between the two.

(c) Assuming there is no friction between the block and the wedge, calculate the time in which the block will hit the incline.




Q. 65. In the system shown in figure, all surfaces are smooth, pulley and strings are massless. Mass of both $A$ and $B$ are equal. The system is released from rest.



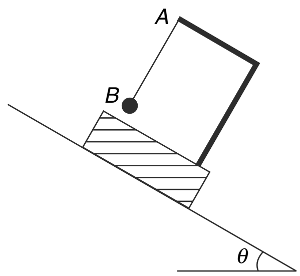

(a) Find the $\vec{a}_A \cdot \vec{a}_B$ immediately after the system is released. $\vec{a}_A$ and $\vec{a}_B$ are accelerations of block $A$ and $B$ respectively.

(b) Find $\vec{a}_A$ immediately after the system is released.

Q.66. A block is placed on an incline having inclination $\theta$. There is a rigid $L$ shaped frame fixed to the block. A plumb line (a ball connected to a thread) is attached to the end $A$ of the frame. The system is released on the inline. Find the angle that the plumb line will make with vertical in its equilibrium position relative to the block when

(a) the incline is smooth

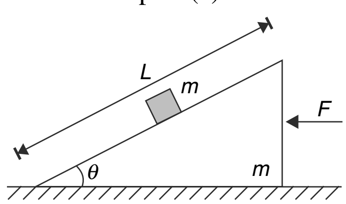
(b) there is friction and the acceleration of the block is half its value when the incline is smooth



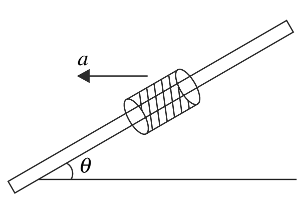

Q. 67. A wedge of mass $m$ is placed on a horizontal smooth table. A block of mass $m$ is placed at the mid point of the smooth inclined surface having length $L$ along its line of greatest slope. Inclination of the inclined surface is $\theta = 45°$. The block is released and simultaneously a constant horizontal force $F$ is applied on the wedge as shown.

(a) What is value of $F$ if the block does not slide on the wedge?

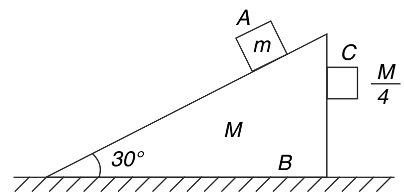
(b) In how much time the block will come out of the incline surface if applied force is 1.5 times that found in part (a)




Q. 68. A rod is kept inclined at an angle $\theta$ with the horizontal $A$ sleeve of mass $m$ can slide on the rod. If the coefficient of friction between the rod and the sleeve is $\mu$, for what values of horizontal acceleration $a$ of the rod, towards left, the sleeve will not slide over the rod?




Q. 69. In the arrangement shown in figure, a block $A$ of mass $m$ has been placed on a smooth wedge $B$ of mass $M$. The wedge lies on a horizontal smooth surface. Another block $C$ of mass $\frac{M}{4}$ has been placed in contact with the wedge $B$ as shown. The coefficient of friction between the block $C$ and the vertical wedge wall is $\mu = \frac{3}{4}$. Find the ratio $\frac{m}{M}$ for which the block $C$ will not slide with respect to the wedge after the system is released?

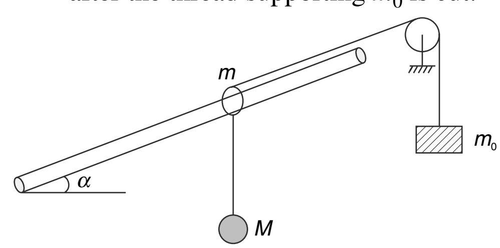



Q. 70. A smooth rod is fixed at an angle $\alpha$ to the horizontal. A small ring of mass $m$ can slide along the rod. A thread carrying a small sphere of mass $M$ is attached to the ring. To keep the system in equilibrium, another thread is attached to the ring which carries a load of mass $m_0$ at its end (see figure). The thread runs parallel to the rod between the ring and the pulley.

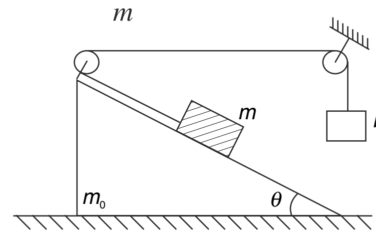
All threads and pulley are massless.

(a) Find $m_0$ so that system is in equilibrium.

(b) Find acceleration of the sphere $M$ immediately after the thread supporting $m_0$ is cut.

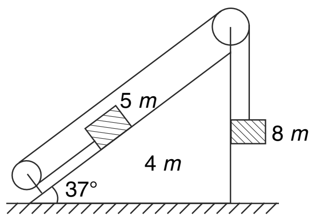



Q. 71. In the system shown in figure all surfaces are smooth and string and pulleys are light. Angle of wedge $\theta = \sin^{-1}\left(\frac{3}{5}\right)$. When released from rest it was found that the wedge of mass $m_0$ does not move. Find $\frac{M}{m}$.

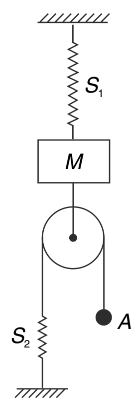



Q. 72. In the last problem take $M = m$ and $m_0 = 2\,m$ and calculate the acceleration of the wedge.

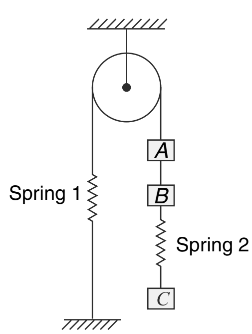
Q. 73. In the system shown in the figure all surfaces are smooth, pulley and string are massless. The string between the two pulleys and between pulley and block of mass 5 m is parallel to the incline surface of the block of mass 4 m. The system is released from rest. Find the acceleration of the block of mass 4 m. $\left[\tan 37° = \frac{3}{4}\right]$




Q. 74. In the system shown in figure, the two springs $S_1$ and $S_2$ have force constant $k$ each. Pulley, springs and strings are all massless. Initially, the system is in equilibrium with spring $S_1$ stretched and $S_2$ relaxed. The end $A$ of the string is pulled down slowly through a distance $L$. By what distance does the block of mass $M$ move?




Q. 75. The system shown in figure is in equilibrium. Pulley, springs and the strings are massless. The three blocks $A$, $B$ and $C$ have equal masses. $x_1$ and $x_2$ are extensions in the spring 1 and spring 2 respectively.

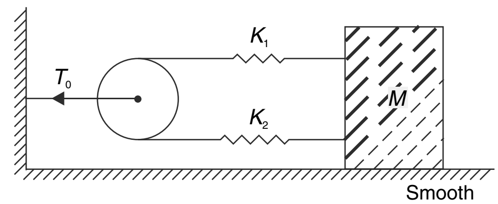



(a) Find the value of $\left|\frac{d^2x_2}{dt^2}\right|$ immediately after spring 1 is cut.

(b) Find the value of $\left|\frac{d^2x_1}{dt^2}\right|$ and $\left|\frac{d^2x_2}{dt^2}\right|$ immediately after string $AB$ is cut.

(c) Find the value of $\left|\frac{d^2x_1}{dt^2}\right|$ and $\left|\frac{d^2x_2}{dt^2}\right|$ immediately after spring 2 is cut.

Q. 76. In the figure shown, the pulley, strings and springs are mass less. The block is moved to right by a distance $x_0$ from the position where the two springs are relaxed. The block is released from this position.

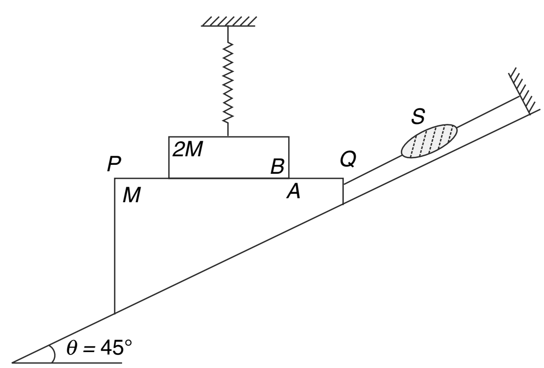



(a) Find the acceleration of the block immediately after it is released.

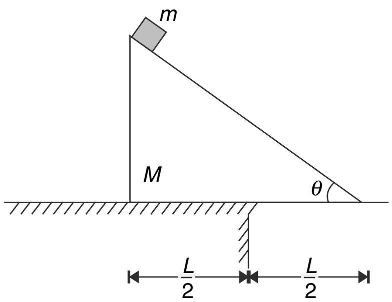
(b) Find tension ($T_0$) in the support holding the pulley to the wall, immediately after the block is released.

Assume no friction.

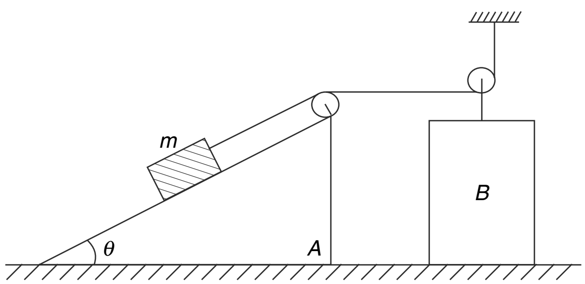
Q.77. The system shown in figure is in equilibrium. Surface $PQ$ of wedge $A$, having mass $M$, is horizontal. Block $B$, having mass $2M$, rests on wedge $A$ and is supported by a vertical spring. The spring balance $S$ is showing a reading of $\sqrt{2}\,Mg$. There is no friction anywhere and the thread $QS$ is parallel to the incline surface. The thread $QS$ is cut. Find the acceleration of $A$ and the normal contact force between $A$ and $B$ immediately after the thread is cut.



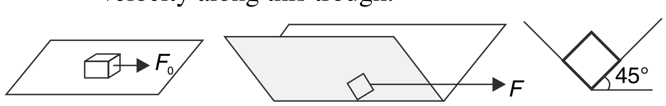

Q.78. A triangular wedge of mass $M$ lies on a smooth horizontal table with half of its base projecting out of the edge of the table. A block of mass $m$ is kept at the top of the smooth incline surface of the wedge and the system is let go. Find the maximum value of $\frac{M}{m}$ for which the block will land on the table. Take $\theta = 60°$.




Q.79. In the system shown in the figure all surfaces are smooth and both the pulleys are mass less. Block on the incline surface of wedge $A$ has mass $m$. Mass of $A$ and $B$ are $M = 4\,m$ and $M_0 = 2\,m$ respectively. Find the acceleration of wedge $A$ when the system is released from rest.

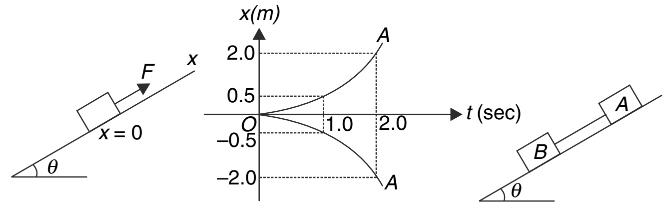



Q.80. A block of mass $m$ requires a horizontal force $F_0$ to move it on a horizontal metal plate with constant velocity. The metal plate is folded to make it a right angled horizontal trough. Find the horizontal force $F$ that is needed to move the block with constant velocity along this trough.

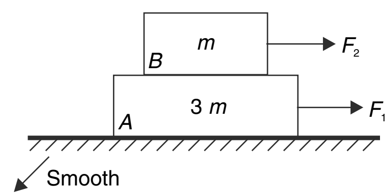



Q.81. Block $A$ of mass $m_A = 200\,g$ is placed on an incline plane and $a$ constant force $F = 2.2\,N$ is applied on it parallel to the incline. Taking the initial position of the block as origin and up along the incline as $x$ direction, the position ($x$) time ($t$) graph of the block is recorded (see figure (b)). The same experiment is repeated with another block $B$ of mass $m_B = 500\,g$. Same force $F$ is applied to it up along the incline and its position – time graph is recorded (see figure (b)). Now the two blocks are connected by a light string and released on the same incline as shown in figure (c). Find the tension in the string.

$$\left[\tan\theta = \frac{3}{4}; g = 10\,m/s^2\right]$$




Q.82. Block $B$ of mass $m$ has been placed on block $A$ of mass $3\,m$ as shown. Block $A$ rests on a smooth horizontal table. $F_1$ is the maximum horizontal force that can be applied on the block $A$ such that there is no slipping between the blocks. Similarly, $F_2$ is the maximum horizontal force that can be applied on the block $B$ so that the two blocks move together without slipping on each other. When $F_1$ and $F_2$ both are applied together as shown in figure.

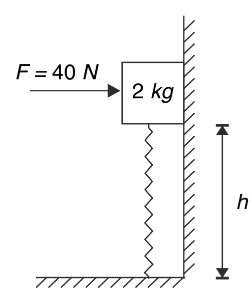



(a) Find the friction force acting between the blocks.

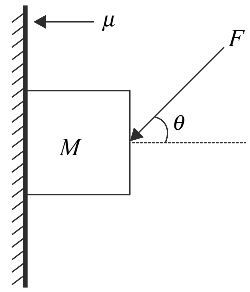
(b) Acceleration of the two blocks.

(c) If $F_2$ is decreased a little, what will be direction of friction acting on $B$.

Q. 83. (i) In the arrangement shown in the figure the coefficient of friction between the 2 $kg$ block and the vertical wall is $\mu = 0.5$. A constant horizontal force of 40 N keeps the block pressed against the wall. The spring has a natural length of 1.0 m and its force constant is $k = 400\,Nm^{-1}$. What should be the height $h$ of the block above the horizontal floor for it to be in equilibrium. The spring is not tied to the block.




(ii) A block of mass $M$ is pressed against a rough vertical wall by applying a force $F$ making an angle of $\theta$ with horizontal (as shown in figure). Coefficient of friction between the wall and the block is $\mu = 0.75$.

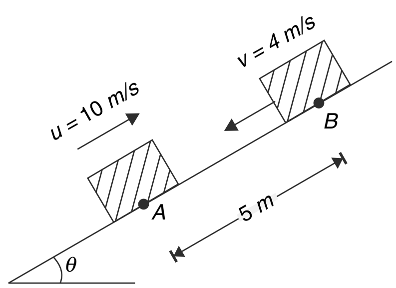



(a) If $F = 2\,Mg$, find the range of values of $\theta$ so that the block does not slide

[Take tan 37° = 0.75; sin 24° = 0.4]

(b) Find the maximum value of $\theta$ above which equilibrium is not possible for any magnitude of force $F$.

Q. 84. A block is projected up along a rough incline with a velocity of $\upsilon = 10\ m/s$. After 4 $s$ the block was at point $B$ at a distance of 5 $m$ from the starting point $A$ and was travelling down at a velocity of $v = 4\ m/s$.

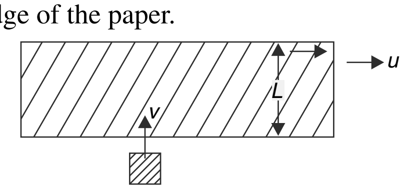



(a) Find time after projection at which the block came to rest.

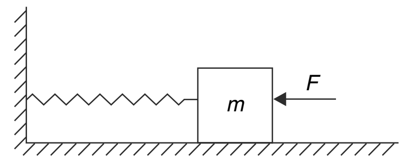
(b) Find the coefficient of friction between the block and the incline.

Take $g = 10\ m/s^2$

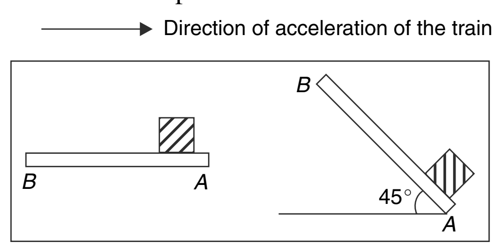
Q. 85. A long piece of paper is being pulled on a horizontal surface with a constant velocity $\upsilon$ along its length. Width of the paper is $L$. A small block moving horizontally, perpendicular to the direction of motion of the paper, with velocity $v$ slides onto the paper. The coefficient of friction between the block and the paper is $\mu$. Find maximum value of $v$ such that the block does not cross the opposite edge of the paper.



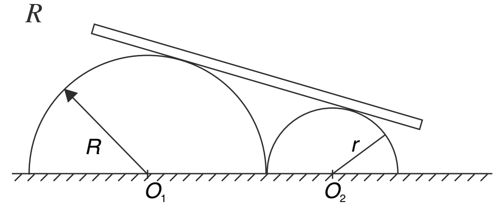

Q. 86. A block of mass $m = 1\ kg$ is kept pressed against a spring on a rough horizontal surface. The spring is compressed by 10 $cm$ from its natural length and to keep the block at rest in this position a horizontal force ($F$) towards left is applied. It was found that the block can be kept at rest if $8\ N \le F \le 18\ N$. Find the spring constant ($k$) and the coefficient of friction ($\mu$) between the block and the horizontal surface.



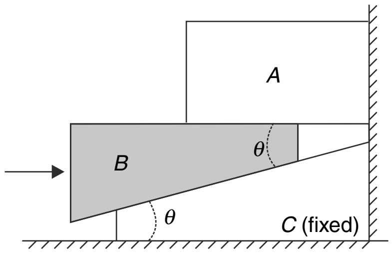

Q. 87. An experimenter is inside a uniformly accelerated train. Train is moving horizontally with constant acceleration $a_0$. He places a wooden plank $AB$ in horizontal position with end $A$ pointing towards the engine of the train. A block is released at end $A$ of the plank and it reaches end $B$ in time $t_1$. The same plank is placed at an inclination of 45° to the horizontal. When the block is released at $A$ it now climbs to $B$ in time $t_2$. It was found that $\frac{t_2}{t_1} = 2^{5/4}$. What is the coefficient of friction between the block and the plank?



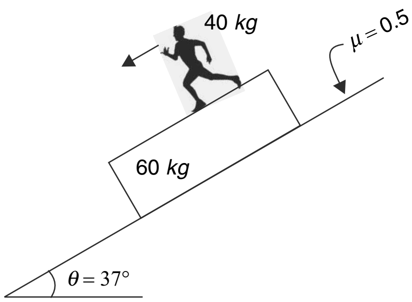

Q. 88. Two hemispheres of radii $R$ and $r$ ($< R$) are fixed on a horizontal table touching each other (see figure). A uniform rod rests on two spheres as shown. The coefficient of friction between the rod and two spheres is $\mu$. Find the minimum value of the ratio $\frac{r}{R}$ for which the rod will not slide.



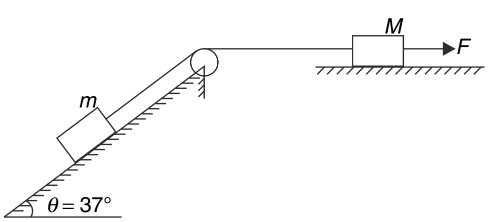

Q. 89. In order to lift a heavy block $A$, an engineer has designed a wedge system as shown. Wedge $C$ is fixed. A horizontal force $F$ is applied to $B$ to lift block $A$. Wedge $B$ itself has negligible mass and mass of $A$ is $M$. The coefficient of friction at all surfaces is $\mu$. Find the value of applied force $F$ at which the block $A$ just begins to rise.



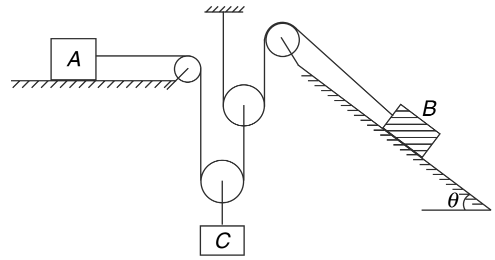

Q.90. A 60 *kg* platform has been placed on a rough incline having inclination $\theta = 37°$. The coefficient of friction between the platform and the incline is $\mu = 0.5$. A 40 *kg* man is running down on the platform so as to keep the platform stationary. What is the acceleration of the man? It is known that the man cannot manage to go beyond an acceleration of 7 $m/s^2$. $\left[\sin 37° = \frac{3}{5}\right]$




Q. 91. In the system show in figure, mass of the block placed on horizontal surface is $M$ = 4 *kg*. A constant horizontal force of $F$ = 40 $N$ is applied on it as shown. The coefficient of friction between the blocks and surfaces is $\mu = 0.5$. Calculate the values of mass $m$ of the block on the incline for which the system does not move. $\left[\sin 37° = \frac{3}{5}; g = 10 m/s^2\right]$



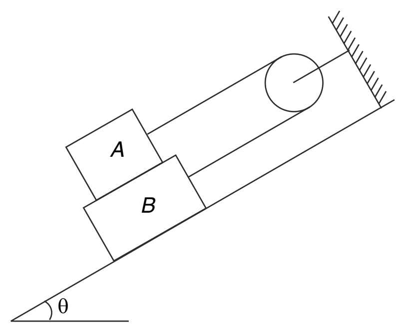

Q. 92. In the arrangement shown in the figure, block $A$ of mass 8 *kg* rests on a horizontal table having coefficient of friction $\mu = 0.5$. Block $B$ has a mass of 6 *kg* and rests on a smooth incline having inclination angle $\theta = \sin^{-1}\left(\frac{2}{5}\right)$. All pulleys and strings are mass less. Mass of block $C$ is $M$. [$g$ = 10 $m/s^2$]



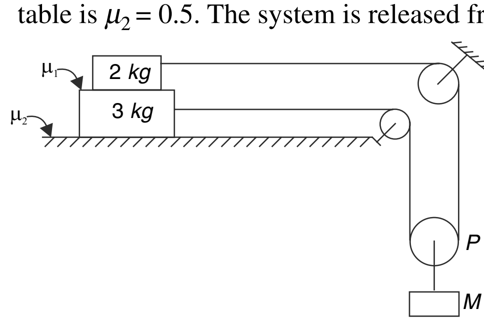

(a) Find value of $M$ for which block $B$ does not accelerate

(b) Find maximum value of $M$ for which $A$ does not accelerate.

Q.93. In the arrangement shown in figure, pulley and string are light. Friction coefficient between the two blocks is $\mu$ whereas the incline is smooth. Block $A$ has mass $m$ and difference in mass of the two blocks is $\Delta m$. Find minimum value of $\mu$ for which the system will not accelerate when released from rest.




Q. 94. In the arrangement shown in figure pulley $P$ can move whereas other two pulleys are fixed. All of them are light. String is light and inextensible. The coefficient of friction between 2 *kg* and 3 *kg* block is $\mu_1 = 0.75$ and that between 3 *kg* block and the table is $\mu_2 = 0.5$. The system is released from rest



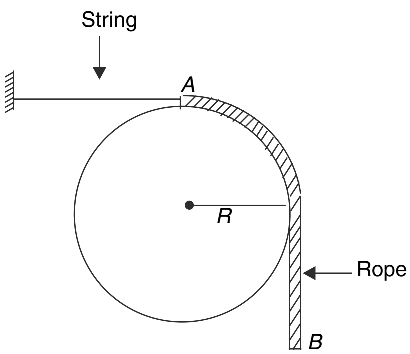

(i) Find maximum value of mass $M$, so that the system does not move. Find friction force between 2 *kg* and 3 *kg* blocks in this case.

(ii) If $M$ = 4 *kg*, find the tension in the string attached to 2 *kg* block.

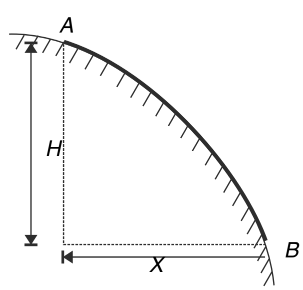
(iii) If $M = 4\ kg$ and $\mu_1 = 0.9$, find friction force between the two blocks, and acceleration of $M$.

(iv) Find acceleration of $M$ if $\mu_1 = 0.75$, $\mu_2 = -0.9$ and $M = 4\ kg$.

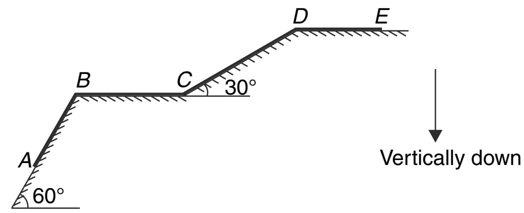
Q. 95. A rope of length $\left(\frac{\pi}{2}+1\right) R$ has been placed on a smooth sphere of radius $R$ as shown in figure. End $A$ of the rope is at the top of the sphere and end $B$ is overhanging. Mass per unit length of the rope is $\lambda$. The horizontal string holding this rope in place can tolerate tension equal to weight of the rope. Find the maximum mass ($M_0$) of a block that can be tied to the end $B$ of the rope so that the string does not break.




Q.96. A uniform rope has been placed on a sloping surface as shown in the figure. The vertical separation and horizontal separation between the end points of the rope are $H$ and $X$ respectively. The friction coefficient ($\mu$) is just good enough to prevent the rope from sliding down. Find the value of $\mu$.




Q.97. A uniform rope $ABCDE$ has mass $M$ and it is laid along two incline surfaces ($AB$ and $CD$) and two horizontal surfaces ($BC$ and $DE$) as shown in figure. The four parts of the rope $AB$, $BC$, $CD$ and $DE$ are of equal lengths. The coefficient of friction ($\mu$) is uniform along the entire surface and is just good enough to prevent the rope from sliding.

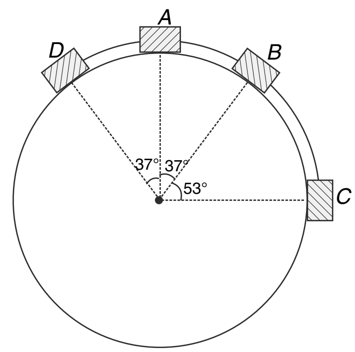



(a) Find $\mu$

(b) $x$ is distance measured along the length of the rope starting from point $A$. Plot the variation of tension in the rope ($T$) with distance $x$.

(c) Find the maximum tension in the rope.

Q. 98. (i) Four small blocks are interconnected with light strings and placed over a fixed sphere as shown. Blocks $A$, $B$ and $C$ are identical each having mass $m = 1\ kg$. Block $D$ has a mass of $m' = 2\ kg$. The coefficient of friction between the blocks and the sphere is $\mu = 0.5$. The system is released from the position shown in figure.

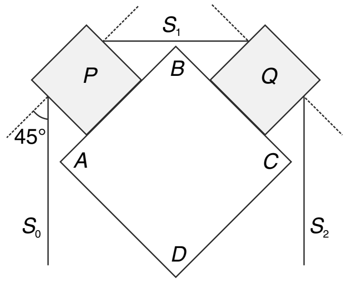



(a) Find the tension in each string. Which string has largest tension?

(b) Find the friction force acting on each block.

$$\left[\text{Take}\tan 37^\circ = \frac{3}{4};\ g = 10\ m/s^2\right]$$

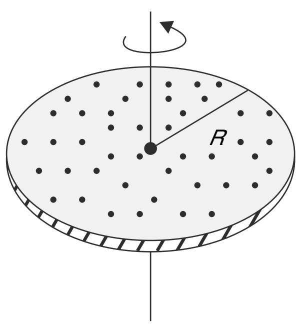
(ii) A fixed square prism $ABCD$ has its axis horizontal and perpendicular to the plane of the figure. The face $AB$ makes 45° with the vertical. On the upper faces $AB$ and $BC$ of the prism there are light bodies $P$ and $Q$ respectively. The two bodies ($P$ and $Q$) are connected using a string $S_1$ and strings $S_0$ and $S_2$ are hanging from $P$ and $Q$ respectively. All strings are mass less, and inextensible. String $S_1$ is horizontal and the other two strings are vertical. The coefficient of friction between the bodies and the prism is $\mu_0$. Assume that $P$ and $Q$ always remain in contact with the prism.




(a) If tension in $S_0$ is $T_0$, find the minimum tension ($T_1$) in $S_1$ to keep the body $P$ at rest.

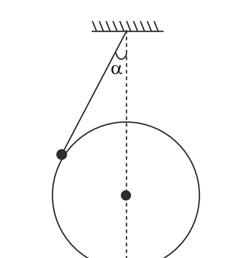
(b) A mass $M_0$ is tied to the lower end of string $S_0$ and another mass $m_2$ is tied to $S_2$. Find the minimum value of $m_2$ so as to keep $P$ and $Q$ at rest.

Q. 99. A metal disc of radius $R$ can rotate about the vertical axis passing through its centre. The top surface of the disc is uniformly covered with dust particles. The disc is rotated with gradually increasing speed. At what value of the angular speed ($\omega$) of the disc the 75% of the top surface will become dust free. Assume that the coefficient of friction between the dust particles and the metal disc is $\mu$ = 0.5. Assume no interaction amongst the dust particles.



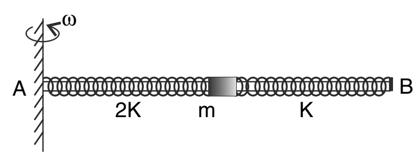

Q.100. In the last question, the axis of the disc is tilted slightly to make an angle $\theta$ with the vertical. Redo the problem for this condition and check the result by putting $\theta = 0$ in your answer.

Q. 101.

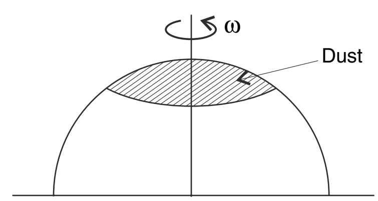



A sphere of mass $M$ is held at rest on a horizontal floor. One end of a light string is fixed at a point that is vertically above the centre of the sphere. The other end of the string is connected to a small particle of mass $m$ that rests on the sphere. The string makes an angle $\alpha = 30°$ with the vertical. Find the acceleration of the sphere immediately after it is released. There is no friction anywhere.

Q. 102. A light rod $AB$ is fitted with a small sleeve of mass $m$ which can slide smoothly over it. The sleeve is connected to the two ends of the rod using two springs of force constant $2k$ and $k$ (see fig). The ends of the springs at $A$ and $B$ are fixed and the other ends (connected to sleeve) can move along with the sleeve. The natural length of spring connected to $A$ is $\ell_0$. Now the rod is rotated with angular velocity $\omega$ about an axis passing through end A that is perpendicular to the rod. Take $\frac{k}{m\omega^2} = \eta$ and express the change in length of each spring (in equilibrium position of the sleeve relative to the rod) in terms of $\ell_0$ and $\eta$.




Q. 103. A metallic hemisphere is having dust on its surface. The sphere is rotated about a vertical axis passing through its centre at angular speed $\omega$ = 10 $rad\ s^{-1}$. Now the dust is visible only on top 20% area of the curved hemispherical surface. Radius of the hemisphere is $R = 0.1\ m$. Find the coefficient of friction between the dust particle and the hemisphere [$g = 10\ ms^{-2}$].




Q. 104. Civil engineers bank a road to help a car negotiate a curve. While designing a road they usually ignore friction. However, a young engineer decided to include friction in his calculation while designing a road. The radius of curvature of the road is $R$ and the coefficient of friction between the tire and the road is $\mu$.

(a) What should be the banking angle ($\theta_0$) so that car travelling up to a maximum speed $V_0$ can negotiate the curve.

(b) At what speed ($V_1$) shall a car travel on a road banked at $\theta_0$ so that there is no tendency to skid. (No tendency to skid means there is no static friction force action on the car).

(c) The driver of a car travelling at speed ($V_1$) starts retarding (by applying brakes). What angle (acute, obtuse or right angle) does the resultant friction force on the car make with the direction of motion?

Q. 105. A turn of radius 100 *m* is banked for a speed of 20 *m/s*

(a) Find the banking angle

(b) If a vehicle of mass 500 *kg* negotiates the curve find the force of friction on it if its speed is – (i) 30 *m/s* (ii) 10 *m/s*

Assume that friction is sufficient to prevent skidding and slipping.

[Take tan 22° = 0.4, sin 22° = 0.375, cos 22° = 0.93, $g = 10\ ms^{-2}$]

Q. 106. A horizontal circular turning has a curved length *L* and radius *R*. A car enters the turn with a speed $V_0$ and its speed increases at a constant rate *f*. If the coefficient of friction is $\mu$,

(a) At what time $t_0$, after entering the curve, will the car skid? (Take it for granted that it skids somewhere on the turning)

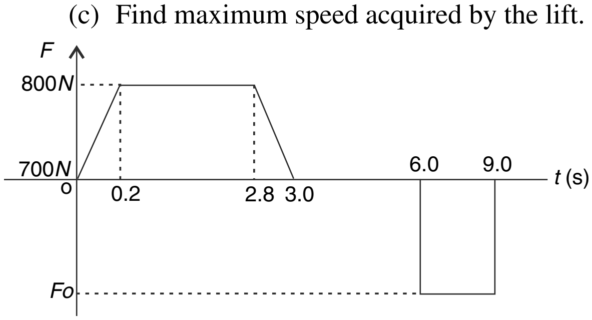
(b) At a time $t$ ($< t_0$) what is the force of friction acting on the car?

Q. 107. A 70 *kg* man enters a lift and stands an a weighing scale inside it. At time $t = 0$, the lift starts moving up and stops at a higher floor at $t = 9.0$ *s*. During the course of this journey, the weighing scale records his weight and given a plot of his weight vs time. The plot is shown in the fig. [Take $g = 10\ m/s^2$]

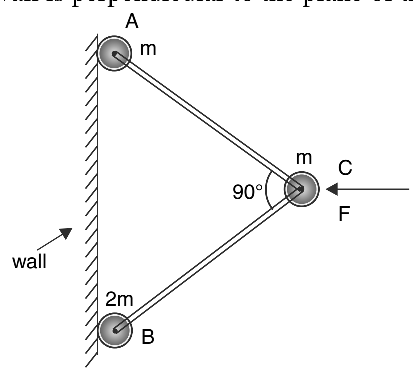
(a) Find $F_0$

(b) Find the magnitude of maximum acceleration of the lift.

(c) Find maximum speed acquired by the lift.

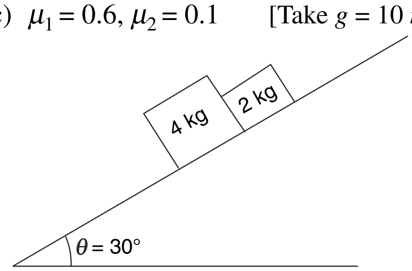



Q.108. Three small discs are connected with two identical massless rods as shown in fig. The rods are pinned to the discs such that angle between them can change freely. The system is placed on a smooth horizontal surface with discs *A* and *B* touching a smooth wall and the angle *ACB* being 90°. A force *F* is applied to the disc C in a direction perpendicular to the wall. Find acceleration of disc *B* immediately after the force starts to act. Masses of discs are $m_A = m$; $m_B = 2\ m$; $m_C = m$ [wall is perpendicular to the plane of the fig.]

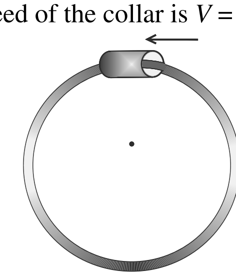



Q. 109. Figure shows two blocks in contact placed on an incline of angle $\theta = 30°$. The coefficient of friction between the block of mass 4 *kg* and the incline is $\mu_1$, and that between 2 *kg* block and incline is $\mu_2$. Find the acceleration of the blocks and the contact force between them if –

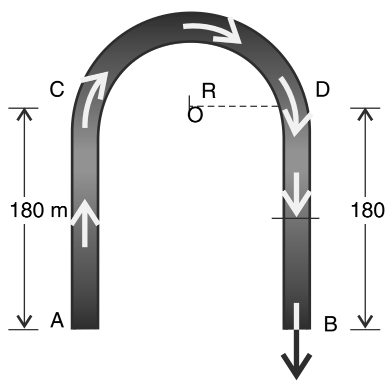
(a) $\mu_1 = 0.5, \mu_2 = 0.8$
(b) $\mu_1 = 0.8, \mu_2 = 0.5$
(c) $\mu_1 = 0.6, \mu_2 = 0.1$  [Take $g = 10\ m/s^2$]




Q. 110. A small collar of mass $m = 100\ g$ slides over the surface of a horizontal circular rod of radius $R = 0.3\ m$. The coefficient of friction between the rod and the collar is $\mu = 0.8$. Find the angle made with vertical by the force applied by the rod on the collar when speed of the collar is $V = 2\ m/s$.



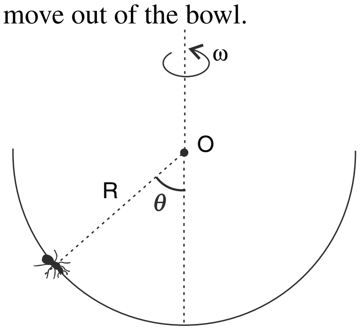
Q. 111. A flat race track consists of two straight section *AC* and *DB* each of length 180 *m* and one semi circular section *DC* of radius $R$ = 150 *m*. A car starting from rest at *A* has to reach *B* in least possible time (the car may cross through point *B* and need not stop there). The coefficient of friction between the tyres and the road is $\mu$ = 0.6 and the top speed that the car can acquire is 180 *kph*. Find the minimum time needed to move from A to B under ideal conditions. Braking is not allowed in the entire journey [$g = 10\ m/s^2$]



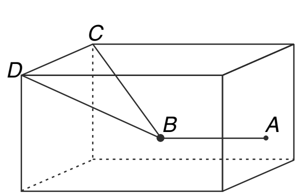

Q. 112. A small insect is climbing slowly along the inner wall of a hemispherical bowl of radius R. The insect is unable to climb beyond $\theta$ = 45°. Whenever it tries to climb beyond $\theta$ = 45°, it slips.

(a) Find the minimum angular speed $\omega$ with which the bowl shall be rotated about its vertical radius so that the insect can climb upto $\theta = 60°$.

(b) Find minimum $\omega$ for which the insect can move out of the bowl.




Q. 113.




A room is in shape of a cube. *A* heavy ball (*B*) is suspended at the centre of the room tied to three inextensible strings as shown. String *BA* is horizontal with A being the centre point of the wall. Find the ratio of tension in the string *BA* and *BC*.

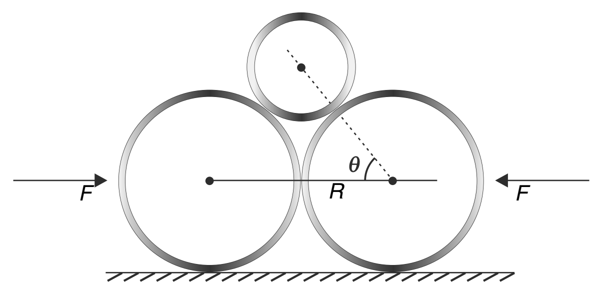
Q.114. Two identical smooth disc of radius *R* have been placed on a frictionless table touching each other. Another circular plate is placed between them as shown in figure. The mass per unit area of each object is $\sigma$, and the line joining the centers of the plate and the disc is $\theta$

(a) Find the minimum horizontal force $F_0$ that must be applied to the two discs to keep them together.

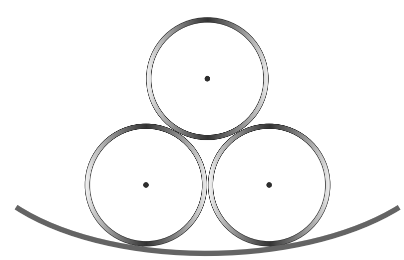
(b) Angle $\theta$ can be changed by changing the size of the circular plate. Find $F_0$ when $\theta \to 0$.

$$\left[\text{use } \cos\theta = 1 - \frac{\theta^2}{2} \text{ and } \sin\theta = \theta \text{ for small } \theta\right]$$

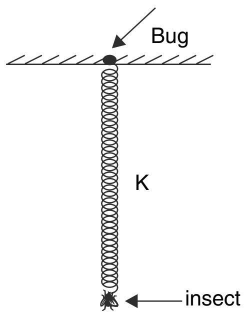
(c) Find $F_0$ when $\theta \to \pi/2$. Explain the result.




Q. 115. Three identical smooth cylinders, each of mass *m* and radius *r* are resting in equilibrium within a fixed smooth cylinder of radius *R* (only a part of this cylinder has been shown in the fig). Find the largest value of *R* in terms of *r* for the small cylinders to remain in equilibrium.



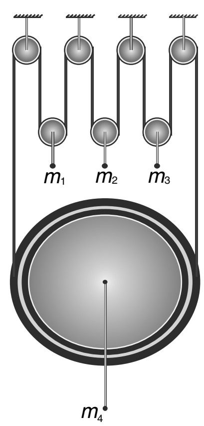
Q. 116. A massless spring of force constant *K* and natural length $\ell_0$ is hanging from a ceiling. An insect of mass *m* is sitting at the lower end of the spring and the system is in equilibrium . The insect starts slowing climbing up the spring so as to eat a bug sitting on the ceiling. Assume that insect climbs without slipping on the spring and $K = \dfrac{mg}{\ell_0}$. Find the length of the spring when the insect is at $\frac{1}{4}$th of its original distance from the bug.



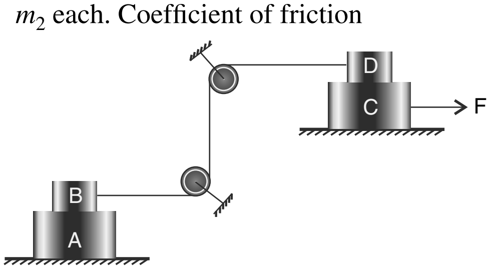

Q. 117. In the system shown in fig., all pulleys are mass less and the string is inextensible and light.

(a) After the system is released, find the acceleration of mass $m_1$

(b) If $m_1 = 1\ kg$, $m_2 = 2\ kg$ and $m_3 = 3\ kg$ then what must be value of mass $m_4$ so that it accelerates downwards?




Q. 118. In the system shown in fig., block $A$ and $C$ are placed on smooth floors and both have mass equal to $m_1$. Blocks $B$ and $D$ are identical having mass $m_2$ each. Coefficient of friction

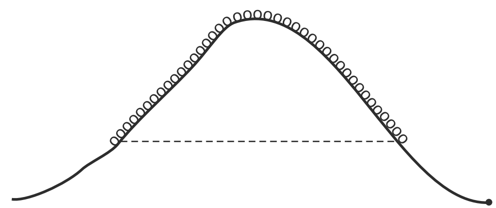



Between $A$ and $B$ and that between $C$ and $D$ are both equal to $\mu$. String and pulleys are light. $A$ horizontal force F is applied on block $C$ and is gradually increased.

(a) Find the maximum value of $F$ (call it $F_0$) so that all the four blocks move with same acceleration.

(b) Will the value of $F_0$ increase or decrease if another block ($E$) of mass $m_2$ is placed above block $D$ and coefficient of friction between $E$ and $D$ is $\mu$?

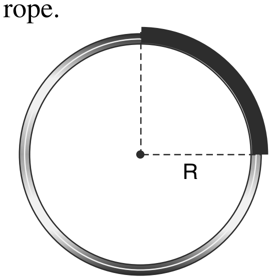
Q. 119. A chain with uniform mass per unit length lies in a vertical plane along the slope of a smooth hill. The two end of the chain are at same height. If the chain is released from this position find its acceleration.



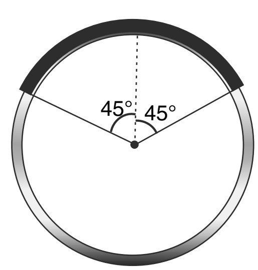
Q. 120. A uniform rope of length $\dfrac{\pi R}{2}$ has been placed on fixed cylinder of radius $R$ as shown in the fig. One end of the rope is at the top of the cylinder. The coefficient of friction between the rope and the cylinder is just enough to prevent the rope from sliding. Mass of the rope is $M$.

(a) At what position, the tension in the rope is maximum?

(b) Calculate the value of maximum tension in the rope.

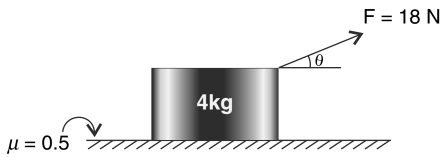



Q. 121. In the last problem, the rope is placed on the cylinder as shown. Find maximum tension in the rope.




Q. 122. A 4 $kg$ block is placed on a rough horizontal surface. The coefficient of friction between the block and the surface is $\mu = 0.5$. A force $F = 18\ N$ is applied on the block making an angle $\theta$ with the horizontal. Find the range of values of $\theta$ for which the block can start moving.

$$\left[\begin{array}{c}\text{Take } g = 10 \text{ m/s}^2, \tan^{-1}(2) = 63° \\ \sin^{-1}\left(\dfrac{10}{9\sqrt{1.25}}\right) = 84°\end{array}\right]$$



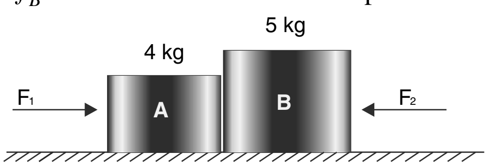

Q. 123. Two rectangular blocks $A$ and $B$ are placed on a horizontal surface at a very small separation. The masses of the blocks are $m_A = 4\ kg$ and $m_B = 5\ kg$. Coefficient of friction between the horizontal surface and both the blocks is $\mu = 0.4$. Horizontal forces $F_1$ and $F_2$ are applied on the blocks as shown. Both the forces vary with time as

$F_1 = 15 + 0.5\ t$

$F_2 = 2t$

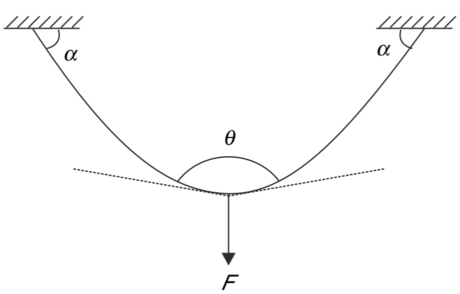
Where '$t$' is time in second.

Plot the variation of friction force acting on the two blocks ($f_A$ and $f_B$) vs time till the motion starts. Take rightward direction to be positive for $f_B$ and leftward direction to be positive for $f_A$.




## LEVEL 3

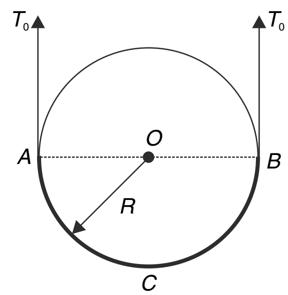
Q. 124. A rope of mass $m$ is hung from a ceiling. The centre point is pulled down with a vertical force $F$. The tangent to the rope at its ends makes an angle $\alpha$ with horizontal ceiling. The two tangents at the lower point make an angle of $\theta$ with each other. Find $\theta$.



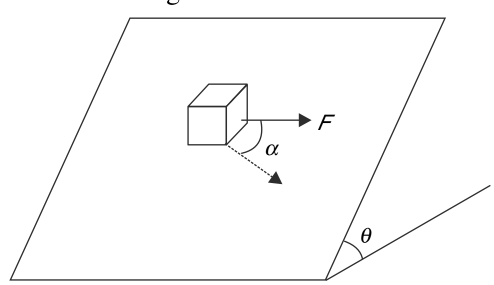

Q. 125. A smooth cylinder is fixed with its axis horizontal. Radius of the cylinder is $R$. A uniform rope ($ACB$) of linear mass density $\lambda$ ($kg$/m) is exactly of length $\pi R$ and is held in semicircular shape in vertical plane around the cylinder as shown in figure. Two massless strings are connected at the two ends of the rope and are pulled up vertically with force $T_0$ to keep the rope in contact with the cylinder.

(a) Find minimum value of $T_0$ so that the rope does not lose contact with the cylinder at any point.

(b) If $T_0$ is decreased slightly below the minimum value calculated in (a), where will the rope lose contact with the cylinder.

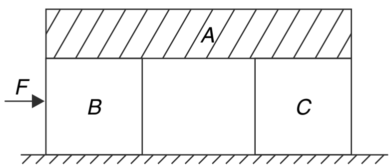



Q. 126. A block of mass $m$ placed on an incline just begins to slide when inclination of the incline is made $\theta_0 = 45°$. With inclination equal to $\theta = 30°$, the block is placed on the incline. A horizontal force ($F$) parallel to the surface of the incline is applied to the block. The force $F$ is gradually increased from zero. At what angle $\alpha$ to the force $F$ will the block first begin to slide?




Q. 127. In the last problem if it is allowed to apply the force $F$ in any direction, find the minimum force $F_{min}$ needed to move the block on the incline.

Q. 128. A block $A$ has been placed symmetrically over two identical blocks $B$ and $C$. All the three blocks have equal mass, $M$ each, and the horizontal surface on which $B$ and $C$ are placed is smooth. The coefficient of friction between $A$ and either of $B$ and $C$ is $\mu$. The block $A$ exerts equal pressure on $B$ and $C$. A horizontal force $F$ is applied to the block *B*.

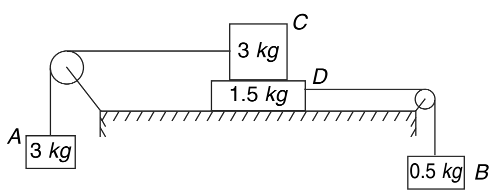



(a) Find maximum value of *F* so that *A* does not slip on *B* or *C* and the three blocks move together.

(b) If *F* is increased beyond the maximum found in (a) where will we see slipping first- at contact of *A* and *B* or at the contact of *A* and *C*.

(c) If *F* is kept half the maximum found in (a), calculate the ratio of friction force between *A* and *B* to that between *A* and *C*. Does this ratio change if *F* is decreased further?

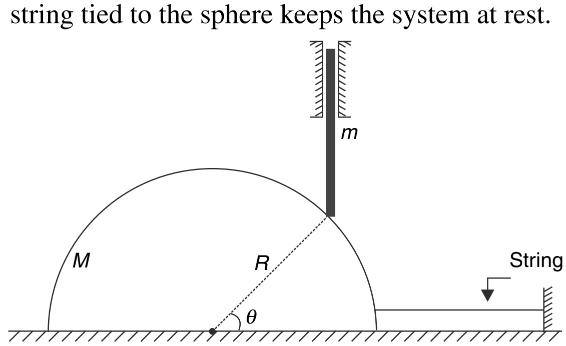
Q. 129. In the arrangement shown in the figure the coefficient of friction between the blocks *C* and *D* is $\mu_1 = 0.7$ and that between block *D* and the horizontal table is $\mu_2 = 0.2$. The system is released from rest. [ Take g = 10 ms$^{-2}$] Pulleys and threads are massless.




(a) Find the acceleration of the block *C*.

(b) Block *B* is replaced with a new block. What shall be the minimum mass of this new block so that block *C* and *D* accelerate in opposite direction?

Q. 130. A hemisphere of mass *M* and radius *R* rests on a smooth horizontal table. A vertical rod of mass *m* is held between two smooth guide walls supported on the sphere as shown. There is no friction between the rod and the sphere. A horizontal string tied to the sphere keeps the system at rest.



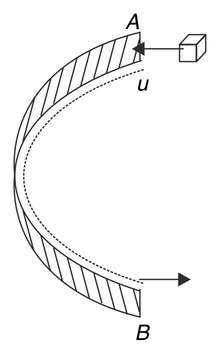

(a) Find tension in the string.

(b) Find the acceleration of the hemisphere immediately after the string is cut.

Q. 131. A semicircular ring of radius *R* is fixed on a smooth horizontal table. A small block is projected with speed *u* so as to enter the ring at end *A*. Initial velocity of the block is along tangent to the ring at *A* and it moves on the table remaining in contact with the inner wall of the ring. The coefficient of friction between the block and the ring is $\mu$.

(a) Find the time after which the block will exit the ring at *B*.

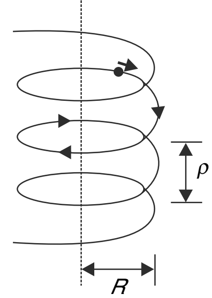
(b) With what speed will the block leave the ring at *B*.




Q. 132. A long helix made of thin wire is held vertical. The radius and pitch of the helix are *R* and $\rho$ respectively. A bead begins to slide down the helix.

(a) Find the normal force applied by the wire on the bead when the speed of the bead is *v*.

(b) Eventually, the bead acquires a constant speed of $v_0$. Find the coefficient of friction between the wire and the bead.

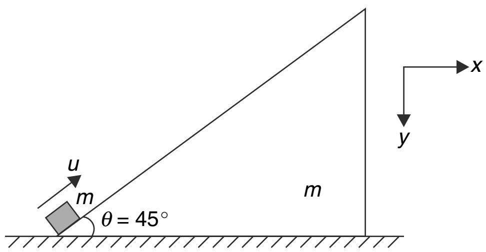



Q. 133. A wedge of mass *m* is kept on a smooth table and its inclined surface is also smooth. A small block of mass *m* is projected from the bottom along the incline surface with velocity *u*. Assume that the block remains on the incline and take $\theta = 45°$, $g = 10\ m/s^2$.

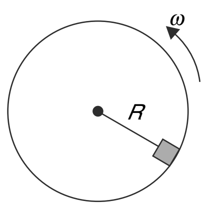
(a) Find the acceleration of the wedge and the $x$ and $y$ components of acceleration of the block.

(b) Draw the approximate path of the block as observed by an observer on the ground. At what angle does the block hit the table?

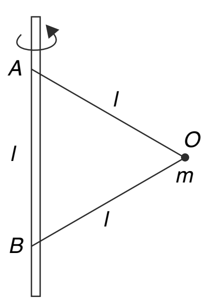
(c) Calculate the radius of curvature of the path of the block when it is at the highest point.




Q. 134. A cylinder with radius $R$ spins about its horizontal axis with angular speed $\omega$. There is a small block lying on the inner surface of the cylinder. The coefficient of friction between the block and the cylinder is $\mu$. Find the value of $\omega$ for which the block does not slip, i.e., stays at rest with respect to the cylinder.




Q. 135. A particle of mass $m$ is attached to a vertical rod with two inextensible strings $AO$ and $BO$ of equal lengths $l$. Distance between $A$ and $B$ is also $l$. The setup is rotated with angular speed $\omega$ with rod as the axis.




(a) Find the values of $\omega$ for which the particle remains at point $B$.

(b) Find the range of values of $\omega$ for which tension ($T_1$) in the string $AO$ is greater than mg but the other string remains slack

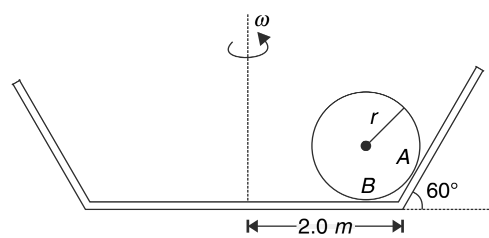
(c) Find the value of $\omega$ for which tension ($T_1$) in string $AO$ is twice the tension ($T_2$) in string $BO$

(d) Assume that both strings are taut when the string $AO$ breaks. What will be nature of path of the particle moment after $AO$ breaks ?

Q. 136. A sphere of mass $m$ and radius $r = \sqrt{3}m$ is placed inside a container with flat bottom and slant sidewall as shown in the figure. The sphere touches the slant wall at point $A$ and the floor at point $B$. It does not touch any other surface. The container, along with the sphere, is rotated about the central vertical axis with angular speed $\omega$. The sphere moves along with the container, i.e., it is at rest relative to the container. The normal force applied by the bottom surface and the slant surface on the sphere are $N_1$ and $N_2$ respectively. There is no friction.

(a) Find the value of $\omega$ above which $N_2$ becomes larger than $N_1$

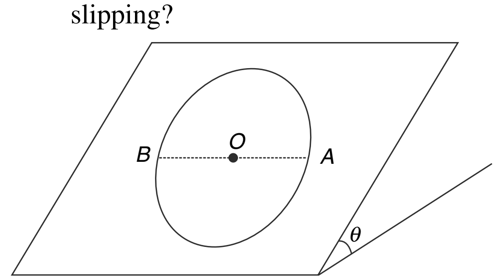
(b) Find the value of $\omega$ above which the sphere leaves contact with the floor.



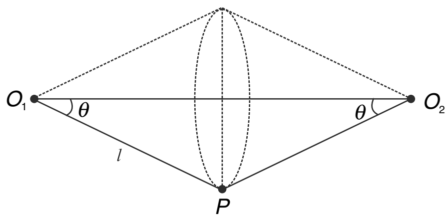

Q. 137. A car is being driven on a tilted ground. The ground makes an angle $\theta$ with the horizontal. The driven drives on a circle of radius $R$. The coefficient of friction between the tires and the ground is $\mu$.

(a) What is the largest speed for which the car will not slip at point $A$? Assume that rate of change of speed is zero.

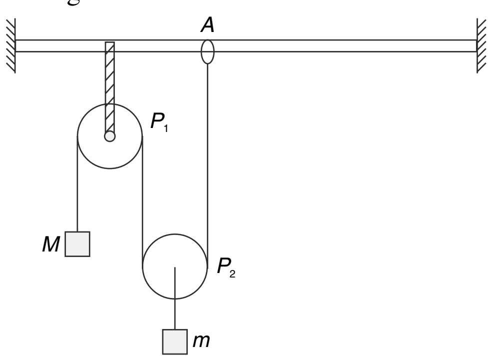
(b) What is the largest constant speed with which the car can be driven on the circle without slipping?




Q. 138. A particle $P$ is attached to two fixed points $O_1$ and $O_2$ in a horizontal line, by means of two light inextensible strings of equal length $l$. It is projected with a velocity just sufficient to make it describe a circle, in a vertical plane, without the strings getting slack and with the angle $< O_2O_1P = <O_1O_2P = \theta$. When the particle is at its lowest point, the string $O_2P$ breaks and the subsequent path of the particle was found to be a circle of radius $l \cos \theta$. Find $\theta$.




Q. 139. The arrangement shown in figure is in equilibrium with all strings vertical. The end $A$ of the string is tied to a ring which can be slid slowly on the horizontal rod. Pulley $P_1$ is rigidly fixed but $P_2$ can move freely. A mass $m$ is attached to the centre of pulley $P_2$ through a thread. Pulleys and strings are mass less.



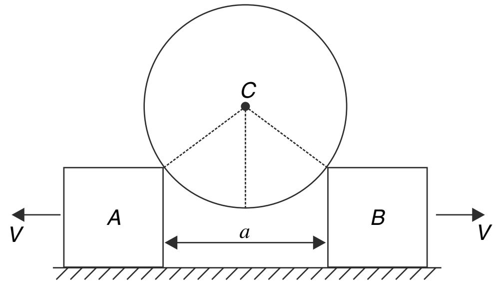

(a) Which block will move up as $A$ is moved slowly to the right?

(b) Will the block of mass $m$ have horizontal displacement?

(c) Is it possible, for a particular position of $A$, that $M$ has no acceleration but $m$ does have an acceleration? If this happens when string from $P_2$ to $A$ makes an angle $\theta$ with vertical, find the acceleration of $m$ at the instant.

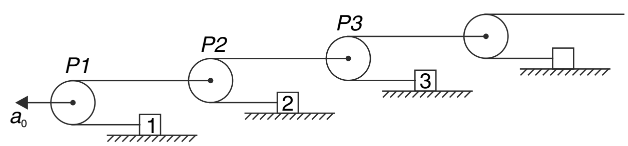
Q. 140. A smooth spherical ball of mass $M = 2\ kg$ is resting on two identical blocks $A$ and $B$ as shown in the figure. The blocks are moved apart with same horizontal velocity $V = 1\ m/s$ in opposite directions (see figure).

(a) Find the normal force applied by each of the blocks on the sphere at the instant separation between the blocks is $a = \sqrt{2}R$; $R = 1.0\ m$ being the radius of the ball.

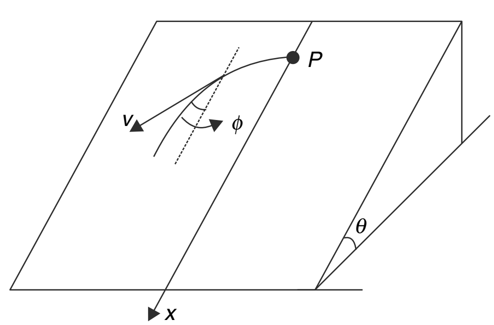



(b) How much force must be applied on each of the two blocks (when $a = \sqrt{2}R$) so that they do not have any acceleration. Assume that the he horizontal surface is smooth.

Q. 141. In the figure all pulleys ($P1$, $P2$, $P3$ …….) are massless and all the blocks (1,2,3 …..) are identical, each having mass $m$. The system consist of infinite number of pulleys and blocks. Strings are light and inextensible and horizontal surfaces are smooth. Pulley $P1$ is moved to left with a constant acceleration of $a_0$. Find the acceleration of block1. Assume the strings to remain horizontal.




Q. 142. A small disc $P$ is placed on an inclined plane forming an angle $\theta$ with the horizontal and imparted an initial velocity $v_0$. Find how the velocity of disc depends on the angle $\phi$ which its velocity vector makes with the $x$ axis (see figure). The coefficient of friction is $\mu = \tan \theta$ and initially $\phi_0 = \frac{\pi}{2}$.



# ANSWERS

**1.** (a) straight line

(b) Parabolic

**2.** 20 $m$

**3.** $\sqrt{\frac{4H}{3g}}$

**4.** $N = 12$; Tension $= \frac{F}{N} = \frac{F}{12}$

**5.** 16 $N$

**6.** (i) (a) True

(b) True

(ii) $\frac{g}{5}$

(iii) $\sqrt{\frac{2\eta h}{(\eta - 1)g}}$

**7.** $t = \sqrt{\frac{2(M-m)L}{Mg}}$

**8.** (a) $a = \frac{g}{4}$

(b) $a = \frac{g}{5}$

**9.** $\frac{4Mmg}{M+m} + M_0 g$

**10.** $m_0 = \frac{4m_1m_2}{m_1+m_2}$; $M = \frac{8m_1m_2}{m_1+m_2}$

All masses will fall down with acceleration $g$

**11.** (a) More than 9/2 $Mg$

(b) Tension in $S2 = Mg/2$, Tension in $S1 = 5\ Mg$

(c) Tension in $S2$ = Mg/6

**12.** $\theta = 45°$, $g/2$

**13.** 73.1 $N$

**14.** 12.5 $s$

**15.** $T = \sqrt{2}\ Mg$; $N = \frac{Mg}{\sqrt{2}}$

**16.** Zero.

**17.** $\mu_{min} = 1$

**18.** (a) 6467 N

(b) 22400 $N$

(c) 190400 $N$

(d) $T_0$ and $F$ do not change. $T$ will increase.

**19.** (a) No

(b) 1 : 4

**20.** $2\sqrt{\frac{2h}{g}}$

**21.** $a_A = a_B = \frac{F}{2m}$; $a_c = 0$

**22.** $3M/5$

**23.** $\theta = 62.5°$

**24.** $R_A : R_B : R_C = 3 : 1 : 2$

**25.** $K = 2$ Sdg

**26.** (a) $\frac{5}{3} cms^{-1}$

(b) 6 $N$

**27.** (a) 2, 4

(b) In both cases acceleration of the frame must be '$g$'.

**28.** 15 $m$

**29.** $\frac{6t}{\sqrt{36-\pi}}$

**30.** $\mu \leq \frac{v_0^2}{gL}$

**31.** 4 $kg$

**32.** 4.8 $kg$

**33.** $\sqrt{174}\ N$

**34.** $1.5\ kg \leq m \leq 9.5\ kg$

**35.** $\frac{5F}{m}$

**36.** (a) At $C$

(b) At $C$

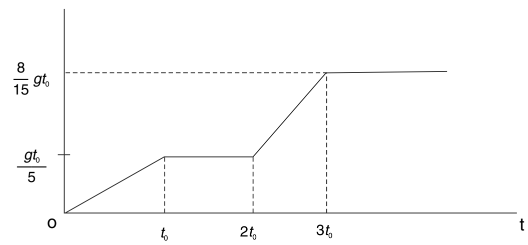
(c) $\sqrt{2}\ m/s^2$ and $\sqrt{2}\ m/s^2$

**37.** (i) $\sqrt{15} < \omega \sqrt{16.67}\ rad/s$

(ii) 500 $rad\ s^{-1}$

**39.** (a) $\frac{8gt_0}{15}$

(b)




**40.** (a) 80 $N$

(b) $\frac{640}{9}N$ for both

**41.** (a) $\frac{m_A}{m_B} = \frac{1}{3}$

(b) $X_{\min} = 0.75\ m$

(c) $V_A' = \left(\frac{3}{2} + \frac{3}{\sqrt{2}}\right) ms^{-1}; V_B' = \left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right) ms^{-1}$

**42.** $V_0 = \frac{Mg}{\mu}\left[4\ell n\left(\frac{4}{3}\right) - 1\right]$

**43.** $t_2$

**44.** $N_{12} = \frac{Mg\cos^2\theta}{1 + 2\sin^2\theta}$

$a = \frac{3g\sin\theta}{1 + 2\sin^2\theta}$

**45.** 1.0 $s$, 1.0 $m$

**46.** (b) $T_P = 21.65\ N$

(c) 3.05 $kg$

**47.** $\frac{\sqrt{5 - 2\sqrt{2}}}{\sqrt{2} - 1}$

**48.** $t = \frac{1}{\sin\theta\sin\phi}\sqrt{\frac{2d}{g}}$

**49.** $\frac{5Mg}{2K}$

**50.** 2.9 $s$

**51.** (a) Force between the wall and the middle ball is maximum. It is 4 $mg$

(b) Force between upper ball and wall is least. It is $\frac{4}{3}$ $mg$.

**52.** $\mu_{\min} = \frac{\sin\theta}{\sqrt{\cos^2\theta + \tan^2\theta}}$

**53.** (a) $K = 2.5\ N/cm$

(b) No

**54.** (a) The block is at height $h = 2.5\ m$

(b) $V = 5\sqrt{2}\ m/s$

(c) 25 $m/s^2$ (↑)

**55.** Zero

**56.** (a) $\frac{2F}{K}$

(b) $\frac{F}{K}$

**57.** With pulley $P_1$ having zero mass, equilibrium is not possible

**58.** $\frac{l}{3}$

**59.** (a) $M$

(b) $\frac{g}{3}$

**60.** (a) $\theta_0 = \frac{\pi}{4}$

(b) $a_{\max} = g\left[\cos\left(\frac{\pi}{8}\right) - \sin\left(\frac{\pi}{8}\right)\right]$

**61.** (a) $t > t^1$

(b) $\theta = \tan^{-1}\left(\frac{1}{12}\right)$

**62.** $a_0 \cos\theta$

**63.** $t = \sqrt{\frac{10}{3}}s$

**64.** (a) $g\sin\theta$

(b) $f = \frac{1}{2}mg\sin 2\theta$

$N = mg\cos^2\theta$

(c) $t = \sqrt{\frac{2h(M + m\sin^2\theta)}{(M + m)g\sin^2\theta}}$

**65.** (a) $\overrightarrow{a_A} \cdot \overrightarrow{a_B} = 0$ immediately after release

(b) $\vec{a}_A = \frac{g}{2}(\leftarrow)$

**66.** (a) $\theta$

(b) $\tan^{-1}\left(\frac{\sin\theta \cdot \cos\theta}{2 - \sin^2\theta}\right)$

**67.** (a) 2 $mg$

(b) $t = \sqrt{\frac{3L}{\sqrt{2}g}}$

**68.** $\frac{g(\sin\theta - \mu\cos\theta)}{(\cos\theta + \mu\sin\theta)} \le a \le \frac{g(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}$

**69.** $\frac{m}{M} = \frac{20}{3\sqrt{3} - 4} = 16.7$

**70.** (a) $(M + m)\, g \sin\alpha$

**(b)** $\frac{(M+m)g\sin^2\alpha}{m + M\sin^2\alpha}$

**71.** $\frac{M}{m} = \frac{1}{5}$

**72.** $a_0 = \frac{48g}{199}$

**73.** $\frac{44g}{205}$

**74.** $\frac{2L}{5}$

**75.** (a) $\left|\frac{d^2x_2}{dt^2}\right| = \frac{3g}{2}$

(b) $\left|\frac{d^2x_1}{dt^2}\right| = 2g;\quad \left|\frac{d^2x_2}{dt^2}\right| = 2g$

(c) $\left|\frac{d^2x_1}{dt^2}\right| = \frac{g}{2};\quad \left|\frac{d^2x_2}{dt^2}\right| = \frac{3g}{2}$

**76.** (a) $\frac{4k_1k_2x_0}{(k_1 + k_2)M}$

(b) $\frac{4k_1k_2x_0}{k_1 + k_2}$

**77.** $a = \frac{g}{\sqrt{2}}$; $N_{AB} = 0$

**78.** 3

**79.** $\frac{6g}{47}$

**80.** $\sqrt{2}\, F_0$

**81.** $T = 0.49\ N$

**82.** (a) Zero

(b) $\frac{F_1}{3m} = \frac{F_2}{m}$

(c) To right

**83.** (a) $\theta \le 13°$

(b) 37°

**84.** (a) $\frac{13}{7}s$

(b) 0.18

**85.** $V_{\max} = \frac{-u^2}{2} + \sqrt{\frac{u^4}{4} + (2\mu gL)^2}$

**86.** $k = 130\ Nm^{-1}$; $\mu = 0.5$

**87.** $\mu = \left(\frac{3a_0 - 4g}{4a_0 + 3g}\right)$

**88.** $\left(\frac{r}{R}\right)_{\min} = \frac{\sqrt{1+\mu^2} - \mu}{\sqrt{1+\mu^2} + \mu}$

**89.** $F = \frac{Mg}{1-\mu^2}\left[\mu + \frac{\mu\cos\theta + \sin\theta}{\cos\theta - \mu\sin\theta}\right]$

**90.** $5\ m/s^2 \le a \le 7\ m/s^2$

**91.** $2\ kg \le m \le 30\ kg$

**92.** (a) $\frac{960}{95}kg$

(b) $\frac{480}{61}kg$

**93.** $\mu_{\min} = \frac{\Delta m}{2m}\tan\theta$

**94.** (i) 2.5 *kg*; 12.5 *N*

(ii) $\frac{50}{3}N$

(iii) $\frac{40}{3}N, a = \frac{5}{3}$ $m/s^2$

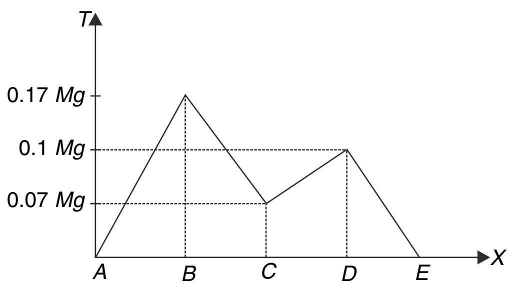
(iv) $\frac{5}{6}m/s^2$

**95.** $M_0 = \lambda R\left(\frac{\pi}{2} - 1\right)$

**96.** $\frac{H}{x}$

**97.** (a) $\mu = \frac{\sqrt{3}+1}{\sqrt{3}+5} = 0.4$

(b)




(c) $T_{max} = 0.17$ *Mg*

**98.** (i) (a) $T_{BC} = 10$ *N*; $T_{AB} = 12$ *N*; $T_{AD} = 7$ *N*

(b) $f_C = 0$; $f_B = 4N$; $f_A = 5N$; $f_D = 5N$

(ii) (a) $T_1 = \left(\frac{1-\mu_0}{1+\mu_0}\right) T_0$

(b) $m_2 = \left(\frac{1-\mu_0}{1+\mu_0}\right)^2 M_0$

**99.** $\omega = \sqrt{\frac{g}{R}}$

**100.** $\omega = \sqrt{\frac{g}{R}(\cos\theta - 2\sin\theta)}$

**101.** $\frac{\sqrt{3}mg}{3m+4M}$

**102.** $x = \frac{l_0}{3\eta - 1}$

**103.** 2.45

**104.** (a) $\theta_0 = \tan^{-1}\left(\frac{\frac{V_0^2}{Rg} - \mu}{1 + \frac{\mu V_0^2}{Rg}}\right)$

(b) $V_1 = \sqrt{rg\tan\theta_0}$

(c) Obtuse

**105.** (a) 22°

(b) (i) 2315 *N*, 1389 *N*

**106.** (a) $t_0 = \frac{\left[R^2\left(\mu^2 g^2 - f^2\right)\right]^{1/4} - V_0}{f}$

(b) $m\sqrt{\frac{(V_0 + ft)^4}{R^2} + f^2}$

**107.** (a) 93.3 N

(b) $\frac{10}{7}$ $m/s^2$

(c) 4 *m/s*

**108.** $\frac{F}{5m}$

**109.** (a) contact force = 0, acceleration of 4 *kg* block is 0.7 $m/s^2$ and that of other block is zero

(b) contact force = 1.4 *N*, acceleration of both = 0

(c ) Contact force = 5.74 *N*, acceleration of both = 1.27 $m/s^2$

**110.** $\theta = \cos^{-1}\left(\frac{3}{\sqrt{41}}\right)$

**111.** 30.1 *s*

**112.** (a) $\sqrt{\frac{g}{R}\frac{2\left(\sqrt{3}-1\right)}{\sqrt{3}\left(\sqrt{3}+1\right)}}$

(b) $\sqrt{\frac{g}{R}}$

**113.** $\frac{2}{\sqrt{3}}$

**114.** (a) $F_0 = \frac{\sigma\pi R^2 g\left(1-\cos\theta\right)^2}{2\sin\theta.\cos\theta}$

(b) $F_0 = 0$

(c) $\infty$

**115.** $R = r\,(1 + 2\sqrt{7})$

**116.** $\dfrac{5l_0}{4}$

**117.** (a) $a_1 = g\left(1 - \dfrac{4M}{m_1}\right)$

(b) $m_4 > \dfrac{18}{11}kg$

**118.** $F_0 = 2\mu m_2\, g\left(\dfrac{m_2 + m_1}{2m_2 + m_1}\right)$; increase

**119.** Zero

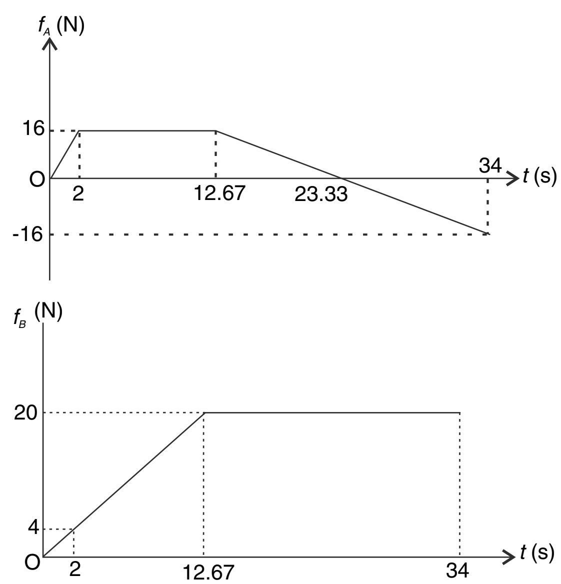
**120.** (a) $\theta = 45°$ from vertical diameter.

(b) $T_{\max} = 2\left(\dfrac{\sqrt{2} - 1}{\pi}\right)Mg$

**121.** Zero

**122.** $21° < \theta < 33°$

**123.**




**124.** $\theta = 2\tan^{-1}\left[\left(1 + \dfrac{mg}{F}\right)\cot\alpha\right]$

**125.** (a) $T_0 = 2\lambda R g$

(b) At the lowest point

**126.** $\sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

**127.** $F_{\min} = \dfrac{mg}{2\sqrt{2}}\left(\sqrt{3} - 1\right)$

**128.** (a) $F_{\max} = \dfrac{3}{4}\mu Mg$

(b) Between $A$ and $B$

(c) 2, No

**129.** (a) $2\ ms^{-1}$

(b) $2.1\ kg$

**130.** (a) $T = N\cos\theta$

(b) $\dfrac{mg}{M\tan\theta + m\cot\theta}$

**131.** (a) $t = \dfrac{R}{\mu u}\left[e^{\pi\mu} - 1\right]$

(b) $V = \dfrac{u}{e^{\pi\mu}}$

**132.** (a) $mg\cos\theta\sqrt{1 + \left(\dfrac{v^2\cos\theta}{Rg}\right)^2}$

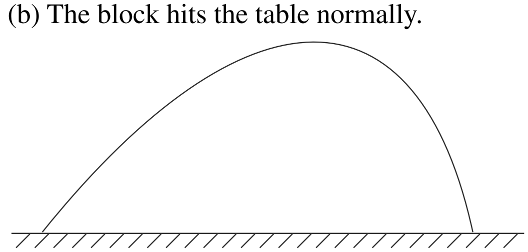
(b) $\dfrac{\tan\theta}{\sqrt{1 + \left(\dfrac{v^2\cos\theta}{Rg}\right)^2}}$ where $\tan\theta = \dfrac{\rho}{2\pi R}$

**133.** (a) $\vec{a}_{wedge} = \dfrac{g}{3}\hat{i}$

$a_{x\ block} = \dfrac{-g}{3}$

$a_{y\ block} = \dfrac{2g}{3}$

(b) The block hits the table normally.



(c) $\dfrac{3u^2}{16g}$

**134.** $\omega \geq \sqrt{\dfrac{g\sqrt{1 + \mu^2}}{R\mu}}$

**135.** (a) $\omega > \sqrt{\frac{g}{l}}$

(b) $\sqrt{\frac{g}{l}} < \omega \le \sqrt{\frac{2g}{l}}$

(c) $\sqrt{\frac{6g}{l}}$

(d) parabolic

**136.** (a) $\sqrt{\frac{g}{\sqrt{3}}}$

(b) $\sqrt{\sqrt{3}g}$

**137.** (a) $[g^2 R^2 (\mu^2 \cos^2 \theta - \sin^2 \theta)]^{1/4}$

(b) $\sqrt{gR(\mu\cos\theta - \sin\theta)}$

**138.** $\theta = \tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$

**139.** (a) Block with mass $M$ will move up.

(b) yes

(c) $g(1 - \cos\theta)$

**140.** (a) $(10\sqrt{2} - 8)\,N$

(b) $(5 - 4\sqrt{2})N$

**141.** $\frac{3a_0}{2}$

**142.** $\frac{v_0}{1+\cos\phi}$

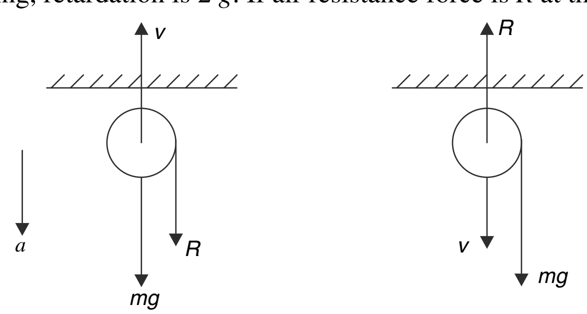
# SOLUTIONS

**1.** (a) Initial velocity is parallel to $\vec{F}$ or anti parallel to $\vec{F}$. Hence particle moves in a straight line and speed may increase or decrease.

(b) Path is parabolic with speed increasing.

In case (a) the particle may retrace its path.

**2.** Just before striking the ceiling, retardation is 2 $g$. If air resistance force is $R$ at this instant, then




$$ma = mg + R$$
$$m(2g) = mg + R$$
$$\Rightarrow R = mg$$

After impact, the air resistance force will be upward but its magnitude will remain mg. This is because speed has not changed.

$\therefore$ After impact net force on the ball = 0

$\therefore$ Ball will fall down with constant speed

$\therefore$ $H = (10\ m/s)(2\ s) = 20\ m$.

# 04 WORK - POWER - ENERGY

## Level 1

Q. 1. (i) The cause of increases in kinetic energy when a man starts running without his feet slipping on ground is asked to two students. Their answers are–

Harshit: Cause of increase in kinetic energy is work done by friction force. Without friction the man cannot run.

Akanksha: Cause of increase in kinetic energy is work done by internal (muscle) forces of the body.

Who is right?

(ii) An inextensible rope is hanging from a tree. A monkey, having mass $m$, climbs to a height $h$ grabbing the rope tightly. The monkey starts from rest and ends up hanging motionlessly on the rope at height $h$.

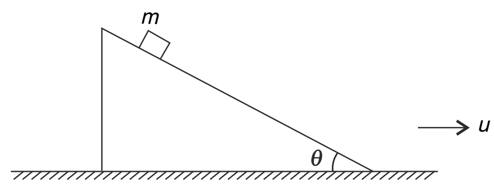
(a) How much work is done by gravity on the monkey?

(b) How much work is done by the rope on the monkey?

(c) Using work – energy theorem, explain the increase in mechanical energy of the monkey.

Q. 2. A man of mass $M$ jumps from rest, straight up, from a flat concrete surface. Centre of mass of the man rises a distance h at the highest point of the motion. Find the work done by the normal contact force (between the man's feet and the concrete floor) on the man.

Q. 3. A block of mass $m = 10\ kg$ is released from the top of the smooth inclined surface of a wedge which is moving horizontally toward right at a constant velocity of $u = 10\ m/s$. Inclination of the wedge is $\theta = 37°$. Calculate the work done by the force applied by the wedge on the block in two seconds in a reference frame attached to -



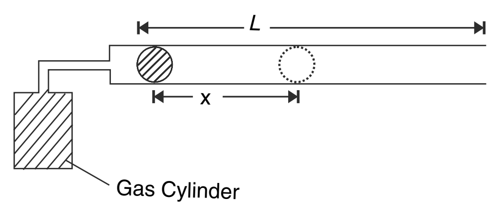

(a) the ground (b) the wedge.

[Take $g = 10\ ms^{-2}$]

Q. 4. In an industrial gun, when the trigger is pulled a gas under pressure is released into the barrel behind a ball of mass $m$. The ball slides smoothly inside the barrel and the force exerted by the gas on the ball varies as

$$F = F_o\left(1 - \frac{x}{L}\right)$$

Where $L$ is length of the end of the barrel from the initial position of the ball and $x$ is instantaneous displacement of the ball from its initial position. Neglect any other force on the ball apart from that applied by the gas. Calculate the speed ($V$) of the ball with which it comes out of the gun.




Q. 5. A particle of mass 3 $kg$ takes 2 second to move from point $A$ to $B$ under the action of gravity and another constant force

$\vec{F} = \left(12\hat{i} - 3\hat{j} + 21\hat{k}\right)N$, where the unit vector $\hat{k}$ is in the direction of upward vertical. The position vector of point $B$ is $\vec{r}_B = \left(15\hat{i} - 7\hat{j} - 6\hat{k}\right)m$ and velocity of the particle when it reaches $B$ is

$$\vec{V}_B = \left(12\hat{i} + \hat{j} - 4\hat{k}\right)m/s.$$

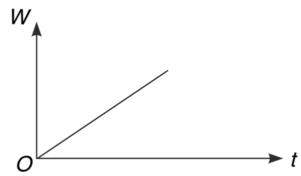
(a) Find the velocity, $\vec{V}_A$ of the particle when it was at $A$.

(b) Find position vector, $\vec{r}_A$ of point $A$.

(c) Find work done by the force $\vec{F}$ as the particle moves from $A$ to $B$.

(d) Find change in gravitational potential energy of the particle as it moves from $A$ to $B$.

Q. 6. A particle can move along a straight line. It is at rest when a force ($F$) starts acting on it directed along the line. Work done by the force on the particle changes with time(t) according to the graph shown in the fig. Can you say that the force acting on the particle remains constant with time?




Q. 7. A particle is moving on a straight line and all the forces acting on it produce a constant power $P$ calculate the distance travelled by the particle in the interval its speed increase from $V$ to $2V$.

Q. 8. Work done and power spent by the motor of an escalator are $W$ and $P$ respectively when it carries a standing passenger from ground floor to the first floor. Will the work and power expended by the motor change if the passenger on moving escalator walks up the staircase at a constant speed?

Q. 9. (i) A block is connected to an ideal spring on a horizontal frictionless surface. The block is pulled a short distance and released. Plot the variation of kinetic energy of the block vs the spring potential energy.

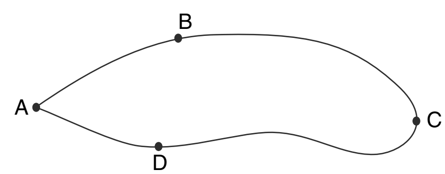
(ii) A ball of mass 200 $g$ is projected from the top of a building 20 $m$ high. The projection speed is 10 m/s at an angle $\theta = \sin^{-1}\left(\frac{3}{5}\right)$ from the horizontal. Sketch a graph of kinetic energy of the ball against height measured from the ground. Indicate the values of kinetic energy at the top and bottom of the building and at the highest point of the trajectory, specifying the heights on the graph. Neglect air resistance and take $g = 10\ m/s^2$

Q. 10. A car of mass $m = 1600\ kg$, while moving on any road, experiences resistance to its motion given by $(m + nV^2)$ newton; where $m$ and $n$ are positive constants. On a horizontal road the car moved at a constant speed of 40 $m/s$ when the engine developed a power of 53 $KW$. When the engine developed an output of 2 $KW$ the car was able to travel on a horizontal road at a constant speed of 10 $m/s$.

(a) Find the power that the engine must deliver for the car to travel at a constant speed of 40 $m/s$ on a horizontal road.

(b) The car is able to climb a hill at a constant speed of 40 $m/s$ with its engine working at a constant rate of 69 $KW$. Calculate the inclination of the hill (in degree)

Q. 11. A particle moves along the loop $A$–$B$–$C$–$D$–$A$ while a conservative force acts on it. Work done by the force along the various sections of the path are – $W_{A \to B} = -50\ J$ ; $W_{B \to C} = 25\ J$; $W_{C \to D} = 60\ J$. Assume that potential energy of the particle is zero at A. Write the potential energy of particle when it is at B and D.




Q. 12. A moving particle of mass m is acted upon by five forces $\vec{F_1}, \vec{F_2}, \vec{F_3}, \vec{F_4}$ and $\vec{F_5}$. Forces $\vec{F_2}$ and $\vec{F_3}$ are conservative and their potential energy functions are $U$ and $W$ respectively. Speed of the particle changes from $V_a$ to $V_b$ when it moves from position $a$ to $b$. Which of the following statement is/are true –

(a) Sum of work done by $\vec{F_1}, \vec{F_4}$ and $\vec{F_5}$ = $U_b - U_a + W_b - W_a$

(b) Sum of work done by $\vec{F_1}, \vec{F_4}$ and $\vec{F_5}$ =

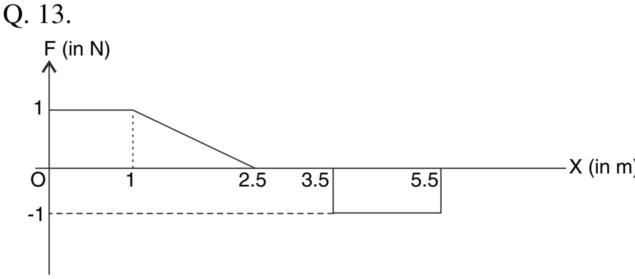
$$U_b - U_a + W_b - W_a + \frac{1}{2}m\left(V_b^2 - V_a^2\right)$$

(c) Sum of work done by all five forces =

$$\frac{1}{2}m\left(V_b^2 - V_a^2\right)$$

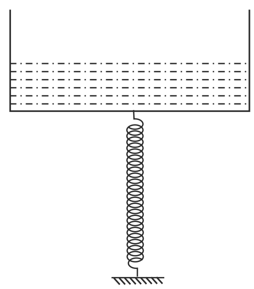
(d) Sum of work done by $\vec{F_2}$ and $\vec{F_3}$ = $(U_b + W_b) - (U_a + W_a)$.

Q. 13.

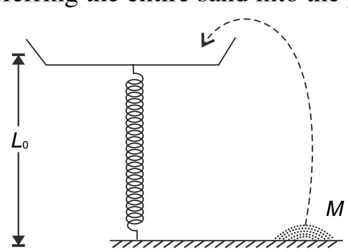


The given graph represents the total force in $x$ direction being applied on a particle of mass $m$ = 2 $kg$ that is constrained to move along $x$ axis. What is the minimum possible speed of the particle when it was at $x = 0$?

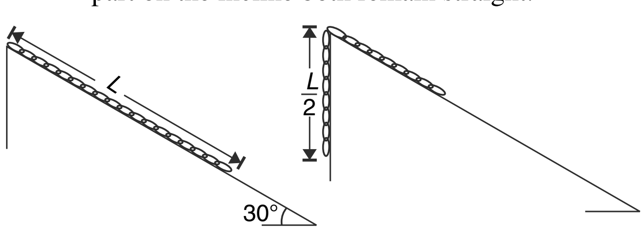
Q. 14. A vertical spring supports a beaker containing some water in it. Water slowly evaporates and the compression in the spring decreases. Where does the elastic potential energy stored in the spring go?



Q. 15. A pan of negligible mass is supported by an ideal spring which is vertical. Length of the spring is $L_o$. A mass $M$ of sand is lying nearby on the floor. A boy lifts a small quantity of sand and gently puts it into the pan. This way he slowly transfers the entire sand into the pan. The spring compresses by $\frac{L_o}{2}$. Assume that height of the sand heap on the floor as well as in the pan is negligible. Calculate the work done by the boy against gravity in transferring the entire sand into the pan.



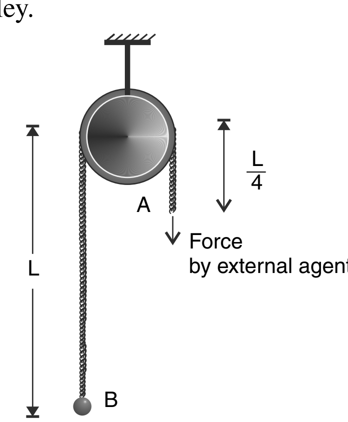

Q. 16. A snake of mass $M$ and length $L$ is lying on an incline of inclination 30°. It craws up slowly and overhangs half its length vertically. Assume that the mass is distributed uniformly along the length of the snake and its hanging part as well as the part on the incline both remain straight.




(a) Find the work done by the snake against gravity ($W_g$)

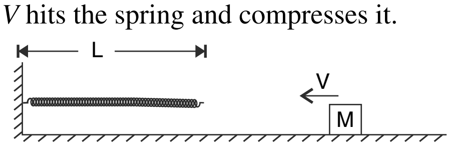
(b) Will the answer to part (a) be different if the snake were of half the length but of same mass.

Q. 17. A uniform rope of linear mass density $\lambda$ ($kg/m$) passes over a smooth pulley of negligible dimension. At one end B of the rope there is a small particle having mass one fifty of the rope. Initially the system is held at rest with length $L$ of the rope on one side and length $\frac{L}{4}$ on the other side of the pulley (see fig). The external agent begins to pull the end A downward. Find the minimum work that the agent must perform so that the small particle will definitely reach the pulley.




Q. 18. A particle of mass $m$ = 100 $g$ is projected vertically up with a kinetic energy of 20 $J$ form a position where its gravitational potential energy is – 50 $J$. Find the maximum height to which the particle will rise above its point of projection. [$g$ = 10 $m/s^2$]

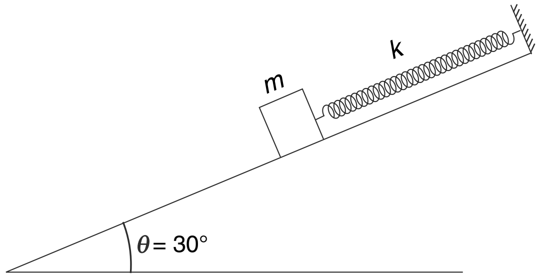
Q. 19. A physics student writes the elastic potential energy stored in a spring as $U = \frac{1}{2}KL^2 + \frac{1}{2}Kx^2$, where $L$ is the natural length of the spring, $x$ is extension or compression in it and $K$ is its force constant. A block of mass $M$ travelling with speed $V$ hits the spring and compresses it.




Find the maximum compression caused.

Q. 20. A block of mass $m$ = 4 $kg$ is kept on an incline connected to a spring (see fig). The angle of the incline is $\theta$ = 30° and the spring constant is

$K = 80\ N/m$. There is a very small friction between the block and the incline. The block is released with spring in natural length. Find the work done by the friction on the block till the block finally comes to rest.[$g = 10\ m/s^2$]

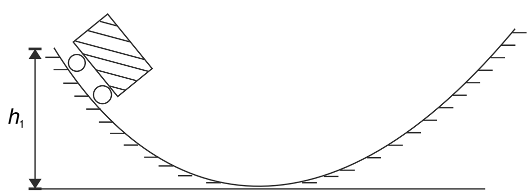



Q. 21. A body is projected directly up a plane which is inclined at an angle $\theta$ to the horizontal. It was found that when it returns to the starting point its speed is half its initial speed.

(a) Was dissipation of mechanical energy of the body, due to friction, higher during ascent or descent?

(b) Calculate the coefficient of friction ($\mu$) between the body and the incline.

Q. 22. A tanker filled with water starts at rest and then rolls, without any energy loss to friction, down a valley. Initial height of the tanker is $h_1$. The tanker, after coming down, climbs on the other side of the valley up to a height $h_2$. Throughout the journey, water leaks from the bottom of the tanker. How does $h_2$ compare with $h_1$?




Q. 23. A stone with weight $W$ is thrown vertically upward into air with initial speed $u$. Due to air friction a constant force $R$ acts on the stone, throughout its flight. Find –

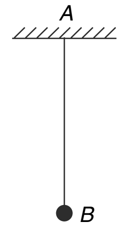
(a) the maximum height reached and

(b) speed of stone on reaching the ground.

Q. 24. A mass $m = 0.1\ kg$ is attached to the end $B$ of an elastic string $AB$ with stiffness $k = 16\ N/m$ and natural length $l_0 = 0.25\ m$. The end $A$ of the string is fixed. The mass is pulled down so that $AB$ is $2l_0 = 0.5\ m$ and then released.

(a) Find the velocity of the mass when the string gets slack for the first time.

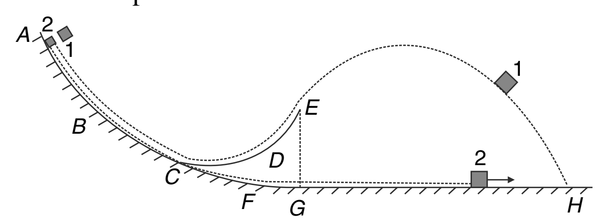
(b) At what distance from $A$ the mass will come to rest for the first time after being released.



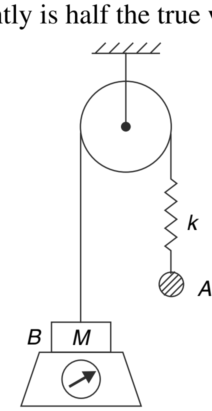

Q. 25. Two blocks 1 and 2 start from same point $A$ on a smooth slide at the same time. The track from $A$ to $B$ to $C$ is common for the two blocks. At $C$ the track divides into two parts. Block 1 takes the route $C$–$D$–$E$ and gets airborne after $E$. Block 2 moves along $CFGH$. Point $E$ is vertically above $G$ and the stretch $GH$ is horizontal. Block 1 lands at point $H$.

(a) Where is the other block at the time block 1 lands at $H$? Has it already crossed $H$ or yet to reach there?

(b) Which block will reach at $H$ with higher speed ?

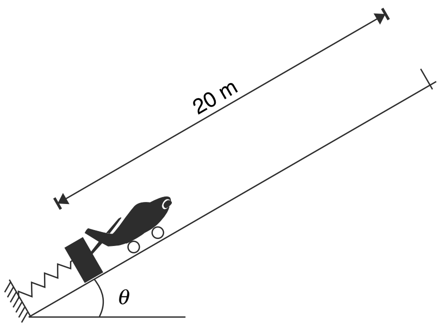



Q. 26 In the arrangement shown in the figure, block $B$ of mass $M$ rests on a weighing scale. Ball $A$ is released from a position where spring is in its natural length and the scale shows the correct weight of block $B$. Find the mass of ball $A$ so that the minimum reading shown by the scale subsequently is half the true weight of $B$.



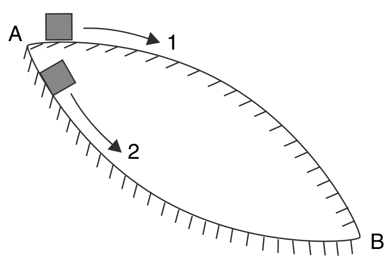

Q. 27 In an aircraft carrier warship the runway is a 20 $m$ long strip inclined at $\theta = 20°$ to the horizontal. The launcher is effectively a large spring that pushes an aircraft of mass $m = 2000$

*kg* for first 5 *m* of the 20 *m* long runway. The jet engine of the plane produces a constant thrust of $6 \times 10^4$ *N* for the entire length of the runway. The plane needs to have a speed of 180 *kph* at the end of the runway. Neglect air resistance and calculate the spring constant of the launcher. [sin 20° = 0.3 and $g = 10\ m/s^2$]




Q. 28 A block of mass *M* is placed on a horizontal surface having coefficient of friction $\mu$. A constant pulling force $F = \frac{Mg}{2}$ is applied on the block to displace it horizontally through a distance *d*. Find the maximum possible kinetic energy acquired by the block.

Q. 29 A small block is made to slide, starting from rest, along two equally rough circular surfaces from *A* to *B* through path 1 and 2. The two paths have equal radii. The speed of the block at the end of the slide was found to be $V_1$ and $V_2$ for path 1 and 2 respectively. Which one is larger $V_1$ or $V_2$?




Q. 30 A particle can move along x axis under influence of a conservative force. The potential energy of the particle is given by $U = 5x^2 - 20x + 2$ joule where *x* is co-ordinate of the particle expressed in meter.

The particle is released at $x = -3\ m$

(a) Find the maximum kinetic energy of the particle during subsequent motion.

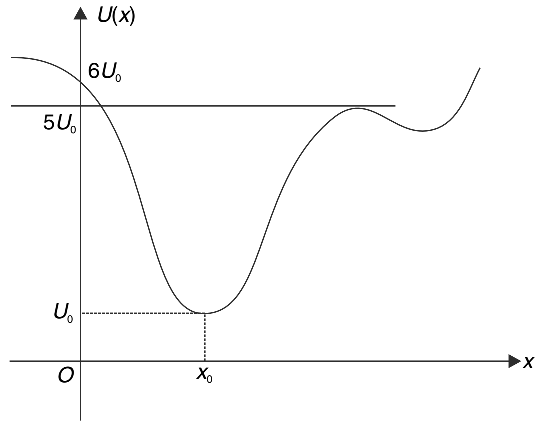
(b) Find the maximum *x* co-ordinate of the particle.

Q. 31 A particle is constrained to move along *x* axis under the action of a conservative force. The potential energy of the particle varies with position *x* as shown in the figure.

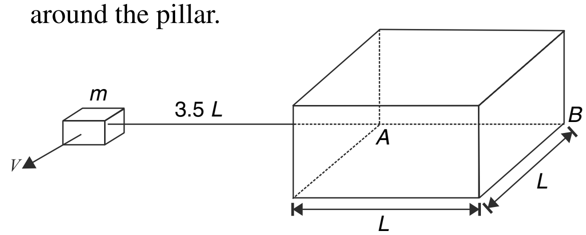
When the particle is at $x = x_0$, it is given a kinetic energy (*k*) such that $0 < k < 4U_0$

(a) Does the particle ever reach the origin?

(b) Qualitatively describe the motion of the particle.




Q. 32 A pillar having square cross section of side length *L* is fixed on a smooth floor. A particle of mass *m* is connected to a corner *A* of the pillar using an inextensible string of length 3.5 *L*. With the string just taut along the line *BA*, the particle is given a velocity *v* perpendicular to the string. The particle slides on the smooth floor and the string wraps around the pillar.




(a) Find the time in which the particle will hit the pillar.

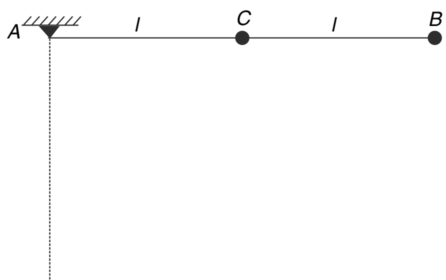
(b) Find the tension in the string just before the particle hits the pillar.

Neglect any energy loss of the particle.

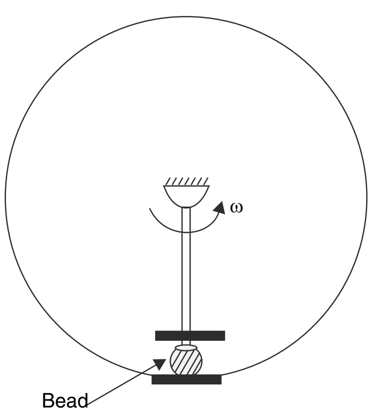
Q. 33 (i) A simple pendulum consist of a small bob of mass *m* tied to a string of length *L*. Show that the total energy of oscillation of the pendulum is $E \simeq \frac{1}{2} mg\, L\theta_0^2$ when it is oscillating with a small angular amplitude $\theta_0$. Assume the gravitational potential energy to be zero of the lowest position of the bob.

(ii) Three identical pendulums *A*, *B* and *C* are suspended from the ceiling of a room. They are swinging in semicircular arcs in vertical planes. The string of pendulum *A* snaps when it is vertical and it was found that the bob fell on the floor with speed $V_1$. The string of $B$ breaks when it makes an angle of 30° to the vertical and the bob hits the floor with speed $V_2$. The string of pendulum $C$ was cut when it was horizontal and the bob falls to the floor hitting it with a speed $V_3$. Which is greatest and which is smallest among $V_1, V_2$ and $V_3$?

Q. 34 $AB$ is a mass less rigid rod of length $2l$. It is free to rotate in vertical plane about a horizontal axis passing through its end $A$. Equal point masses ($m$ each) are stuck at the centre $C$ and end $B$of the rod. The rod is released from horizontal position. Write the tension in the rod when it becomes vertical.




Q. 35 A rigid mass less rod of length $L$ is rotating in a vertical plane about a horizontal axis passing through one of its ends. At the other end of the rod there is a mass less metal plate welded to the rod. This plate supports a heavy small bead that can slide on the rod without friction. Just above the bead there is another identical metal plate welded to the rod. The bead remains confined between the plates. The gap between the plates is negligible compared to $L$. The angular speed of the rod when the bead is at lowest position of the circle is $\omega = 2\sqrt{\frac{g}{L}}$ . How many times a clink of the bead hitting a metal plate is heard during one full rotation of the rod ?



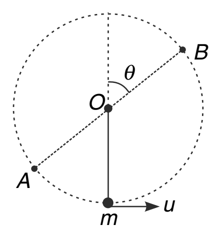

Q. 36 A child of mass $m$ is sitting on a swing suspended by a rope of length $L$. The swing and the rope have negligible mass and the dimension of child can be neglected. Mother of the child pulls the swing till the rope makes an angle of $\theta_0 = 1$ radian with the vertical. Now the mother pushes the swing along the arc of the circle with a force $F = \frac{mg}{2}$ and releases it when the string gets vertical. How high will the swing go?

[Take cos(1 radian) $\simeq$ 0.5]

Q. 37. A particle of mass $m$ is suspended by a string of length $l$ from a fixed rigid support. Particle is imparted a horizontal velocity $u = \sqrt{2gl}$ . Find the angle made by the string with the vertical when the acceleration of the particle is inclined to the string by 45°?

Q. 38 A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration $a_c$ is varying with time $t$ as $a_c = k^2rt^2$, where $k$ is a constant. Calculate the power delivered to the particle by the force acting on it.

Q. 39 A ball is hanging vertically by a light inextensible string of length $L$ from fixed point $O$. The ball of mass $m$ is given a speed u at the lowest position such that it completes a vertical circle with centre at $O$ as shown. Let $AB$ be a diameter of circular path of ball making an angle $\theta$ with vertical as shown. ($g$ is acceleration due to gravity)



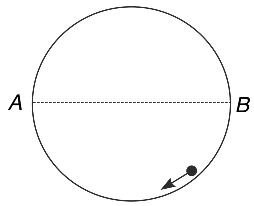

(a) Let $T_A$ and $T_B$ be tension in string when ball is at $A$ and $B$ respectively, then find $T_A - T_B$.

(b) Let $\vec{a}_A$ and $\vec{a}_B$ be acceleration of ball when it is at $A$ and $B$ respectively, then find the value of $|\vec{a}_A + \vec{a}_B|$.

Q. 40 A ball suspended by a thread swings in a vertical plane so that the magnitude of its total acceleration in the extreme position and lowest position are equal. Find the angle $\theta$ that the thread makes with the vertical in the extreme position.

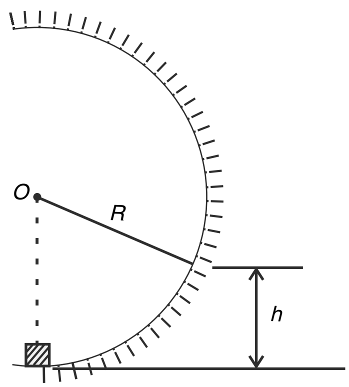
Q. 41 A particle of mass $m$ oscillates inside the smooth surface of a fixed pipe of radius $R$. The axis of the pipe is horizontal and the particle moves from $B$

to $A$ and back. At an instant the kinetic energy of the particle is $K$ (say at position of the particle shown in the figure). What is the force applied by particle on the pipe at this instant?



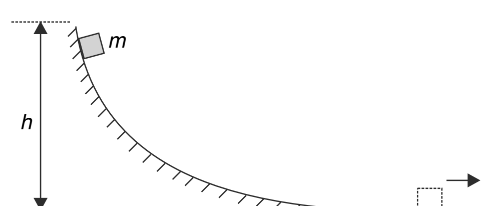

## LEVEL 2

Q. 42.

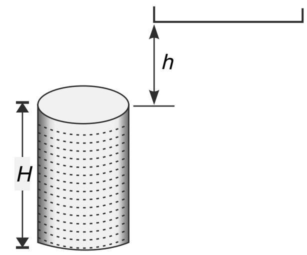



(i) There is a vertical loop of radius $R$. A small block of mass $m$ is slowly pushed along the loop from bottom to a point at height $h$. Find the work done by the external agent if the coefficient of friction is $\mu$. Assume that the external agent pushes tangentially along the path.

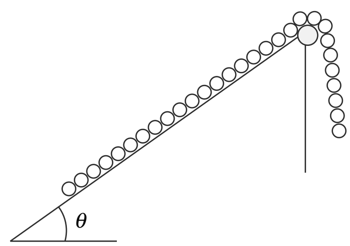
(ii) A block of mass $m$ slides down a smooth slope of height $h$, starting from rest. The lower part of the track is horizontal. In the beginning the block has potential energy $U = mgh$ which gets converted into kinetic energy at the bottom. The velocity at bottom is $v = \sqrt{2gh}$ . Now assume that an observer moving horizontally with velocity $v = \sqrt{2gh}$ towards right observes the sliding block. She finds that initial energy of the block is $E = mgh + \frac{1}{2}mv^2$ and the final energy of the block when it reaches the bottom of the track is zero. Where did the energy disappear?



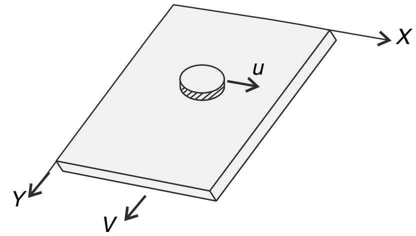

Q. 43. A completely filled cylindrical tank of height $H$ contains water of mass $M$. At a height $h$ above the top of the tank there is another wide container. The entire water from the tank is to be transferred into the container in time $t_0$ such that level of water in tank decreases at a uniform rate. How will the power of the external agent vary with time?




Q. 44. A uniform chain of mass $m_0$ and length $l$ rests on a rough incline with its part hanging vertically as shown in the fig. The chain starts sliding up the incline (and hanging part moving down) provided the hanging part equals $\eta$ times the chain length ($\eta < 1$). What is the work performed by the friction force by the time chain slides completely off the incline. Neglect the dimension of pulley and assume it to be smooth.




Q. 45. A large flat board is lying on a smooth ground. A disc of mass $m = 2\ kg$ is kept on the board. The coefficient of friction between the disc and the board is $\mu = 0.2$ . The disc and the board are moved with velocity $\vec{u} = 2\hat{i}\ \ ms^{-1}$ and $\vec{V} = 2\hat{j}\ ms^{-1}$ respectively [in reference frame of the ground]. Calculate the power of the external force applied on the disc and the force applied on the board. At what rate heat is being dissipated due to friction between the board and the disc? [$g = 10\ ms^{-2}$]




Q. 46. A car can pull a trailer of twice its mass up a certain slope at a maximum speed $V$. Without the trailer the maximum speed of the car, up the same slope is 2 $V$. The resistance to the motion is proportional to mass and square of speed. If the car (without trailer) starts to move down the same slope, with its engine shut off, prove that eventually it will acquire a constant speed. Find this speed.

Q. 47 Force acting on a particle in a two dimensional $XY$ space is given as $\vec{F} = \dfrac{3(X\hat{i} + Y\hat{j})}{(X^2 + Y^2)^{3/2}}$. Show that the force is conservative.

Q. 48. In a two dimensional space the potential energy function for a conservative force acting on a particle of mass $m$ = 0.1 $kg$ is given by $U = 2\,(x + y)$ joule ($x$ and $y$ are in m). The particle is being moved on a circular path at a constant speed of $V = 1\ ms^{-1}$. The equation of the circular path is $x^2 + y^2 = 4^2$.

(a) Find the net external force (other than the conservative force) that must be acting on the particle when the particle is at (0, 4).

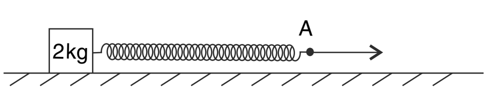
(b) Calculate the work done by the external force in moving the particle from (4, 0) to (0, 4).

Q. 49. A particle of mass $m$ moves in $xy$ plane such that its position vector, as a function of time, is given by $\vec{r} = b(kt - \sin kt)\hat{i} + b(kt + \cos kt)\hat{j}$; where $b$ and $k$ are positive constants.

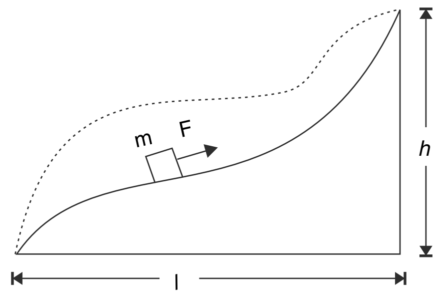
(a) Find the time $t_0$ in the interval $o \le t \le \dfrac{\pi}{k}$ when the resultant force acting on the particle has zero power.

(b) Find the work done by the resultant force acting on the particle in the interval $t_o \le t \le \dfrac{\pi}{k}$

Q.50. A block of mass 2 kg is connect to an ideal spring and the system is placed on a smooth horizontal surface. The spring is pulled to move the block and at an instant the speed of end A of the spring and speed of the block were measured to be 6 $m/s$ and 3 $m/s$ respectively. At this moment the potential energy stored in the spring is increasing at a rate of 15 $J/s$. Find the acceleration of the block at this instant.

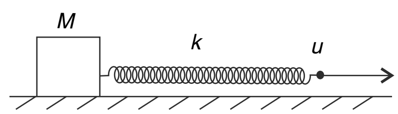



Q. 51. A body of mass $m$ is slowly hauled up a rough hill as shown in fig by a force $F$ which acts tangential to the trajectory at each point. Find the work performed by the force, if the height of hill is $h$, the length of its base $l$ and coefficient of friction between the body and hill surface is $\mu$. What is the work done if body is moved along some alternative path shown by the dotted line, friction coefficient being same.

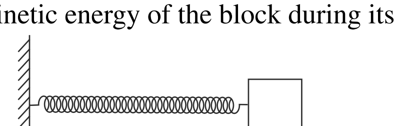



Q. 52. In previous problem what is the work done by $\vec{F}$ if the body started at rest at the base and has a velocity $v$ on reaching the top?

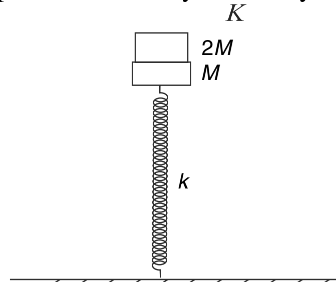
Q. 53. A block of mass $M$ is placed on a horizontal smooth table. It is attached to an ideal spring of force constant $k$ as shown. The free end of the spring is pulled at a constant speed $u$. Find the maximum extension ($x_o$) in the spring during the subsequent motion.




Q. 54. A spring block system is placed on a rough horizontal floor. Force constant of the spring is $k$. The block is pulled to right to give the spring an elongation equal to $x_0$ and then it is released. The block moves to left and stops at the position where the spring is relaxed. Calculate the maximum kinetic energy of the block during its motion.



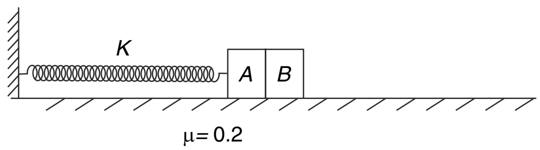
Q. 55. In the fig shown, a block of mass $M$ is attached to the spring and another block of mass $2M$ has been placed over it. The system is in equilibrium. The block are pushed down so that the spring compresses further by $\dfrac{9Mg}{K}$. System is released.



(a) At what height above the position of release, the block of mass $2M$ will lose contact with the other block?

(b) What is maximum height attained by $2M$ above the point of release?

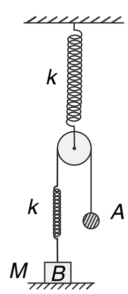
Q. 56. Block $A$ and $B$ are identical having 1 kg mass each. $A$ is tied to a spring of force constant $k$ and B is placed in front of $A$ (touching it). Block '$B$' is pushed to left so as to compress the spring by $0.1\,m$ from its natural length. The system is released from this position. Coefficient of friction for both the blocks with horizontal surface is $\mu = 0.2$.



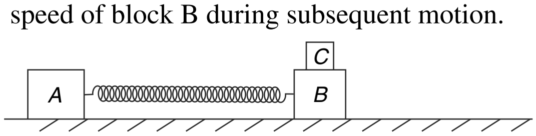

(a) Take $k = \frac{200}{3}\,N/m$. Kinetic energy of the system comprising of the two blocks will be maximum after travelling through a distance $x_0$ from the initial position. Find $x_0$. Find the contact force between the two blocks when they come to rest.

(b) Take $k = 100\,N/m$. What distance ($x_1$) will the block travel together, after being released, before $B$ separates from $A$.

Q. 57. In the arrangement shown in the fig. string, springs and the pulley are mass less. Both the springs have a force constant of $k$ and the mass of block B resting on the table is M. Ball A is released from rest when both the springs are in natural length and just taut. Find the minimum value of mass of A so that block B leaves contact with the table at some stage.



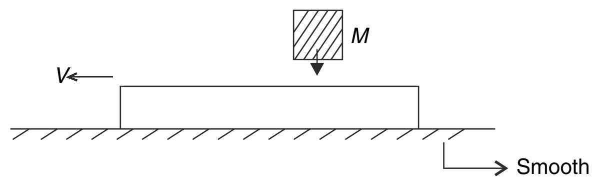

Q. 58. Two block $A$ and $B$ are connected to a spring (force constant $k = 480\,N/m$) and placed on a horizontal surface. Another block $C$ is placed on $B$. The coefficient of friction between the floor and block $A$ is $\mu_1 = 0.5$, whereas there is no friction between $B$ and the floor. Coefficient of friction between $C$ and $B$ is $\mu_2 = 0.85$. Masses of the blocks are $M_A = 50\,kg$; $M_B = 28\,kg$ and $M_C = 2\,kg$. The system is held at rest with spring compressed by $x_0 = 0.5\,m$. After the system is released, find the maximum speed of block B during subsequent motion.




Q. 59. A plank is moving along a smooth surface with a constant speed $V$. A block of mass $M$ is gently placed on it. Initially the block slips and then acquires the constant speed ($V$) same as the plank. Throughout the period, a horizontal force is applied on the plank to keep its speed constant.

(a) Find the work performed by the external force.

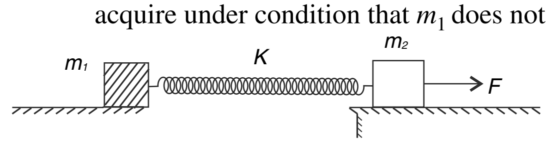
(b) Find the heat developed due to friction between the block and the plank.



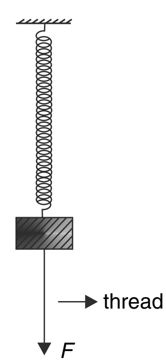

Q. 60. A block of mass $m_1$ is lying on the edge of a rough table. The coefficient of friction between the block and the table is $\mu$. Another block of mass $m_2$ is lying on another horizontal smooth table. The two block are connected by a horizontal spring of force constant $K$. Block of mass $m_2$ is pulled to the right with a constant horizontal force $F$.

(a) Find the maximum value of $F$ for which the block of mass $m_1$ does not fall off the edge.

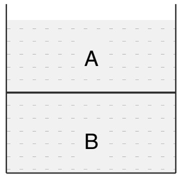
(b) Calculate the maximum speed that $m_2$ can acquire under condition that $m_1$ does not fall.




Q. 61. A vertical spring supports a block in equilibrium. The spring is designed to break when extension in it crosses a limit. There is a light thread attached to the block as shown. The thread is pulled down with a force $F$ which gradually increases from zero. The spring breaks when the force becomes $F_0$. Instead of gradually increasing the force, if the thread were pulled by applying a constant force, for what minimum value of the constant force the spring will break?




Q. 62 Two liquid $A$ & $B$ having densities $2\rho$ and $\rho$ respectively, are kept in a cylindrical container separated by a partition as shown in figure. The height of each liquid in the container is $h$ and area of cross section of the container is $A$. Now the partition is removed. Calculate change in gravitational potential energy ($\Delta U$) of the system



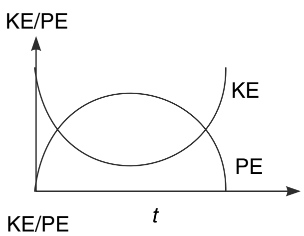

(a) assuming that the two liquids mix uniformly.

(b) Assuming that the two liquids are immiscible.

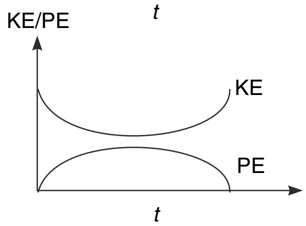
What do you conclude from the sign of $\Delta U$ in the above two cases?

Q. 63. A particle is projected at an angle $\theta$ = 30º with the horizontal. Two students A and B have drawn the variation of kinetic energy and gravitational potential energy of the particle as a function of time taking the point of projection as the reference level for the gravitational potential energy. Who is wrong and why?

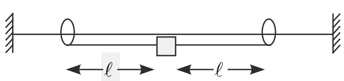
(a)




(b)



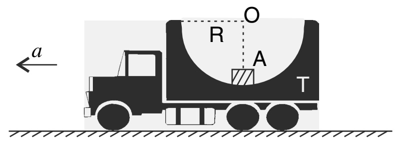

Q. 64. Two small rings each of mass '$m$' are connected to a block of same mass '$m$' through inextensible light strings. Rings are constrained to move along a smooth horizontal rod. Initially system is held at rest (as shown in figure) with the strings just taut. Length of each string is '$\ell$'. The system is released from the position shown. Find the speed of the block ($v$) and speed of the rings ($u$) when the strings make an angle of $\theta$ = 60º with vertical. (Take $g$ = 10 $m/s^2$)



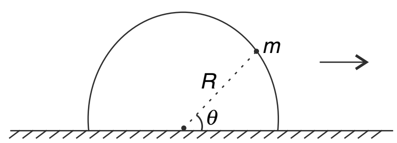

Q. 65. A toy truck $T$ at rest, has a hemispherical trough of radius $R$ in it [$O$ is the centre of the hemisphere]. A small block $A$ is kept at the bottom of the trough. The truck is accelerated horizontally with an acceleration $a$.

(i) Find the minimum value of $a$ for which the block is able to move out of the trolley.

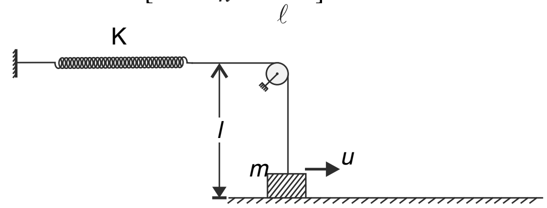
(ii) If magnitude of $a$ is twice the minimum value found in (i), find the maximum height (measured from its original level at the bottom of the trough) to which the block will rise.



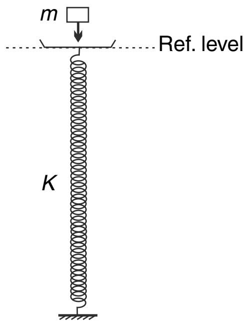

Q. 66. A semicircular wire frame of radius $R$ is standing vertical on a horizontal table. It is pulled horizontally towards right with a constant acceleration. A bead of mass m remain in equilibrium (relative to the semicircular wire) at a position where radius makes an angle $\theta_0$ with horizontal. There is no friction between the wire and the bead. The bead is displaced a little bit in upward direction and released. Calculate the speed of the bead relative to the wire at the instant it strikes the table. Assume that all throughout the semicircular wire keeps moving with constant acceleration.



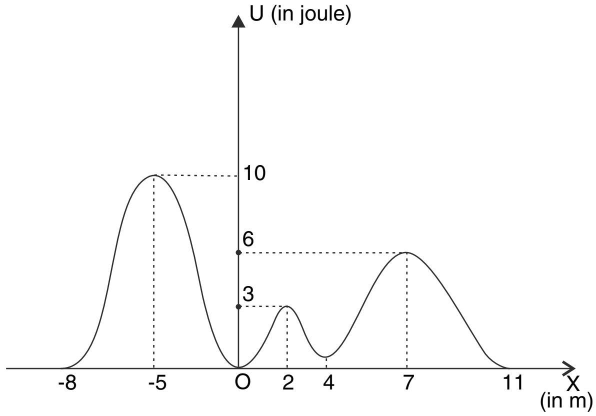

Q. 67. A ideal spring of force constant $k$ is connected to a small block of mass $m$ using an inextensible light string (see fig). The pulley is mass less and friction coefficient between the block and the horizontal surface is $\mu = \frac{1}{\sqrt{3}}$. The string between the pulley and the block is vertical and has length $l$. Find the minimum velocity $u$ that must be given to the block in horizontal direction shown, so that subsequently it leaves contact with the horizontal surface. [Take $k = \frac{2mg}{\ell}$ ]




Q. 68. A light spring is vertical and a mass less pan is attached to it. Force constant of the spring is $k$. A block of mass $m$ is gently dropped on the pan. Plot the variation of spring potential energy, gravitation potential energy and the total potential energy of the system as a function of displacement ($x$) of the block. For gravitational potential energy take reference level to be the initial position of the pan.




Q. 69. A particle of mass $m = 1.0\ kg$ is free to move along the $x$ axis. It is acted upon by a force which is described by the potential energy function represented in the graph below. The particle is projected towards left with a speed $v$, from the origin. Find minimum value of $v$ for which the particle will escape far away from the origin.




Q. 70. A particle of mass $m = 1\ kg$ is free to move along $x$ axis under influence of a conservative force. The potential energy function for the particle is

$$U = a\left[\left(\frac{x}{b}\right)^4 - 5\left(\frac{x}{b}\right)^2\right]\text{joule}$$

Where $b = 1.0\ m$ and $a = 1.0\ J$. If the total mechanical energy of the particle is zero, find the co-ordinates where we can expect to find the particle and also calculate the maximum speed of the particle.

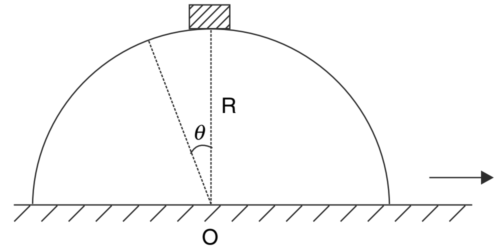
Q. 71. A particle of mass $m$ moves under the action of a central force. The potential energy function is given by $U(r) = mkr^3$

Where $k$ is a positive constant and $r$ is distance of the particle from the centre of attraction.

(a) What should be the kinetic energy of the particle so that it moves in a circle of radius $a_0$ about the centre of attraction?

(b) What is the period of this circular motion?

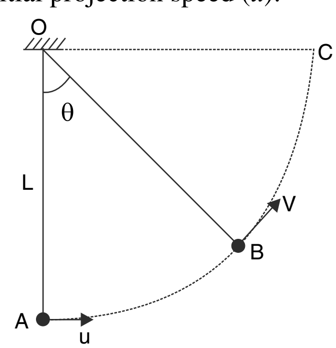
Q. 72. A small block is placed on the top of a smooth inverted hemispherical bowl of radius $R$.




(a) The bowl is given a sudden impulse so that it begins moving horizontally with speed $V$. Find minimum value of $V$ so that the block immediately loses contact with the bowl as it begins to move.

(b) The bowl is given a constant acceleration '$a$' in horizontal direction. Find maximum value of '$a$' so that the block does not lose contact with the bowl by the time it rotates through an angle $\theta = 1°$ relative to the bowl. You can make suitable mathematical approximations justified for small value of angle $\theta$.

Q. 73 A pendulum bob is projected form its lowest position with velocity ($u$), in horizontal direction, that is just enough to make the string horizontal (position $OC$). At angular position $\theta$, at point $B$, the speed ($V$) of the bob was observed to be half its initial projection speed ($u$).

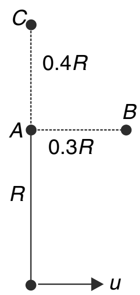



(a) Find $\theta$

(b) Plot variation of magnitude of tangential acceleration with $\theta$.

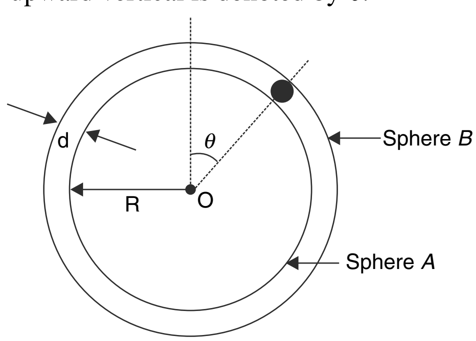
(c) Let the travel time from $A$ to $B$ be $t_1$ and that from $B$ to $C$ be $t_2$. Looking at the graph obtained in part ($b$), tell which is larger $-t_1$ or $t_2$?

Q. 74. A small ball is attached to an end of a light string of length $R$. It is suspended in vertical plane supported at point $A$. $B$ and $C$ are two nails (of negligible thickness) at a horizontal distance 0.3 $R$ from $A$ and $a$ vertical distance 0.4 $R$ above $A$ respectively. The ball is given a horizontal velocity $u = \sqrt{5gR}$ at its lowest point. Subsequently, after the string hitting the nails, the nails become the centre of rotation. Assume no loss in kinetic energy when the string hits the nails. It is known that the string will break if tension in it is suddenly increased by 200% or more.




Will the string break during the motion? If yes, where? What is tension in the string at the instant the string breaks?

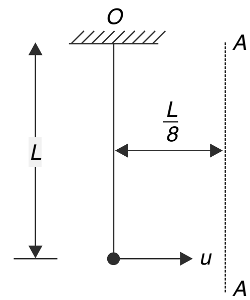
Q. 75. A spherical ball of mass $m$ is kept at the highest point in space between two fixed concentric spheres $A$ and $B$ (see figure). The smaller sphere has a radius $R$ and the space between the two spheres has a width $d$. The ball has diameter just less than $d$. All surfaces are frictionless. The ball is given a gentle push (towards the right). The angle made by the radius vector of the ball with upward vertical is denoted by $\theta$.




(a) Express the total normal reaction force exerted by the spheres as a function of $\theta$.

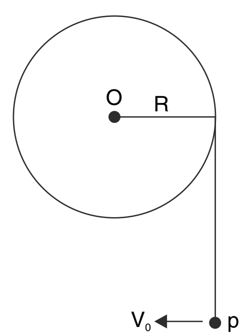
(b) Let $N_A$ and $N_B$ denote the magnitudes of normal reaction forces on the ball exerted by the spheres $A$ and $B$ respectively. Sketch the variations of $N_A$ and $N_B$ as function of cos $\theta$ in the range of $0 \leq \theta \leq \pi$ by drawing two separate graphs.

Q. 76. A particle is suspended vertically from a point $O$ by an inextensible mass less string of length $L$. A vertical line $AB$ is at a distance of $\frac{L}{8}$ from $O$ as shown in figure. The particle is given a horizontal velocity $u$. At some point, its motion ceases to be circular and eventually the object passes through the line $AB$. At the instant of crossing $AB$, its velocity is horizontal. Find $u$.




Q. 77 A simple pendulum has a bob of mass $m$ and string of length $R$. The bob is projected from lowest position giving it a horizontal velocity just enough for it to complete the vertical circle. Let the angular displacement of the pendulum from its initial vertical position be represented by $\theta$. Plot the variation of kinetic energy ($kE$) of the bob and the tension ($T$) in the string with $\theta$. Plot the graph for one complete rotation of the pendulum.

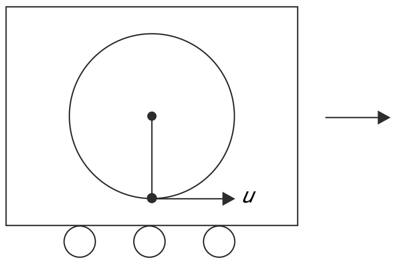
Q. 78 A light thread is tightly wrapped around a fixed disc of radius $R$. A particle of mass $m$ is tied to the end $P$ of the thread and the vertically hanging part of the string has length $\pi R$. The particle is imparted a horizontal velocity $V = \sqrt{\frac{4\pi gR}{3}}$. The string wraps around the disc as the particle moves up. At the instant the velocity of the particle makes an angle of $\theta = 60°$ with horizontal, calculate.



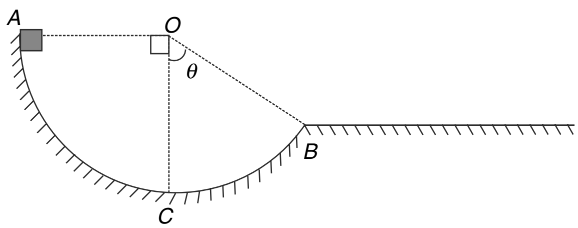

(a) speed of the particle

(b) tension in the string

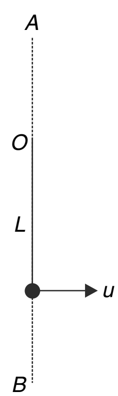
Q. 79 An experimenter is inside a train. He observes that minimum speed at lowest position needed by a pendulum bob to complete a vertical circle is 10 $m/s$. Calculate the minimum speed ($u$) needed at the lowest position so as to complete the vertical circle when the train is moving horizontally at an acceleration of $a$ = 7.5 $m/s^2$. Find the maximum tension in the string during the motion. [$g = 10\ m/s^2$].




Q. 80 A track (*ACB*) is in the shape of an arc of a circle. It is held fixed in vertical plane with its radius *OA* horizontal. *A* small block is released on the inner surface of the track from point *A*. It slides without friction and leaves the track at *B*. What should be value of $\theta$ so that the block travels the largest horizontal distance by the time it returns to the horizontal plane passing through *B*?



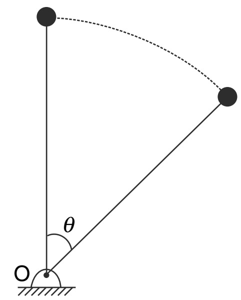

Q. 81 Bob of a simple pendulum of length *L* is projected horizontally with a speed of $u = \sqrt{4gL}$ , from the lowest position. Find the distance of the bob from vertical line *AB*, at the moment its tangential acceleration becomes zero.



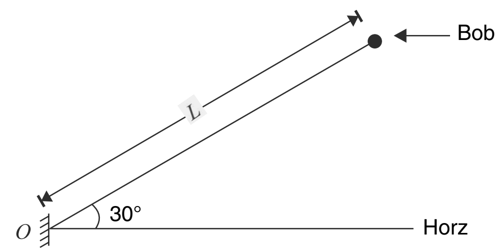

Q. 82 A light rigid rod has a bob of mass *m* attached to one of its end. The other end of the rod is pivoted so that the entire assembly can rotate freely in a vertical plane. Initially, the rod is held vertical as shown in the figure. From this position it is allowed to fall.

(a) When the rod has rotated through $\theta = 30°$, what kind of force does it experience– compression or tension?

(b) At what value of $\theta$ the compression (or tension) in the rod changes to tension (or compression)?




Q. 83 A pendulum has length $L = 1.8\ m$. The bob is released from position shown in the figure. Find the tension in the string when the bob reaches the lowest position. Mass of the bob is 1 *kg*.



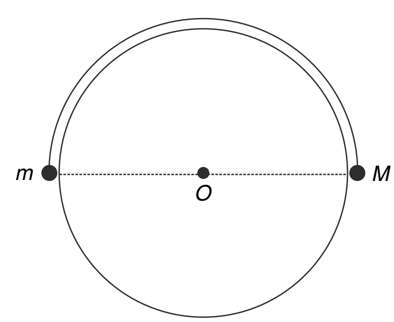

Q. 84 A small body of mass *m* lies on a horizontal plane. The body is given a velocity $v_0$, along the plane.

(a) Find the mean power developed by the friction during the whole time of motion, if friction coefficient is $\mu = 0.3$; $m = 2.0\ kg$ and $v_0 = 3\ m/s$.

(b) Find the maximum instantaneous power developed by the friction force, if the friction coefficient varies as $\mu = \alpha x$, where $\alpha$ is a constant and *x* is distance from the starting point.

Q. 85 Two particles of masses *M* and *m* ($M > m$) are connected by a light string of length $\pi R$.

The string is hung over a fixed circular frame of radius *R*.



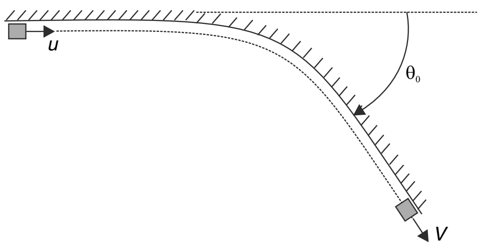

Initially the particles lie at the ends of the horizontal diameter of the circle (see figure). Neglect friction.

(a) If the system is released, and if $m$ remains in contact with the circle, find the speed of the masses when $M$ has descended through a distance $R\theta$ ($\theta < \pi$).

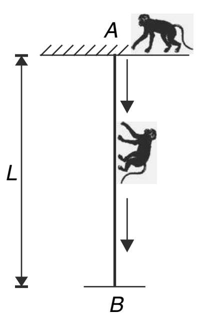
(b) Find the reaction force between the frame and $m$ at this instant.

(c) Prove that $m_1$ will certainly remain in contact with the frame, just after the release, if $3m > M$.

Q. 86 A small object is sliding on a smooth horizontal floor along a vertical wall. The wall makes a smooth turn by an angle $\theta_0$. Coefficient of friction between the wall and the block is μ. Speed of the object before the turn is $u$. Find its speed ($V$) just after completing the turn. Does your answer depend on shape of the curve? [The turn is smooth and there are no sharp corners.]

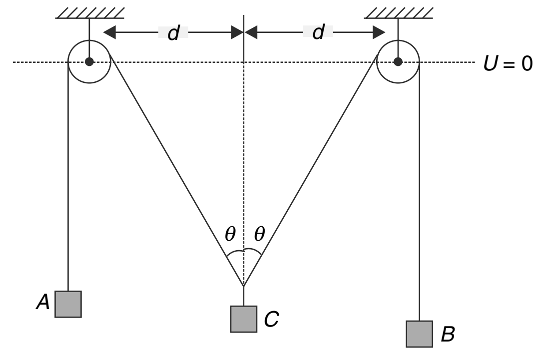



Q. 87 $AB$ is a vertically suspended elastic cord of negligible mass and length $L$. Its force constant is $k = \frac{4mg}{L}$. There is a massless platform attached to the lower end of the cord. A monkey of mass $m$ starts from top end $A$ and slides down the cord with a uniform acceleration of $\frac{g}{2}$. Just before landing on the platform, the monkey loses grip on the cord. After landing on the platform the monkey stays on it. Calculate the maximum extension in the elastic cord.




## LEVEL 3

Q. 88 In the arrangement shown in the fig. all the three blocks have equal mass $m$. The length of the strings connecting $A$ to $C$ and $B$ to $C$ is $L$ each. Assume the gravitational potential energy of any mass at the level of the pulleys to be zero. Neglect dimension of the pulley and treat the strings to be massless. Distance between the pulleys is $2d$.

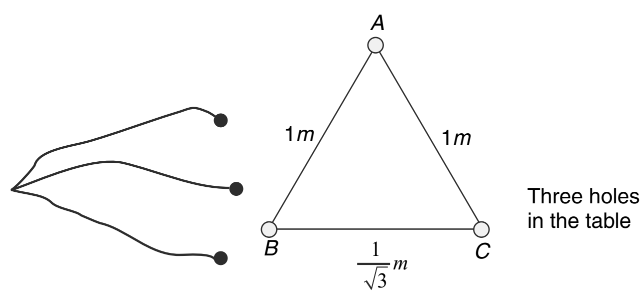



(a) Write the potential energy of the system as a function of angle $\theta$.

(b) Knowing that potential energy of the system will be maximum or minimum in equilibrium position, find value of $\theta$ for equilibrium.

(c) Tell if the equilibrium is stable or unstable.

Q. 89 Three identical masses are attached to the ends of light strings, the other ends of which are connected together as shown in the figure. Each of the three strings has a length of 3 $m$. The three masses are dropped through three holes in a table and the system is allowed to reach equilibrium.




(a) What is total length of the strings lying on the table in equilibrium?

(b) Select a point $K$ inside the $\Delta ABC$ such that $AK + BK + CK$ is minimum, use the result obtained in (a) and the fact that potential energy of the system will be minimum when it is in equilibrium.

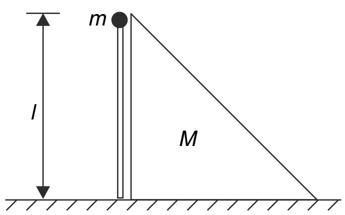
Q. 90 A particle of mass $m$ is attached to an end of a light rigid rod of length $a$. The other end of the rod is fixed, so that the rod can rotate freely in vertical plane about its fixed end. The mass $m$ is given a horizontal velocity $u$ at the lowest point.

(a) Prove that when the radius to the mass makes an angle $\theta$ with the upward vertical the horizontal component of the acceleration of the mass (measured in direction of $u$) is

$[g(2 + 3\cos\theta) - u^2/a]\sin\theta$

(b) If $4ag < u^2 < 5ag$, show that there are four points at which horizontal component of acceleration is zero. Locate the points.

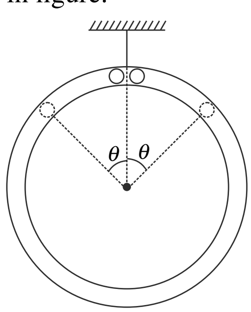
Q. 91 A weightless rod of length $l$ with a small load of mass $m$ at one of its end is held vertical with its lower end hinged on a horizontal surface. The load touches a wedge of mass $M$ in this position. A slight jerk towards right sets the system in motion (see figure), with rod rotating freely in vertical plane about its lower end. There is no friction.




(a) For what mass ratio $\frac{M}{m}$ will the rod form an angle $\theta = \pi/3$ with the vertical at the moment the load separates from the wedge?

(b) What is speed of the wedge at that moment? Neglect friction.

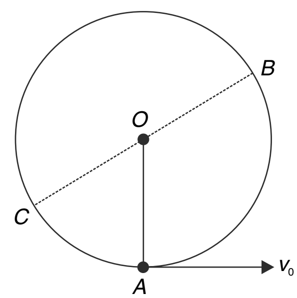
Q. 92 A tube of mass $M$ hangs from a thread and two balls of mass $m$ slide inside it without friction (see figure). The balls are released simultaneously from the top of the ring and slide down on opposite sides. $\theta$ defines the positions of balls at any time as shown in figure.




(a) Show that ring will start to rise if $m \geq \frac{3M}{2}$.

(b) If $M = 0$, find the angle $\theta$ at which the tube begins to rise.

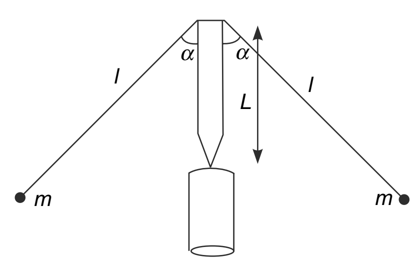
Q. 93 A heavy particle is attached to one end of a light string of length $l$ whose other end is fixed at $O$. The particle is projected horizontally with a velocity $v_0$ from its lowest position $A$. When the angular displacement of the string is more than 90°, the particle leaves the circular path at $B$. The string again becomes taut at $C$ such that $B, O, C$ are collinear. Find $v_0$ in terms of $l$ and $g$.




Q. 94. The teeter toy consists of two identical weights hanging from a peg on dropping arms as shown. The arrangement is surprisingly stable. Let us consider only oscillatory motion in the vertical plane. Consider the peg and rods (connecting the weights to the peg) to be very light. The length of each rod is $l$ and length of the peg is $L$. In the position shown the peg is vertical and the two weights are in a position lower than the support point of the peg. Angle $\alpha$ that the rods make with the peg remains fixed.

(a) Assuming the zero of gravitational potential energy at the support point of the peg evaluate the potential energy ($U$) when the peg is tilted to an angle $\theta$ to the vertical. The tip of the peg does not move.

(b) Knowing that U shall be minimum in stable equilibrium position prove that $\theta = 0$ is the stable equilibrium position for the toy if the two weights are in a position lower than the support point of the peg.



# ANSWERS

**1.** (i) Akanksha is right.

(ii) (a) $-mgh$

(b) 0

(c) internal (muscle) forces of the body perform work

**2.** Zero

**3** (a) 960 $J$

(b) zero

**4.** $V = \sqrt{\frac{F_o L}{m}}$ –

**5.** (a) $\overrightarrow{V_A} = \left(4\hat{i} + 3\hat{j} + 2\hat{k}\right) m/s$

(b) $\overrightarrow{r_A} = \left(-\hat{i} + 3\hat{j} - 4\hat{k}\right) m$

(c) $W = 138 J$

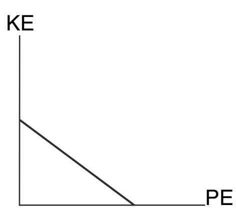
(d) $\Delta U_g = -60\ J$

**6.** No, the force is decreasing with time.

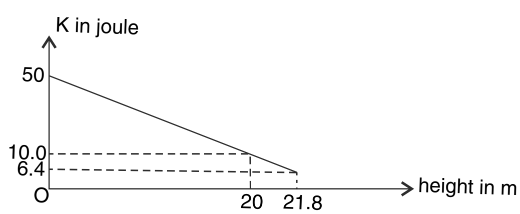
**7.** $x = \frac{7mv^3}{3P}$

**8.** The power will not change but work done will decrease.

**9.** (i)




(ii)




**10.** (a) 53 KW

(b) $\theta = 1.43°$

**11.** $U_B = 50\ J$ ; $U_D = -35\ J$

**12.** (b), (c)

**13.** 0.5 $m/s$

**14.** When a small amount of water evaporates, the spring relaxes a little bit. Water remaining in the beaker gains gravitational potential energy. Therefore, the spring energy gets converted into the gravitational potential energy of beaker + water.

**15.** $\frac{3}{4} Mg L_o$

**16.** (a) $\frac{MgL}{16}$ (b) Yes.

**17.** $\frac{\lambda L g}{4}$

**18.** 20 m

**19.** $\sqrt{\frac{M}{K}}.V$

**20.** $-5\ J$

**21.** (a) Same in both

(b) $\mu = \frac{3}{5}\tan\theta$

**22.** $h_2 = h_1$

**23.** (a) $\frac{Wu^2}{2g(W+R)}$

(b) $v = u\left(\frac{W-R}{W+R}\right)^{1/2}$

**24.** (a) $\sqrt{5}\ m/s$

(b) zero

**25.** (a) Block 2 has already crossed $H$.

(b) Both reach $H$ with same speed.

**26.** $\frac{M}{4}$

**27.** $k = 2.096 \times 10^5\ N/m$

**28.** $Mgd\left[\frac{\sqrt{\mu^2+1}}{2} - \mu\right]$

**29.** $V_1$

**30.** (a) $K_{max} = 125\ J$

(b) $X_{max} = 7m$

**31.** (a) No

(b) Oscillations about $x_0$

**32.** (a) $t = \frac{4\pi L}{v}$

(b) $T = \frac{2mv^2}{L}$

**33.** (i) $V_1 = V_2 = V_3$

**34.** Tension in segment $AC = \frac{28}{5}mg$

Tension in Segment $BC = \frac{17}{5}mg$

**35.** 2

**36.** The swing gets horizontal

**37.** $\tan^{-1}2$

**38.** $mk^2r^2t$

**39.** (a) $6\,mg\cos\theta$

(b) $g\sqrt{4+12\cos^2\theta}$

**40.** $2\tan^{-1}\left(\frac{1}{2}\right)$

**41.** $\frac{3K}{R}$

**42.** (i) $mgh+\mu mg\sqrt{2Rh-h^2}$

**43.** $P=\frac{Mg}{t_0}\left(h+H\frac{t}{t_0}\right)$

**44.** $-\frac{l(1-\eta)[\eta-(1-\eta)\sin\theta}{2}m_0g$

**45.** $4\sqrt{2}\;W,\;4\sqrt{2}\,W\;;\;8\sqrt{2}\,W$

**46.** $u=\sqrt{5}\,V$

**48.** (a) $\left(2\hat{i}+1.975\hat{j}\right)N$

(b) Zero

**49.** (a) $t_o=\frac{\pi}{4k}$

(b) $W=\left(\frac{1+2\sqrt{2}}{2}\right)mb^2k^2$

**50.** 2.5 $m/s^2$

**51.** $mg\left(h+\mu\ell\right)$. Work done is path independent and will be same for the alternative path

**52.** $\frac{1}{2}mv^2+mg\left(h+\mu\ell\right)$

**53.** $x_o=\sqrt{\frac{M}{k}}\;u$

**54.** $K_{\max}=\frac{1}{8}kx_o^2$

**55.** (a) $\frac{12Mg}{k}$

(b) $\frac{24Mg}{k}$

**57.** $m=\frac{M}{2}$

**58.** 4 $m/s$

**59.** (a) $MV^2$

(b) $\frac{1}{2}MV^2$

**60.** (a) $\frac{\mu m_1 g}{2}$

(b) $\frac{\mu m_1 g}{2}\;\frac{1}{\sqrt{m_2k}}$

**61.** $\frac{F_0}{2}$

**62.** (a) $\frac{1}{2}\rho Ah^2g$

(b) $-\rho Ah^2 g$

The positive sign of $\Delta U$ means external work will be required to mix the two liquids uniformly.

$\Delta U$ is negative in second case which means the heavier liquid will automatically move to lower side.

**63.** A is wrong. Under given conditions the two curves cannot touch.

**64.** $u=\sqrt{\frac{g\ell}{5}},\;v=\sqrt{\frac{3g\ell}{5}}$

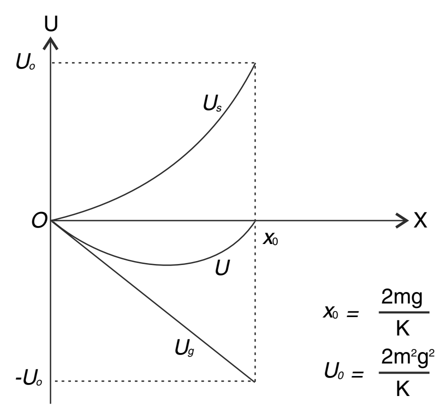
**65.** (i) $a_{\min}=g$

(ii) $2R$

**66.** $\sqrt{2gR\left(\frac{1+\cos\theta_0}{\sin\theta_0}\right)}$

**67.** $\mu=2\sqrt{gl}$

**68.**




**69.** $2\sqrt{3}$ $m/s$

**70.** $-\sqrt{5}\le x\le\sqrt{5};\;v_{\max}=5/\sqrt{2}\;ms^{-1}$

**71.** (a) $\frac{3}{2}km\,a_0^3$

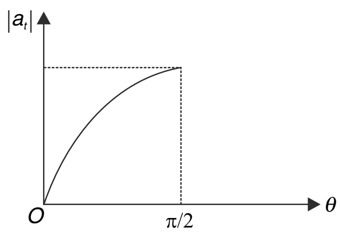
(b) $\frac{2\pi}{\sqrt{3ka_0}}$

**72.** (a) $\sqrt{Rg}$

(b) $\frac{60g}{\pi}$

**73.** (a) $\theta = \cos^{-1}\left(\frac{1}{4}\right)$

(b)

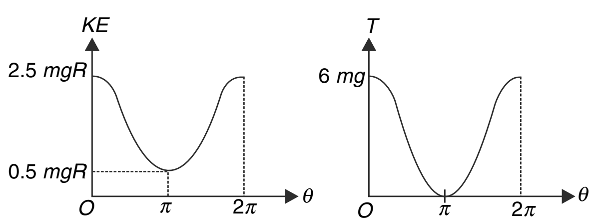



(c) $t_1 > t_2$

**74.** The string will break on hitting the second nail at $C$. $T = 8.6\,mg$

**76.** $\sqrt{gL\left(2+\frac{3\sqrt{3}}{2}\right)}$

**77.**




**78.** (a) $\sqrt{\sqrt{3}gR}$

(b) $mg\left(\sqrt{3}+\frac{1}{2}\right)$

**79.** $u = \sqrt{115}\ m/s$

**80.** $\theta = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

**81.** $\frac{5\sqrt{5}}{27}L$

**82.** (a) Compression

(b) $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

**83.** 35 $N$

**84.** (a) –9 Watt

(b) $-\frac{1}{2}mv_0^2\sqrt{\alpha g}$

**85.** (a) $\sqrt{2gR\frac{(M\theta - m\sin\theta)}{M+m}}$

(b) $\frac{mg}{M+m}\left[(M+3m)\sin\theta - 2M\theta\right]$

**86.** $V = ue^{-\mu\theta_0}$; No the answer does not depend on the shape of the curve.

**87.** $\frac{L}{8}\left(2+\sqrt{19}\right)$

**88.** (a) $U = -2mgL + mgd\left(\frac{2-\cos\theta}{\sin\theta}\right)$

(b) $\theta = 60°$

(c) Stable

**89.** (a) $\frac{1}{2}\left[1+\sqrt{\frac{11}{3}}\right]$

**90.** The four points are represented by –

$\theta = 0$, $\pi$, $\cos^{-1}\left(\frac{u^2-2ag}{3ag}\right)$ and

$\left[2\pi - \cos^{-1}\left(\frac{u^2-2ag}{3ag}\right)\right]$,

**92.** $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

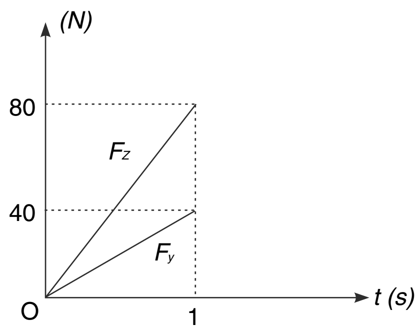
**93.** $\frac{(4+3\sqrt{2})}{2}gl$

**94.** (a) 2 mg cos $\theta$ [L – $l$ cos α]

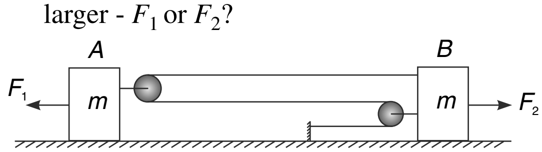
# 05 MOMENTUM AND CENTER OF MASS

## LEVEL 1

Q. 1. A particle is acted upon by a force for 1 second whose $X$ component remains constant at $F_x$ = 30 $N$ but $y$ and $z$ components vary with time as shown in the graph. Calculate the magnitude of change in momentum of the particle in 1 $s$. What angle does the change in momentum ($\Delta \vec{P}$ )make with $X$ axis?




Q. 2. Two block $A$ and $B$ of equal mass are connected using a light inextensible string passing over two light smooth pulleys fixed to the blocks (see fig). The horizontal surface is smooth. Every segment of the string (that is not touching the pulley) is horizontal. When a horizontal force $F_1$ is applied to $A$ the magnitude of momentum of the system, comprising of $A + B$, changes at a rate $R$. When a horizontal force $F_2$ is applied to $B$ ($F_1$ not applied) the magnitude of momentum of the system $A + B$ once again changes at the rate $R$. Which force is larger - $F_1$ or $F_2$?




Q. 3. A particle of mass $m$ = 1 $kg$ is moving in space in $X$ direction with a velocity of 10 $ms^{-1}$. A 4 $N$ force acting in $Y$ direction is applied on it for a time interval of 5.0 $s$. Later a 5 $N$ force was applied on it in Z direction for 4.0 $s$

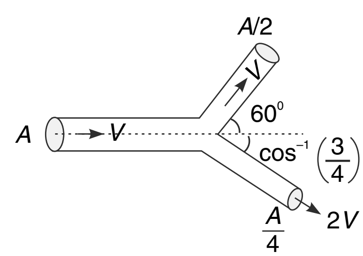
(a) Calculate the total work done by both the forces.

(b) Which force performed greater work?

Q. 4. An observer $O_1$ standing on ground finds that momentum of a projectile of mass 2 $kg$ changes with time as $\vec{P}_{01} = \left(4t\ \hat{i} + 20\,t\,\hat{k}\right) kg\ m/s$ Acceleration due to gravity is $\vec{g} = \left(10\hat{k}\right)$ $m/s^2$ and there is a wind blowing in horizontal direction. Another observer $O_2$ driving a car observes that momentum of the same projectile changes with time as -

$\vec{P}_{02} = \left(8t\,\hat{i} - 16t^2\ \hat{j} + 20t\,\hat{k}\right) kg\,m/s$. Find the acceleration of the car at $t = \dfrac{1}{8} s$

Q. 5. Water flows through a tube assembly as shown in the fig. Speed of flow (marked as $V$ and $2V$), cross sectional area ($A$, $A/2$ and $A/4$) and the angles between segments has been shown in fig. Calculate the force applied by the water flow on the tube. Take density of water to be $\rho$.

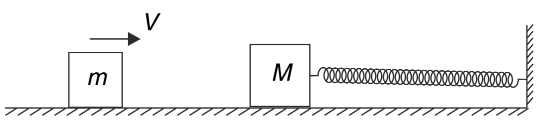



Q. 6. A man is running along a road with speed $u$. On his chest there is a paper of mass $m$ and area $S$. There is a wind blowing against the man at speed $V$. Density of air is $\rho$. Assume that the air molecules after striking the paper come to rest relative to the man. Find the minimum coefficient of friction between the paper and the chest so that the paper does not fall?

Q. 7. Two particles $A$ and $B$ of mass 2 $m$ and m respectively attract each other by mutual gravitational force and no other force acts on them. At time $t = 0$, A was observed to be at rest and B was moving away from A with a speed u. At a later time $t$ it was observed that $B$ was moving towards $A$ with speed u. Assume no collision has taken place by then. Find work done by the gravitational force in the time interval 0 to $t$.

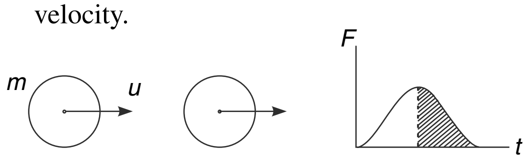
Q. 8. (i) A block of mass m moving towards right with a velocity $V$ strikes (head on) another block of mass $M$ which is at rest connected to a spring. The coefficient of restitution for collision between the blocks is $e = 0.5$.

Find the ratio $\frac{M}{m}$ for which the subsequent compression in the spring is maximum. There is no friction.



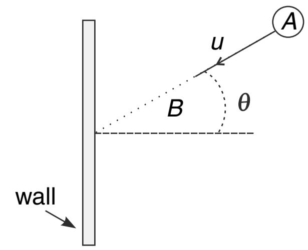

(ii) Ball A collides head on with another identical ball B at rest. Find the coefficient of restitution if ball $B$ has 80% of the total kinetic energy of the system after collision.

Q. 9. A ball having mass $m$ and velocity $u$ makes a head on collision with another ball. After collision velocity of the ball of mass m was found to be $V$ in the direction of its original motion. The interaction force between the two balls during their collision has been shown in the graph. The area of the shaded part of the graph is same as the area of the not shaded part. Find the velocity of the balls at the instant they were having equal velocity.

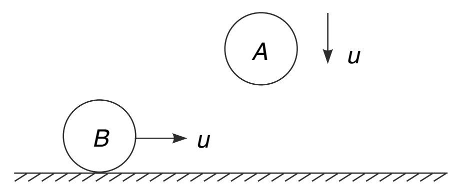



Q. 10. Ball A is about to hit a wall at an angle of incidence of $\theta = 30°$. But before hitting the wall it made a head on collision with another identical ball $B$. The ball B then collides with the wall. The coefficient of restitution for collision between two balls is $e_1 = 0.8$ and that between a ball and the wall is $e_1 = 0.6$. Find the final velocity of ball $B$. Initial velocity of $A$ was $u = 5\ ms^{-1}$. Neglect friction.

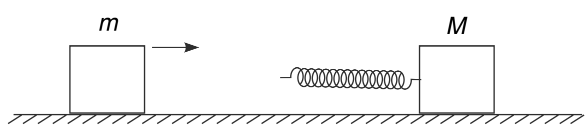
$$\left[\tan^{-1}\left(\frac{2.25}{2.34}\right) = 44°\right]$$



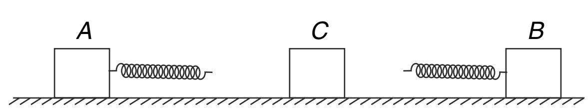

Q. 11. Two identical balls $A$ and $B$ are moving as shown in the fig. Ball $A$ hits a smooth floor head on with a velocity $u$ and at the same instant ball B strikes $A$ head on with a horizontal velocity $u$. The collision between $A$ and $B$ is perfectly inelastic whereas the coefficient of restitution for collision between $A$ and the floor is $e = 0.5$. At what time the two balls will collide again? Assume friction to be absent everywhere.




Q. 12. Two blocks of mass $m$ and $M$ are lying on a smooth table. A spring is attached with the block of mass $M$ (see fig). Block of mass m is given a velocity towards the other block. Find the value of $\frac{M}{m}$ for which the kinetic energy of the system will never fall below one third of the initial kinetic energy imparted to the block of mass $m$.



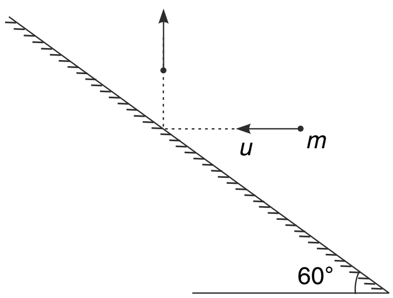

Q. 13. Two identical blocks $A$ and $B$ have two identical springs fixed to them (see fig). Mass of each block is $M$ and force constant of each spring is $K$. The two blocks have been placed on a smooth table. Another block $C$ of mass $m$ ($<< M$) is placed between $A$ and $B$ and is held close to $A$ so as to compress the spring attached to A by $X_0$. From this position the system is released. $C$ moves to push $B$ and then is back to push $A$. The sequence continues until all interactions between the blocks cease. Find the speeds eventually acquired by $A$ and $B$.



Q. 14 A particle of mass m is flying horizontally at velocity $u$. It strikes a smooth inclined surface and its velocity becomes vertical.

(a) Find the loss in kinetic energy of the particle due to impact if the inclination of the incline is 60° to the horizontal.

(b) Can the particle go vertically up after collision if inclination of the incline is 30°?

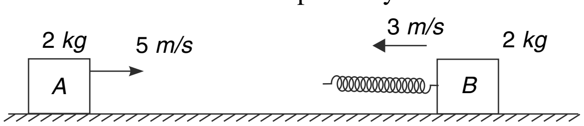



Q. 15. A block $A$ of mass 2 $kg$ is moving to right with a speed of 5 $m/s$ on a horizontal smooth surface. Another block $B$ of mass 2 $kg$, with a mass less spring of force constant $K$ =200 $N/m$ attached to it, is moving to left on the same surface with a speed of 3 $m/s$. Block $A$ collides with the spring attached to $B$. Calculate

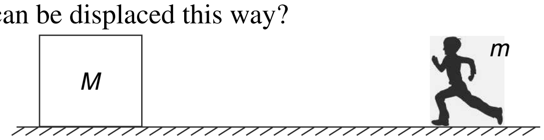
(a) the final velocity of the block $A$.

(b) the minimum kinetic energy of the system of two blocks during subsequent motion.

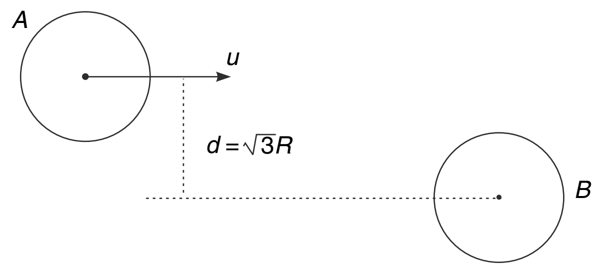
(c) Repeat part (b) if there is no spring and the two blocks collide head on. Assume that the blocks are made of perfectly elastic material.




Q. 16. A box of mass $M$ is at rest on a horizontal surface. A boy of mass $m$ (< $M$) wants to push the box by applying a horizontal force on it. The boy knows that he will not be able to push the box as the coefficient of friction $\mu$ between his shoes and ground is almost equal to that between the box and the ground. He decides to run, acquire a speed $u$ and then bang into the box. After hitting the box, the boy keeps pushing as hard as possible. What is the maximum distance through which the box can be displaced this way?




Q. 17. A smooth ball $A$ travels towards another identical ball $B$ with a velocity $u$. Ball $B$ is at rest and the impact parameter $d$ is equal to $\sqrt{3}\,R$ where $R$ is radius of each ball. Due to impact the direction of motion of ball $A$ changes by 30°. Find the velocity of $B$ after the impact. It is given that collision is elastic.

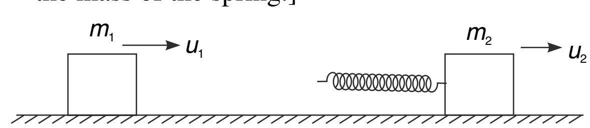



Q. 18. (a) Two identical balls are moving along $X$ axis and undergo an elastic collision. Plot the position time graph for the two balls.

(b) Consider five identical balls moving along $X$ axis. What is the maximum number of collisions that is possible? Assume that more than two balls do not collide at the same time and collisions are elastic.

Q. 19. Two particles of mass $m$ each are attached to the end of a mass less spring. This dumb-bell is moving towards right on a smooth horizontal surface at speed V with the spring relaxed. Another identical dumb-bell is moving along the same line is opposite direction with the same speed. The two dumb-bells collide head on and collision is elastic. Assuming collisions to be instantaneous, how many collisions will take place?


Q. 20. Two blocks of mass $m_1$ and $m_2$ are moving along a smooth horizontal floor. A non-ideal spring is attached at the back of mass $m_2$. Initial velocities of the blocks are $u_1$ and $u_2$ as shown; with $u_1 > u_2$. After collision the two blocks were found to be moving with velocities $V_1$ and $V_2$ respectively. Find the ratio of impulse (on each block) during the deformation phase of the spring and that during its restoration phase. [By non ideal spring we mean that it does not completely regain its original shape after deformation. You can neglect the mass of the spring.]



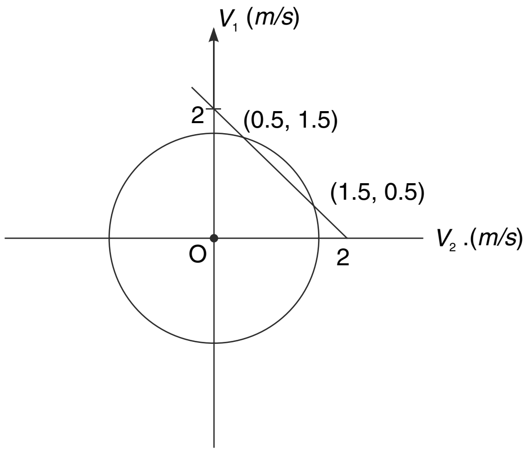
Q. 21. A ball moving with velocity $V_0$, makes a head on collision with another identical ball at rest. The velocity of incident ball and the other ball after collision is $V_1$ and $V_2$ respectively.

(a) Using momentum conservation write an equation having $V_1$ and $V_2$ as unknowns. Plot a graph of $V_1$ vs $V_2$ using this equation.

(b) Assuming the collision to be elastic write an equation for kinetic energy. Plot a graph of $V_1$ vs $V_2$ using this equation.

(c) The intersection point of the above two graphs gives solution. Find $V_1$ and $V_2$.

(d) In a particular collision, the plot of graphs mentioned above is as shown in figure

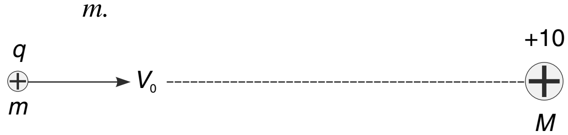



Find $V_1$ and $V_2$ for this collision. Also write the percentage loss in kinetic energy during the collision.

Q. 22. A particle having charge $+q$ and mass $m$ is approaching (head on) a free particle having mass M and charge 10 $q$. Initially the mass $m$ is at large distance and has a velocity $V_0$, whereas the other particle is at rest.

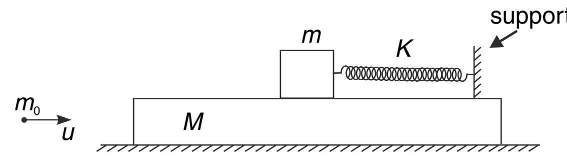
(a) Find the final velocity of the two particles when $M = 20\ m$.

(b) Find the final state of the two particles if $M = m$.

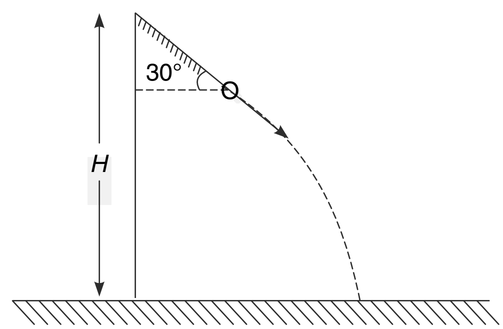



Q. 23. In the system shown in fig. block of mass $M$ is placed on a smooth horizontal surface. There is a mass less rigid support attached to the block. Block of mass $m$ is placed on the first block and it is connected to the support with a spring of force constant $K$. There is no friction between the blocks. A bullet of mass $m_0$, moving with speed $u$ hits the block of mass $M$ and gets embedded into it. The collision is instantaneous. Assuming that $m$ always stays over $M$, calculate the maximum extension in the spring caused during the subsequent motion.

$K = 8960\ N/m$ ; $u = 400\ m/s$



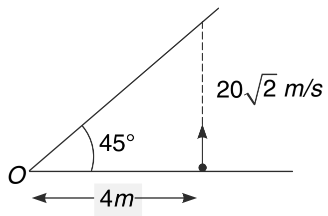

Q. 24. Starting from a height $H$, a ball slips without friction, down a plane inclined at an angle of 30° to the horizontal (fig.). After leaving the inclined plane it fall under gravity on a parabolic path and hits the horizontal ground surface. The impact is perfectly elastic (It means that there is no change in horizontal component of ball's velocity and its vertical velocity component gets inverted. There is no change in speed due to collision). Will the ball rise to a height equal to $H$ or less than $H$ after the impact?




Q. 25. Hailstones are observed to strike the surface of the frozen lake at an angle of 30° with the vertical and rebound at 60° with the vertical. Assuming the contact to be smooth, find the coefficient of restitution.

Q. 26. A ball of mass $m$ approaches a heavy wall of mass $M$ with speed 4 $m/s$ along the normal to the wall. The speed of wall before collision is 1m/s towards the ball. The ball collides elastically with the wall. What can you say about the speed of the ball after collision? Will it be slightly less than or slightly higher than 6 $m/s$ ?

Q. 27. A particle is thrown upward with speed $20\sqrt{2}$ $m/s$. It strikes the inclined surface as shown in the figure. Collision of particle and inclined surface is perfectly inelastic. What will be maximum height (in $m$) attained by the particle from the ground ($g = 10\ m/s^2$)



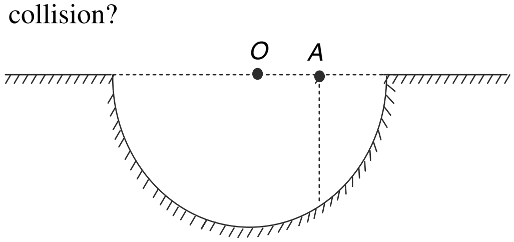

Q.28. 2n identical cubical blocks are kept in a straight line on a horizontal smooth surface. The separation between any two consecutive blocks is same. The odd numbered blocks 1, 3, 5,.....($2n–1$) are given velocity $v$ to the right whereas blocks 2, 4, 6,......$2n$ are given velocity $v$ to the left. All collisions between blocks are perfectly elastic. Calculate the total number of collisions that will take place.


Q.29. A small ball with mass $M = 0.2$ kg rests on top of a vertical column with height $h = 5\ m$. A bullet with mass $m = 0.01$ kg, moving with velocity $v_0 = 500\ m/s$, passes horizontally through the center of the ball. The ball reaches the ground at a horizontal distance $s = 20\ m$ from the column. Where does the bullet reach the ground? What part of the kinetic energy of the bullet was converted into heat when the bullet passed through the ball? Neglect resistance of the air. Assume that $g = 10\ m/s^2$.

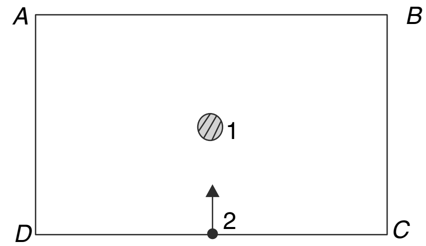
Q. 30. Figure shows a circular frictionless track of radius $R$, centred at point $O$. A particle of mass $M$ is released from point $A$ ($OA = R/2$). After collision with the track, the particle moves along the track.

(a) Find the coefficient of restitution $e$.

(b) What will be value of $e$ if the velocity of the particle becomes horizontal just after collision?

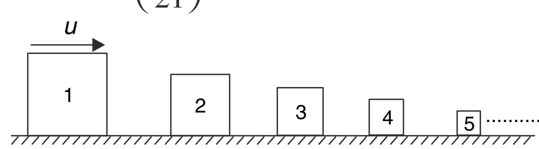



Q. 31. A rectangular billiard table has dimensions $AB = 4\sqrt{3}$ feet and $BC = 2$ feet. Ball 1 is at the centre of the table. Ball 2 moving perpendicular to $CD$ hits ball 1. After the collision ball 2 itself goes straight into the hole at $A$. Prove that ball 1 will fall into the hole at $C$. Assume that the balls are identical and their dimensions are too small compared to the dimensions of a hole. All collisions are elastic

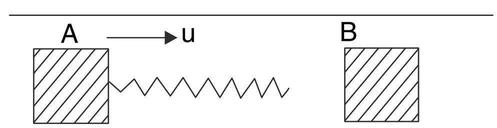



Q. 32. Blocks shown in figure have been placed on a smooth horizontal surface and mass of $(n + 1)^{th}$ block is $\frac{1}{20}$ times the mass of $n^{th}$ block (where $n = 1, 2, 3, 4,.....$). The first block is given an initial velocity $u$ towards the second block. All collision are head on elastic collisions. If $u = 10$ m/s then how many blocks must be kept so that the last one acquires speed equal to or greater than the escape speed ($= 11.0\ km\ s^{-1}$)

[**Take** $\log_{10}\left(\frac{40}{21}\right) = 0.28$ and $\log_{10}(11) = 1.04$]



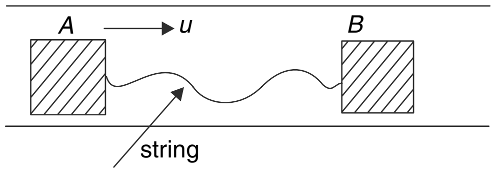

Q. 33. There is a long narrow and smooth groove in a horizontal table. Two identical blocks $A$ and $B$ each of mass $m$ are placed inside the groove at some separation. An ideal spring is fixed to $A$ as shown. Block $A$ is given a velocity $u$ to the right and it interacts with $B$ through the spring.



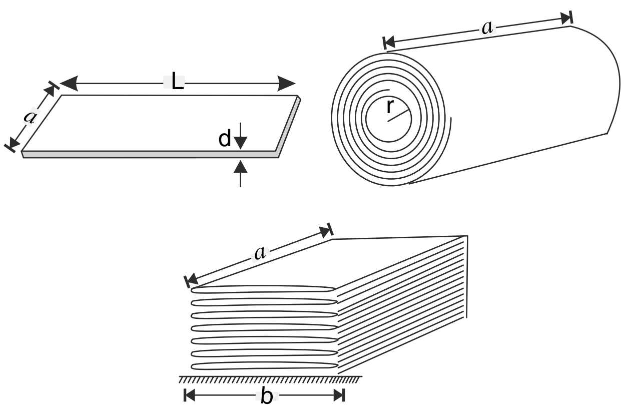

(a) What will be final state of motion of the two blocks?

(b) During their course of interaction what is the minimum kinetic energy of the system?

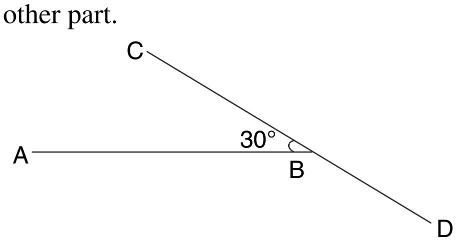
(c) The spring is removed and the two blocks are tied using a mass less string. Now $A$ is set into motion with speed $u$. What will be the final state of motion of the two blocks in this case ? How much kinetic energy is lost by the system? Where goes this energy?




Q. 34. A carpet lying on ground has length $L$, width $a$ and a small thickness $d$. It is rolled over a light cylindrical pipe of radius $r = \frac{L}{100\sqrt{\pi}}$ and kept on a level ground. Increase in gravitational potential energy of the carpet is $\Delta U_1$ (compared to its initial position when it was lying flat). In another experiment the carpet was folded to give it a shape of a cuboid (see figure) having width $b$. When this is placed on level ground its gravitational potential energy is $\Delta U_2$ higher than its initial position (flat on ground). It is given that $d = 10^{-4} L$. Find $b$ for which $\Delta U_1 = \Delta U_2$. [Take $\sqrt{\frac{\pi}{2}} = 1.25$]




Q. 35. Two identical thin rods are welded as shown in the fig. $B$ is midpoint of rod $CD$. Now the system is cut into two parts through its center of mass $M$. The part $AM$ weights 4 kg. Find the mass of the other part.



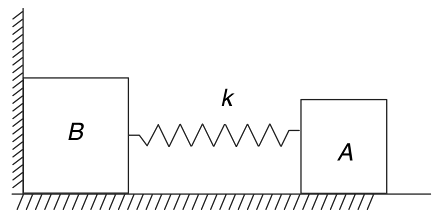

Q. 36. (i) A regular polygon has 2016 sides and $r$ is the radius of the circle circumscribing the polygon. Particles of equal mass are placed at 2015 vertices of the polygon. Find the distance of the centre of mass of the particle system from the centre of the polygon.

(ii) In the last problem you have been asked to remove any one particle from the system so that the centre of mass of the remaining 2014 particles lies farthest from the geometrical centre of the polygon. Which particle will you remove?

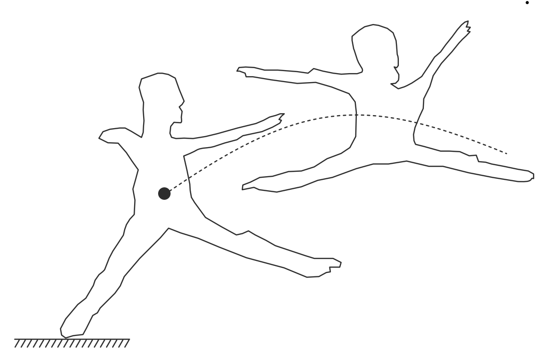
Q. 37. Two identical block $A$ and $B$ each having mass $m$, are connected with a spring of force constant $k$. The floor is smooth and $A$ is pushed so as to compress the spring by $x_0$. The system is released from this position

(a) Calculate the maximum speed of the centre of mass of the system during subsequent motion.

(b) What is acceleration of the centre of mass at the instant it acquires half its maximum speed?




Q. 38. A dancer leaps off the floor with her centre of mass having a velocity of 5 $m/s$ making an angle of $\theta = 37°$ to the horizontal. At the top of the trajectory the dancer has her legs stretched so that the centre of mass gets closer to head by a vertical distance of 0.25 $m$. By how much does the head rises vertically from its initial position? $\left[\sin 37° = \frac{3}{5}\right]$.

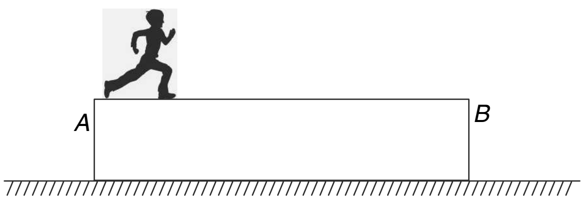


Q. 39. In order to make a jump straight up, a 60 kg player starts the motion crouched down at rest. He pushes hard against the ground, raising his centre of mass by a height $h_0$ = 0.5 m. Assume that his legs exert a constant force $F_0$ during this motion. At this point, where his centre of mass has gone up by $h_0$ his feet leave the ground and he has an upward velocity of $v$. Centre of mass of his body rises further by $h = 0.8\ m$ before falling down [Take $g = 10\ m/s^2$]

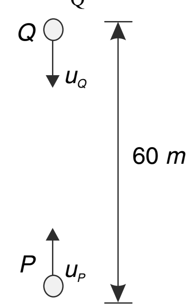
(a) Find $v$.

(b) Find the normal force applied by the ground on his feet just before he left the ground.

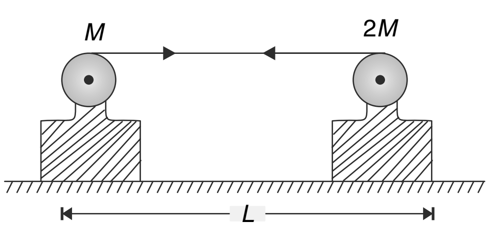
Q. 40. A platform is kept on a rough horizontal surface. At one end $A$ of the platform there is a man standing on it. The man runs towards the end $B$ and the platform is found to be moving. In which direction will the platform be moving after the man abruptly comes to rest on the platform at $B$?




Q. 41. Two particles $P$ and $Q$ have mass 1 $kg$ and 2 $kg$ respectively. They are projected along a vertical line with velocity $u_p = 20\ m/s$ and $u_Q = 5\ m/s$ when separation between them was 60 $m$. $P$ was projected vertically up while $Q$ was projected vertically down. Calculate the maximum height attained by the centre of mass of the system of two particles, measured from the initial position of $P$. Assume that the particles do not collide and that the ground is far below their point of projection [$g = 10\ m/s^2$]

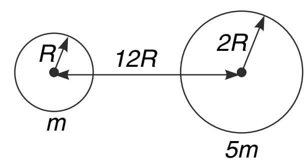



Q. 42. Two small motors are kept on a smooth table at a separation $L$. The motors have mass $M$ and $2M$ and are connected by a light thread. The motors begin to wrap the thread and thereby move closer to each other. The tension in the thread is maintained constant at $F$. Find the time after which the two motors will collide. Neglect the dimensions of the motors and their stands.




Q. 43. Consider a uniform rectangular plate. If a straight line is drawn, passing through its centre of mass (in the plane of the plate), so as to cut the plate in two parts – the two parts obtained are of equal mass irrespective of the orientation of the line. Can you also say that a straight line passing through the centre of mass of a triangular plate, irrespective of its orientation, will also divide the triangle into two pieces of equal mass?

Q. 44. Two spherical bodies of masses $m$ and $5\ m$ and radii $R$ and $2R$ respectively are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only then find the distance covered by smaller sphere just before collision.



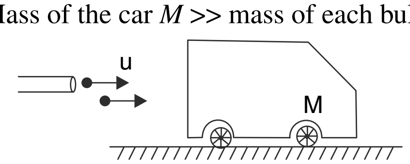

Q. 45. A shell is fired vertically upward with a speed of 60 $m/s$. When at its maximum height it explodes into large number of fragments. Assume that the fragments fly in every possible direction and all of them have same initial speed of 25 $m/s$
[Take $g = 10\ m/s^2$]

(a) Prove that after the explosion all the fragments will lie on an expanding sphere. What will be speed of the centre of the sphere thus formed – one second after explosion?

(b) Find the radius of the above mentioned sphere at the instant the bottom of the sphere touches the ground.

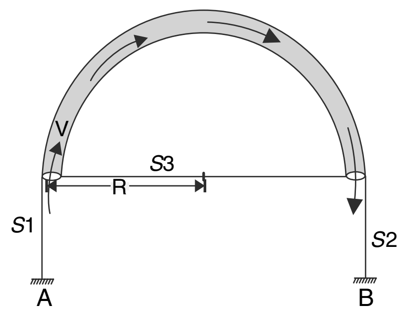
## LEVEL 2

Q. 46. A car of mass $M$ is free to move on a frictionless horizontal surface. A gun fires bullets on the car. The bullets leave the stationary gun with speed $u$ and mass rate $b\ kg\ s^{-1}$. The bullets hit the vertical rear surface of the car while travelling horizontally and collisions are elastic. If the car starts at rest find its speed and position as a function of time. Mass of the car $M >>$ mass of each bullet.




Q. 47. (i) Liquid of density $\rho$ flows at speed $\upsilon$ along a flexible pipe bent into a semicircle of radius

$R$. The cross sectional area of the pipe is $A$ and its cross sectional radius is small compared to $R$. Three strings $S1$, $S2$ and $S3$ keep the pipe in place. $S3$ ties the two ends of the pipe and the other two string have their ends secured at $A$ and $B$. Strings $S1$and $S2$ are perpendicular to the string $S3$. The entire system is in horizontal plane. Find the tension in the three strings.



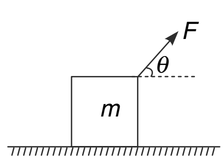

(ii) A car of mass $M$ is moving with a velocity $V_0$ on a smooth horizontal surface. Bullets, each of mass $m$, are fired horizontally perpendicular to the velocity of the car with a speed $u$ relative to the car. After firing $n$ bullets it was found that the car was travelling with velocity $V_0$ in a direction opposite to its original direction of motion. Assume that $mu << MV_0$ and also that $nm << M$. Find $n$ in terms of other given parameters.

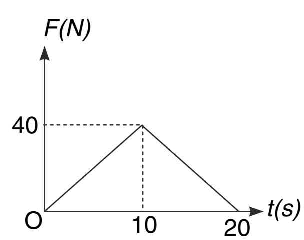
Q. 48. A block of mass $m$ = 4.4 kg lies on a horizontal rough surface. The coefficient of friction between the block and the surface is $\mu$ = 0.5. A force $F$ starts acting on the block making an angle $\theta = 37°$ to the horizontal. The force changes with time as shown in the graph.

(a) At what time the block begins to move?

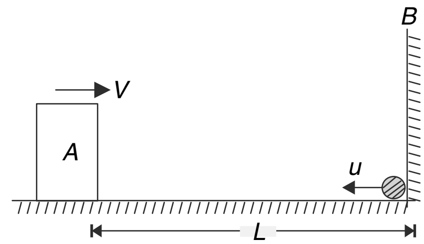
(b) Calculate the maximum speed attained by the block.

$$\left[\tan 37° = \frac{3}{4}\ ;\ g = 10 ms^{-2}\right]$$

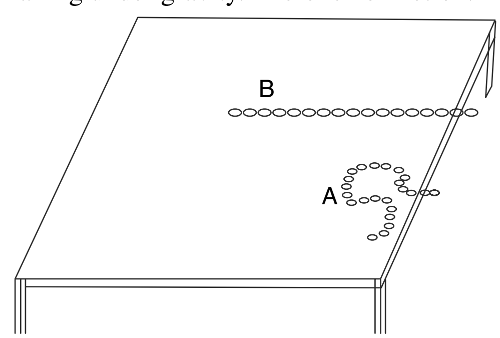



Q. 49. A heavy block $A$ is made to move uniformly along a smooth floor with velocity $V = 0.01$ *m/s* towards left. A ball of mass $m$ = 50 $g$ is projected towards the block with a velocity of $u$ = 100 *m/s*. The ball keeps bouncing back and forth between the block $A$ and fixed wall $B$. Each of the collisions is elastic. After the ball has made 1000 collisions with the block and wall each, the distance between the block and the wall was found to be $L$ = 1.2 *m*. Calculate the average force being experienced by the block due to collision at this instant. All collision are instantaneous.



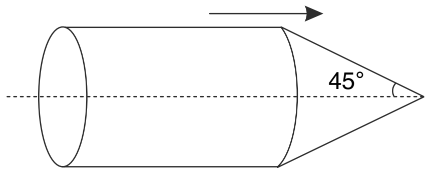

Q. 50. A chain ($A$) of length $L$ is coiled up on the edge of a table. Another identical chain ($B$) is placed straight on the table as shown. A very small length of both the chains is pushed off the edge and it starts falling under gravity. There is no friction.




(a) Find the acceleration of the chain $B$ at the instant $\frac{L}{2}$ length of it is hanging. Assume no kinks in the chain so that the entire chain moves with same speed.

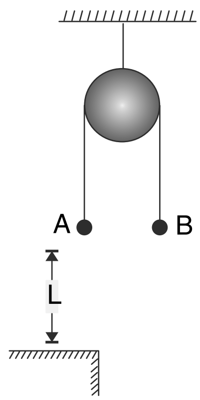
(b) For chain A assume that velocity of each element remains zero until it is jerked into motion with a velocity equal to that of the falling section. Find acceleration of the hanging section at the instant a length $l_0$ has slipped off the table and its speed is known to be $v_0$ at the instant.

Q. 51 To understand the effect of air resistance on the motion of a bullet let's consider a bullet of the shape shown in the fig. The bullet is flying horizontally. The cross section of cylindrical part is $A$ and the conical part has a semi vertical angle of 45°. Assume that the bullet is fired with initial velocity $u$ and moves in a gaseous medium in which molecules are at rest (do you think this assumption is necessary?). Collisions of the molecules with the bullet are elastic. Take mass of bullet to be $M$, density of gaseous medium as $\rho$ and disregard gravity.



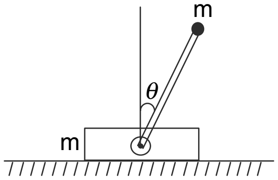

(a) Consider two bullets one small and other large, made of same material. Which will experience larger retardation due to air resistance?
(b) Write the speed of bulled after time $t$.
(c) Write distance travelled by the bullet in time $t$.

Q. 52. Two particle $A$ and $B$, of mass $3m$ and $2m$ respectively, are attached to the ends of a light inextensible string which passes over a smooth fixed pulley of negligible mass. After the system is released and $A$ falls through a distance $L$, it hits a horizontal inelastic table so that its speed is immediately reduced to zero. Assume that $B$ never hits the table or the pulley. Find



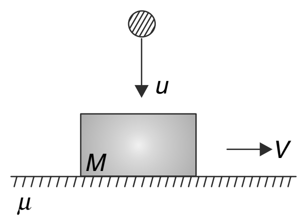

(a) the time for which $A$ is resting on the table after the first collision and before it is jerked off,
(b) the difference between the total kinetic energy of the system immediately before $A$ first hits the table and total kinetic energy immediately after $A$ starts moving upwards for the first time. Explain the loss in kinetic energy.

Q. 53. A light rod of length $L$ is hinged to a plank of mass $m$. The plank is lying on the edge of a horizontal table such that the rod can swing freely in the vertical plane without any hindrance from the table. A particle of mass $m$ is attached to the end of the rod and system is released from $\theta = 0°$ position (see figure)



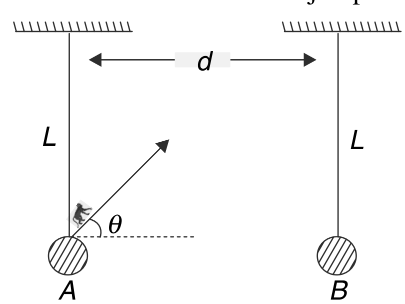

(a) Assume that friction between the plank and the table is large enough to prevent it from slipping and calculate the smallest normal force applied by the plank on the table.
(b) Assume that friction is absent everywhere and calculate the speed of the plank when the rod makes $\theta = 180°$.

Q. 54. A block of mass $M = 5\ kg$ is moving on a horizontal table and the coefficient of friction is $\mu = 0.4$. A clay ball of mass $m = 1\ kg$ is dropped on the block, hitting it with a vertical velocity of $u = 10\ m/s$. At the instant of hit, the block was having a horizontal velocity of $v = 2\ m/s$. After an interval of $\Delta t$, another similar clay ball hits the block and the system comes to rest immediately after the hit. Assume that the clay balls stick to the block and collision is momentary. Find $\Delta t$. Take $g = 10\ m/s^2$.



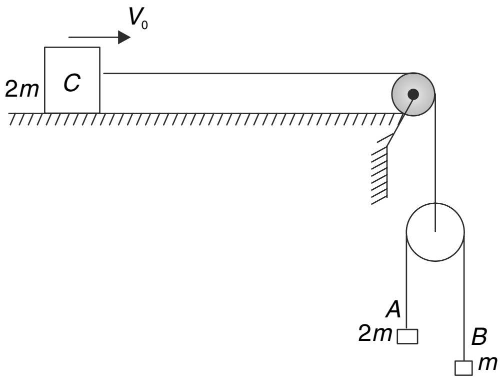

Q. 55. Vertical strings of same length $L$ support two balls $A$ and $B$ of mass $2m$ each. There is a small monkey of mass $m$ sitting on ball $A$. Suddenly, the monkey jumps off the ball $A$ at an angle $\theta = 45°$ to the horizontal and lands exactly on the ball $B$. Thereafter, the monkey and the ball $B$ just manage to complete the vertical circle.

(a) Find distance $d$ between the two string and the speed with which the monkey jumped of the ball $A$.
(b) Find the impulse of the string tension on ball $A$ during the small period when the monkey interacted with the ball to jump off it.

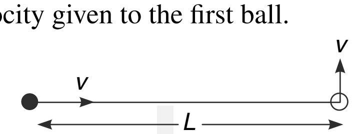



Q. 56. In the shown figure, pulleys and strings are ideal and horizontal surface is smooth. The block $C$ (mass $2m$) is given a horizontal velocity of

$V_0 = 3\ m/s$ towards right and the entire system is let go. Find the velocity of three blocks, just after the strings regain tension. Mass of *A* and *B* are $2m$ and *m* respectively and take $g = 10\ m/s^2$.



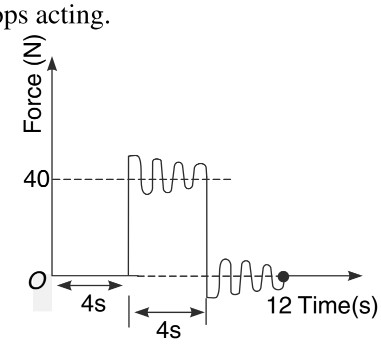

Q. 57. Two identical small balls are interconnected with a light and inextensible thread having length *L*. The system is on a smooth horizontal table with the thread just taut. Each ball is imparted a velocity *v*, one towards the other ball and the other in a direction that is perpendicular to the velocity given to the first ball.




(a) After how much time the thread will become taut again?

(b) Calculate the kinetic energy of the system after the string gets taut.

Q. 58. A particle of mass 1 kg is moving with a velocity of 200 m/s. An impulsive force of 4 s duration acts on the particle in a direction opposite to its motion. The force fluctuates a little bit around 40 N magnitude and then it dies out in next 4s showing small fluctuations. An oscilloscope records the force as shown. The two oscillating components in the graph are identical except that one is mirror image of the other. Find the magnitude of velocity of particle after the force stops acting.

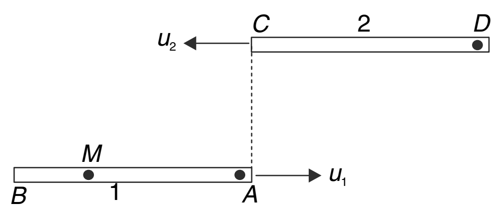



Q. 59. A moving particle of mass *m* collides elastically with a stationary particle of mass 2*m*. After collision the two particles move with velocity $\vec{v}_1$ and $\vec{v}_2$ respectively. Prove that $\vec{v}_2$ is perpendicular to $(2\vec{v}_1 + \vec{v}_2)$

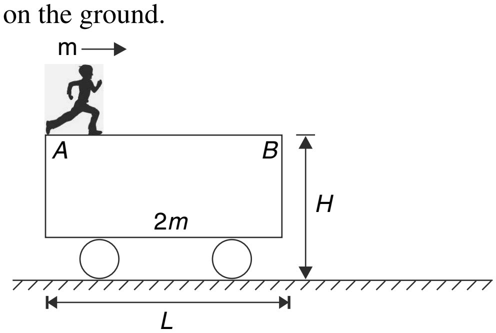
Q. 60. Two identical carts are moving on parallel smooth tracks with velocities $u_1 = 10\ ms^{-1}$ and $u_2 = 15\ ms^{-1}$. The empty carts (with drivers) have mass 3m each. Each cart has a sack of mass *m* kept at end *A* and end *D* (see figure). At the instant the carts being to cross, the sack in cart 1 is the thrown perpendicular (relative to cart 1) with some unknown velocity and it lands on cart 2 at its end *D* after a time $t_0$. Immediately after the sack lands into cart 2, the original sack in cart 2 is thrown perpendicularly (relative to cart 2) towards cart 1 in identical fashion. The sack lands on cart 1 at point *M*, a time $t_0$ after the throw. Assume that the carts are constrained to move in straight lines.

(i) Find length *BM* if length of each cart is *L*

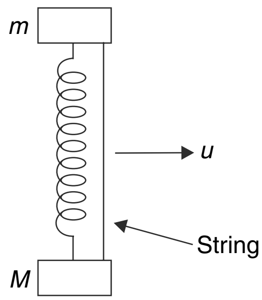
(ii) Find the velocity of cart 1 after the sack thrown from cart 2 lands on it.




Q. 61. A man of mass *m* is standing on the flat top of a cart of mass 2*m*. The length and height of the cart is *L* and *H* respectively and it is at rest on a smooth horizontal ground. The man starts running from end *A*, speeds up and jumps out of the cart at point *B* with a velocity *u* relative to the cart in horizontal direction. Calculate the total horizontal distance covered by the man by the time he lands on the ground.



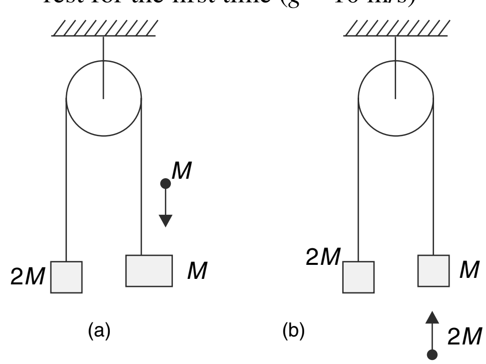

Q. 62. Two blocks of masses $m = 2\ kg$ and $m = 8\ kg$ are connected to a spring of force constant $K = 1\ kN/m$. The spring is compressed by 20 *cm* and the two blocks are held in this position by a string. The system is placed on a horizontal smooth surface and given a velocity $u$ = 3 m/s perpendicular to the spring. The string snaps while moving. Find the speed of the block of mass $m$ when the spring regains its natural length.



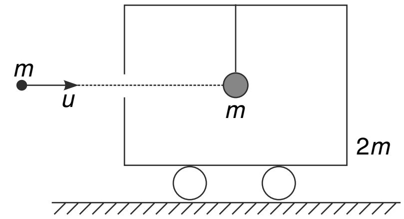

Q. 63. Two blocks of mass $M$ and $2M$ are connected to the two ends of a light, inextensible string passing over an ideal pulley as shown in figure. The system is released from rest.

(a) One second after the system is released, a particle of mass $M$ hits the block of mass $M$ and sticks to it. The particle hits the block with a speed of 10 m/s while travelling downward. Find the total distance travelled by block of mass $2M$ after it is released.

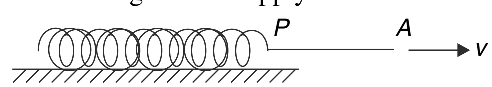
(b) One second after the system is released, a particle of mass $2M$ hits the block of mass $M$ and sticks to it. The particle hits the block with a speed of 10 m/s while travelling in upward direction. Find distance travelled by the block of mass $2M$ after it is released to the time it comes to rest for the first time ($g$ = 10 m/s)$^2$



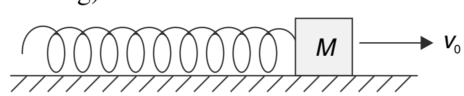

Q. 64. A toy car of mass 2 $m$ is at rest on a smooth horizontal surface. A small bob of mass $m$ is suspended by a mass less string of length $L$ from the roof of the car. A horizontally flying bullet of mass $m$ enters into the car through a small window and sticks to the bob. Speed of the bullet is $u$. Find minimum value of $u$ (call it $u_0$) for which the bob can touch the roof of the car.




Q. 65. A heap of rope is lying on a horizontal surface. One free end $A$ of the rope is pulled horizontally with a constant velocity $v$. Assume that the heap does not move and the moving part of the rope remains straight and horizontal (i.e. there is no sag). Mass per unit length of the rope is $\lambda$. Find the tension at point $P$ where the straightened part of the rope meets the heap. How much force the external agent must apply at end $A$?



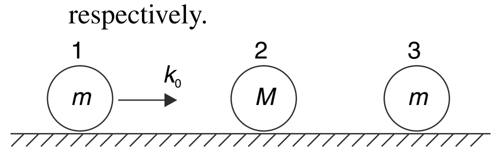

Q. 66. In the last problem, the free end $A$ of the rope is tied to a block of mass $M$ and the block is given a horizontal velocity $v_0$ (see figure). Calculate the following quantities at the instant the block is at a distance $x$ from the right end of the heap (here 'heap' means the coiled part of the rope that is not moving).




(i) Speed of the block.

(ii) Tension force applied by the rope on the block.

Q. 67. (i) Three balls 1, 2 and 3 lie on a smooth horizontal table. Ball 1 is given a velocity towards ball 2. Kinetic energy given to ball 1 is $k_0$. It collides with 2 and in turn ball 2 hits ball 3. All collisions are head on elastic. Masses of the balls are $m$, $M$ and $m$ respectively.

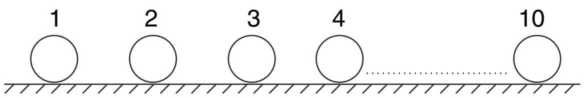



(a) Calculate the kinetic energy ($k_3$) of ball 3 after ball 2 hits it.

(b) Draw the variation of $k_3$ as a function of $M$.

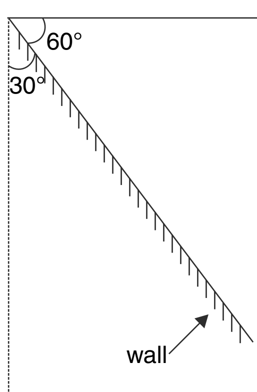
(ii) Consider 10 balls laid on a smooth surface with masses $m, \frac{m}{2}, \frac{m}{4}, \frac{m}{8} \ldots\ldots \frac{m}{512}$ and first ball is pushed towards the second with kinetic energy $k_0$. [All collisions are elastic and head on]. The kinetic energy acquired by the last ball is $k_{10}$. In a separate experiment the $10^{th}$ ball is pushed towards $9^{th}$ ball with kinetic energy $k_0$. This time the kinetic energy acquired by $1^{st}$ ball is $k_1$.

Compare $k_{10}$ and $k_1$.

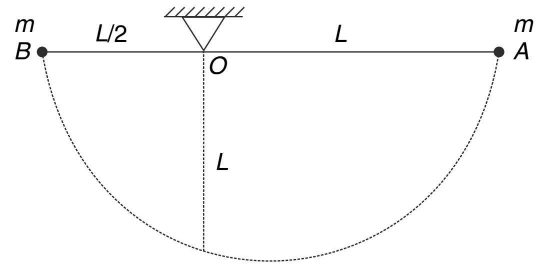



Q. 68. A simple pendulum is suspended from a peg on a wall which is inclined at an angle of 30° with the vertical. The pendulum is pulled away from the wall to a horizontal position (with string just taut) and released. The bob repeatedly bounces off the wall, the coefficient of restitution being $e = \frac{2}{\sqrt{5}}$.

Find the number of collisions of the bob with the wall, after which the amplitude of oscillation (measured from the wall) becomes less than 30°.



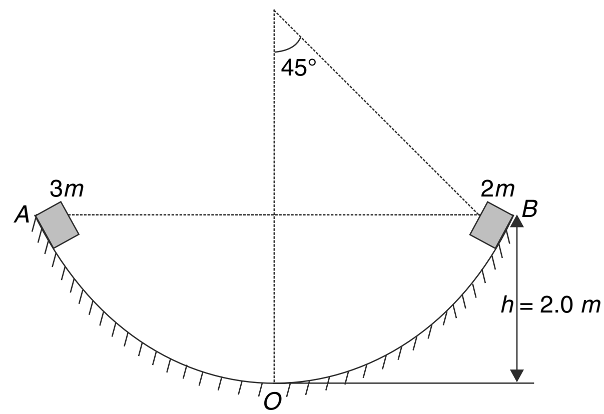

Q. 69. Two particles $A$ and $B$, having same mass $m$ are tied to a common point of suspension $O$. $A$ is tied with the help of an inextensible string of length $L$ and $B$ is tied using an elastic string of unstretched length $\frac{L}{2}$. The two particles are released from horizontal positions as shown in figure. The particles have been released at a time gap so that both the string and the elastic cord become vertical simultaneously. It was observed that the length of the cord became equal to that of the string at this moment and the two particles collided. The particles got stuck together and their velocity just after the collision was observed to be $\frac{\sqrt{gL}}{2}$.




(a) In which direction will the combined mass move immediately after collision – right or left?

(b) Find tension in the string immediately after the collision.

Q. 70. A smooth track, fixed to the ground, is in the shape of a quarter of a circle. Two small blocks of mass $3m$ and $2m$ are released from the two edges $A$ and $B$ of the circular track. The masses slide down and collide at centre $O$ of the track. Vertical height of $A$ and $B$ from $O$ is $h = 2m$. Collision is elastic. Find the maximum height (above $O$) attained by the block of mass $2m$ after collision.

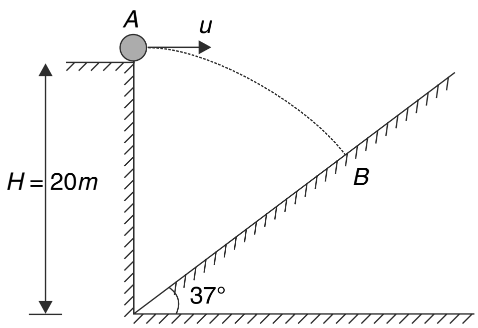



Q. 71. A man stands on a frictionless horizontal ground. He slides a 10 *kg* block on the surface with a speed of 3 *m/s* relative the ground, towards a vertical massive wall. The wall itself it moving towards the man at a constant speed of 2 *m/s*. The block makes a perfectly head on elastic collision with the wall, rebounds and reaches back to the man 3 second after the throw. At the moment the block was thrown, the wall was at a distance of 10 *m* from the man.

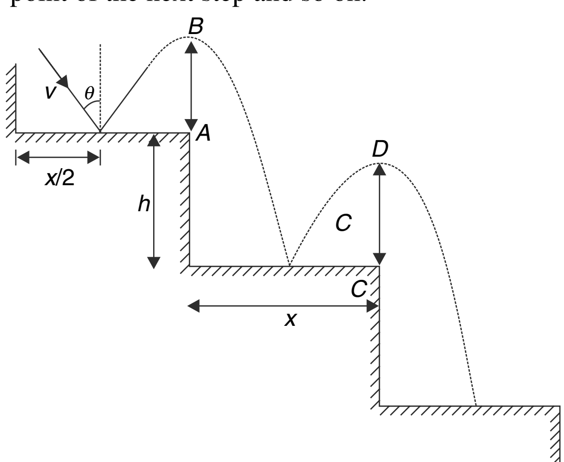
(a) Find the mass of the man.

(b) Find the ratio of work done by the man in throwing the block to the work done by the wall on the block.

Q. 72. A ball is projected from point A in horizontal direction with a velocity of $u = 28$ *m/s*. It hits the incline plane at point $B$ and rebounds. Show that whatever be the coefficient of restitution between the ball and the incline, the ball will always hit the incline for the second time at a point above $B$ (i.e., it will not hit the incline below $B$). Assume the incline to be smooth and take $g = 10\ m/s^2$ $\left[\sin 37° = \frac{3}{5}\right]$




Q. 73. A staircase has each step of height $h$ and width $x$. A ball strikes the centre point of a step with velocity $v$ making an angle $\theta$ with the vertical. It rebounds and strikes the centre point of the next step. Once again it rebounds and hits the centre point of the next step and so on.




Assume that there is no friction between the ball and the steps and coefficient of restitution is $e$.

(a) Show that each time after hitting a step, the ball climbs to the same height (i.e., heights like $AB$ and $CD$ shown in figure are equal).

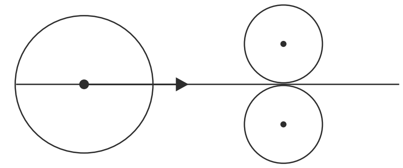
(b) Find $h$ and $x$.

Q. 74. Two identical discs are initially at rest in contact on a horizontal table. A third disc of same mass but of double radius strikes them symmetrically and comes to rest after the impact.

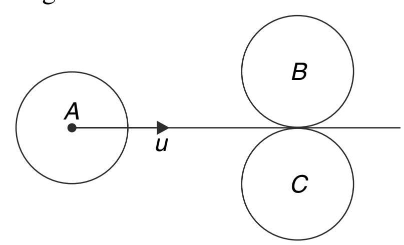
(a) Find the coefficient of restitution for the impact.

(b) Find the minimum kinetic energy of the system (as a percentage of original kinetic energy before collision) during the process of collision.

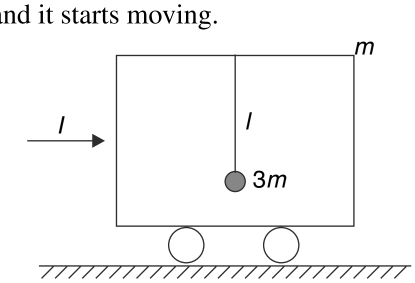
Treat the collision to be instantaneous.



Q. 75. On a billiard table two balls $B$ and $C$ are at rest touching each other. A third ball $A$, travelling with speed u, strikes the two balls elastically (see fig.). Somehow, $A$ hits $B$ first and within a fraction of a second hits ball $C$. You may assume that $B$ and $C$ are placed symmetrically with respect to the line of motion of $A$ and that all the balls are identical. What angle does the final velocity of $A$ make with its original direction of motion.



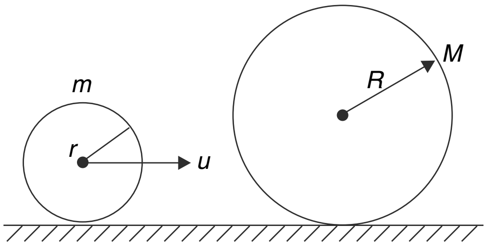

Q. 76. A toy car of mass $m$ is placed on a smooth horizontal surface. A particle of mass $3m$ is suspended inside the car with the help of a string of length $l$. Initially everything is at rest. A sudden horizontal impulse $I = 2m\sqrt{gl}$ is applied on the car and it starts moving.




(a) Find the maximum angle $\theta_0$ that the string will make with the vertical subsequently.

(b) Find tension in the string when it makes angle $\theta_0$ with the vertical.

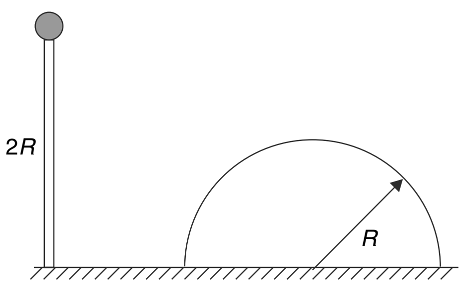
Q. 77. A smooth ball of mass $M$ and radius $R$ is lying on a smooth horizontal table. A smaller ball of radius $r$ and mass $m$ travelling horizontally on the table with velocity $u$ hits the larger ball. Collision is elastic. During the interaction of the balls the larger ball does not lose contact with the table at any instant.



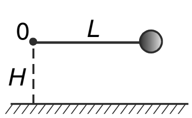

(i) Calculate the velocity of the balls after collision.

(ii) Calculate the maximum possible interaction force between the balls during collision.

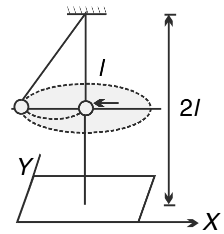
Q. 78. A light rigid rod has a small ball of mass $m$ attached to its one end. The other end is hinged on a table and the rod can rotate freely in vertical plane. The rod is released from vertical position and while falling the ball at its end strikes a hemisphere of mass $m$ lying freely on the table. The collision between the ball and the hemisphere is elastic. The radius of hemisphere and length of the rod are $R$ and $2R$ respectively. Find the velocity of the hemisphere after collision.




Q. 79. (i) $O$ is a fixed peg at a height $H$ above a perfectly inelastic smooth horizontal plane. A light inextensible string of length $L$ (> $H$) has one end attached to $O$ and the other end is attached to a heavy particle. The particle is held at the level of $O$ with string horizontal and just taut and released from rest. Find the height of the particle above the plane when it comes to rest for the first time after the release.




(ii) The bob of a pendulum has mass m and the length of pendulum is $l$. It is initially at rest with the string vertical and the point of suspension at a height $2l$ above the floor. A particle P of mass $\frac{m}{2}$ moving horizontally along –ve $x$-direction with velocity $\sqrt{2gl}$ collides with the bob and comes to rest. The bob swings and when it comes to rest for the first time, another particle $Q$ of mass m moving horizontally along y direction collides with the bob and sticks to it. It is observed that the bob now moves in a horizontal circle.




(a) Find tension in string just before the second collision.

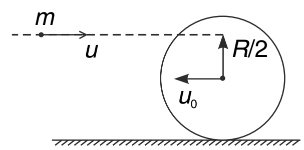
(b) Find the height of the circular path above the floor.

(c) Find the time period of the circular motion.

(d) The string breaks during the circular motion at time $t = 0$. At what time the bob will hit the floor?

Q. 80. A billiard ball collides elastically with an identical stationary ball. The collision is not head on. Show that the directions of motion of the two balls are at right angles after the collision. Solve the problem in centre of mass frame as well as in lab frame.

Q. 81.

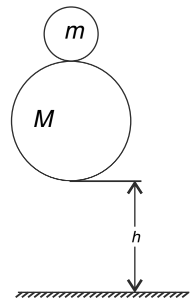



A heavy ball of radius $R$ is travelling on a smooth horizontal surface with a velocity of $u_0$ towards left. A horizontally moving small ball of mass $m$ strikes it at a height $\frac{R}{2}$ above the centre while travelling with velocity $u$ towards right.

(a) After collision the small ball moves in vertically upwards direction with velocity $u$. Prove that this can happen only if $u > \sqrt{3}u_0$

(b) Find the velocity of small ball after collision if the collision is elastic and the balls are smooth.

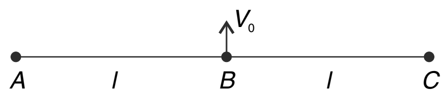
Q. 82. Two elastic balls of masses $M$ and $m$ ($M >> m$) are placed on top of each other with a small gap between them. The balls are dropped on to the ground with the bottom of the lower ball at height $h$ above the ground. The lower ball has a radius $R$ and the upper ball has negligible dimension.



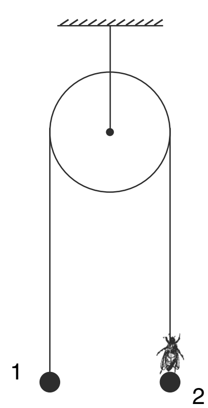

(a) Up to what height the ball of mass $m$ will bounce above the ground ?

(b) Does the result obtained above violates the low of conservation of mechanical energy?

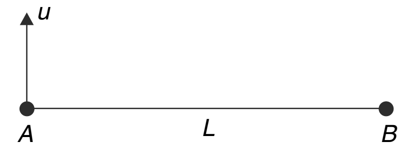
Q. 83. Three identical particles are placed on a horizontal smooth table, connected with strings as shown. The particle $B$ is imparted a velocity $V_0 = 9$ $m/s$ in horizontal direction perpendicular to the line $ABC$. Find speed of particle A when it is about to collide with C.



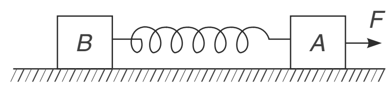

Q. 84. A light inextensible string, passing over a pulley, supports two particles 1 and 2 at its ends. An insect of mass m is sitting on particle 2 and the system is in equilibrium. The sum of masses of particles and the insect is $M$. Now the insect crawls a distance $x$ up relative to the string. Find the displacement of centre of mass of the system of two particles and the insect. In which direction does the centre of mass move and why?



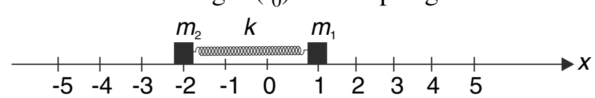

Q. 85. Two particles (A and B) of masses m and 2m are joined by a light rigid rod of length L. The system lies on a smooth horizontal table. The particle (A) of mass m is given a sharp impulse so that it acquires a velocity u perpendicular to the rod. Calculate maximum speed of particle B during subsequent motion. By what angle $\theta$ will the rod rotate by the time the speed of particle B become maximum for the first time?




Q. 86. Two blocks $A$ and $B$, each of mass m, are connected by a spring of force constant $K$. Initially, the spring is in its natural length. A horizontal constant force $F$ starts acting on block $A$ at time $t$=0 and at time $t$ , the extension in the spring is seen to be $\ell$. What is the displacement of the block $A$ in time $t$?




Q. 87. Two blocks of mass $m_1$ and $m_2$ are connected to the ends of a spring. The spring is held compressed and the system is placed on a smooth horizontal table. The block of mass $m_1 = 2$ $kg$ is kept at $x = 1$ cm mark and the other block is at $x = 2$ cm mark. The system is released from this position. It was observed that at the instant $m_1$ was at $x = 5$ cm mark its velocity was zero and at that moment $m_2$ was located at $x = -4$ cm. Find mass $m_2$ and unstretched length ($l_0$) of the spring.




Q. 88. Two particles having masses $m_1$ and $m_2$ are moving with velocities $\vec{V}_1$ and $\vec{V}_2$ respectively. $\vec{V}_0$ is velocity of centre of mass of the system.

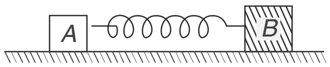
(a) Prove that the kinetic energy of the system in a reference frame attached to the centre of mass of the system is $KE_{cm} = \frac{1}{2}\mu V^2_{rel}$. Where $\mu = \frac{m_1 m_2}{m_1 + m_2}$ and $V_{rel}$ is the relative speed of the two particles.

(b) Prove that the kinetic energy of the system in ground frame is given by

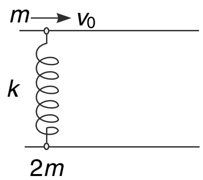
$$KE = KE_{cm} + \frac{1}{2}(m_1 + m_2)V_0^2$$

(c) If the two particles collide head on find the minimum kinetic energy that the system has during collision.

Q. 89. Two blocks $A$ and $B$ of mass m and 2m respectively are connected by a light spring of force constant $k$. They are placed on a smooth horizontal surface. Spring is stretched by a length $x$ and then released. Find the relative velocity of the blocks when the spring comes to its natural length.



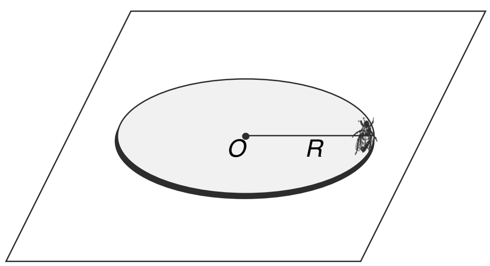

Q. 90. Two ring of mass $m$ and $2m$ are connected with a light spring and can slide over two frictionless parallel horizontal rails as shown in figure. Ring of mass m is given velocity '$v_0$' in horizontal direction as shown. Calculate the maximum stretch in spring during subsequent motion.



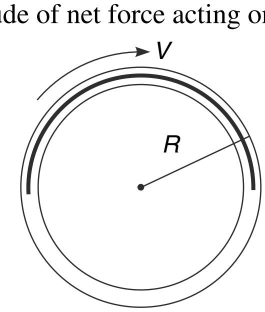

Q. 91. A disc of mass $M$ and radius $R$ is kept flat on a smooth horizontal table. An insect of mass m alights on the periphery of the disc and begins to crawl along the edge.

(a) Describe the path of the centre of the disc.

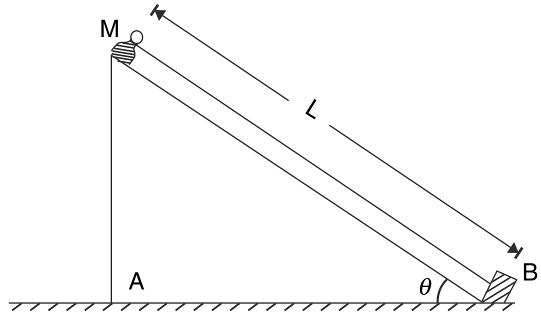
(b) For what value of $\frac{m}{M}$ the centre of the disc and the insect will follow the same path?




Q. 92. A metal wire having mass $M$ is bent in the shape of a semicircle of radius $R$ and is sliding inside a smooth circular grove of radius $R$ present in a horizontal table. The wire just fits into the groove and is moving at a constant speed $V$. Find the magnitude of net force acting on the wire.

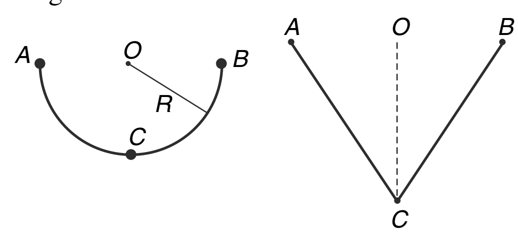



Q. 93. A triangular wedge (A) has inclined surface making an angle $\theta = 37°$ to the horizontal. A motor ($M$) is fixed at the top of the wedge. Mass of the wedge plus motor system is $3m$. A small block (B) of mass $m = 1$kg is placed at the bottom of the incline and is connected to the motor using a light string. The motor is switched on and it slowly hauls block B through a distance $L = 2.0$ meter along the incline. Calculate the work done by the string tension force on the wedge plus motor system. All surfaces are frictionless.




Q. 94. An ice cream cone of mass $M$ has base radius $R$ and height $h$. Assume its wall to be thin and uniform. When ice cream is filled inside it (so as to occupy the complete conical space) its mass becomes 5 $M$. Find the distance of the centre of mass of the ice cream filled cone from its vertex.

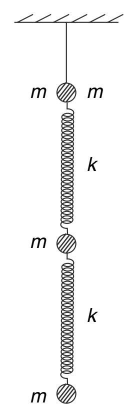
Q. 95. A flexible rope is in the shape of a semicircle ACB with its centre at O. Ends $A$ and $B$ are fixed. Radius of the semicircle is $R$. The midpoint C is pulled so that the rope acquires $V$ shape as shown in the figure.




(a) Make a guess whether the centre of mass of the rope moves closer to O or moves away from it when it is pulled?

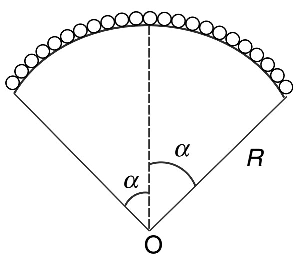
(b) Calculate the shift in position of the centre of mass of the rope.

Q. 96. Three small balls of equal mass ($m$) are suspended from a thread and two springs of same force constant ($K$) such that the distances between the first and the second ball and the second third ball are the same. Thus the centre of mass of the whole system coincides with the second ball. The thread supporting the upper ball is cut and system starts a free fall. Find the distance of the centre of mass of

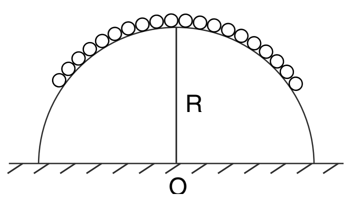


the system from the second ball when both the springs acquire their natural length in the falling system.

Q. 97. (a) A uniform chain is lying in form of on arc of a circle of radius R. The arc subtends an angle of $2\alpha$ at the centre of the circle. Find the distance of the centre of mass of the chain from the centre of the circle.



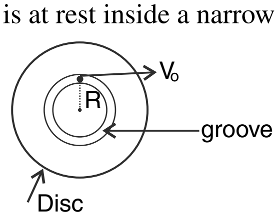

(b) A uniform chain of length $\frac{\pi R}{2}$ is lying symmetrically on the top of a fixed smooth half cylinder (see figure) of radius $R$. The chain is pulled slightly from one side and released. It begins to slide. Find the speed of the chain when its one end just touches the floor. What is speed of centre of mass of the chain at this instant?




(c) In part (b) assume that the half cylinder is not fixed and can slide on the smooth floor. Find the displacement of the cylinder by the time one end of the chain touches the floor. Mass of cylinder is equal to that of the chain.

For part (b) and (c) assume that the chain remains in contact with the cylinder all the while.

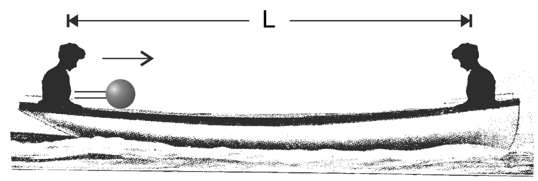
Q. 98. A small body of mass m is at rest inside a narrow groove carved in a disc. Groove is a circle of radius R concentric to the disc. Mass of the disc is also m . The disc lies on a smooth horizontal floor. The small body is given a sharp impulse so that it acquires a tangential velocity $V_o$ at time $t = 0$.




(a) The velocity of the centre of the disc becomes zero for the first time at time $t_0$. Find $t_0$.

(b) Find speed of the small body at time $\frac{t_0}{3}$.

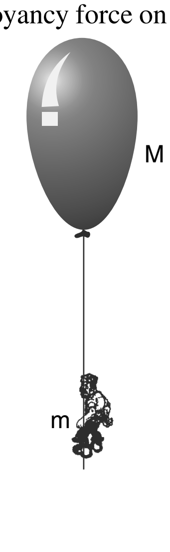
Q. 99. Laila and Majnu are on a boat for a picnic. The boat is initially at rest. Laila has a big watermelon which she throws towards Majnu. The man catches the melon and eats half of it. He throws back the remaining half to Laila. She eats the half of the melon that she receives & throws the remaining part to Majnu. Majnu again eats half of what he receives and returns the remaining part back to Laila. This continues till the melon lasts. The two are sitting at the two ends of the boat which has a length L. Combined mass of the boat and the two lovers is $M_0$ and the mass of the water melon is M. Assume that the boat can move horizontally on water without any resistive force. Find the displacement of the boat when the watermelon gets finished.




Q. 100. A hot air balloon (mass M) has a passenger (mass m) and is stationary in the mid air. The passenger climbs out and slides down a rope with constant velocity u relative to the balloon.

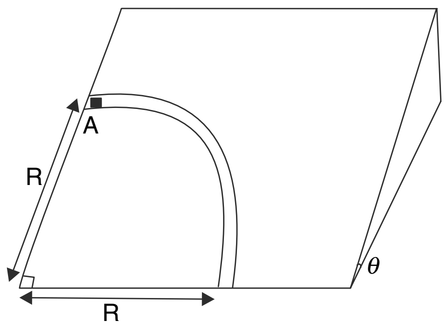
(a) Show that when the passenger is sliding down, there is no change in mechanical energy (kinetic + gravitational potential energy) of the system (Balloon + passenger). Calculate the speed of balloon.

(b) Calculate the power of the buoyancy force on the system when the man is sliding. For easy calculation, assume that volume of man is negligible compared to the balloon.




(c) If buoyancy force is doing positive work, where is this work done lost? You have proved that sum of kinetic and potential energy of the system remains constant.

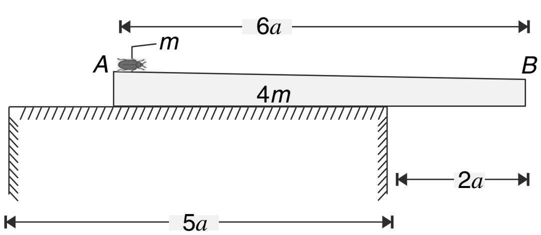
Q.101. A wooden wedge of mass 10 $m$ has a smooth groove on its inclined surface. The groove is in shape of quarter of a circle of radius $R$ = 0.55 m. The inclined face makes an angle $\theta = \cos^{-1}\left(\frac{\sqrt{11}}{5}\right)$ with the horizontal. A block '$A$' of mass $m$ is placed at the top of the groove and given a gentle push so as to slide along the groove. There is no friction between the wedge and the horizontal ground on which it has been placed. Neglect width of the groove.




(a) Find the magnitude of displacement of the wedge by the instant the block A reaches the bottom of the groove.

(b) Find the velocity of the wedge at the instant the block A reaches the bottom of the groove.

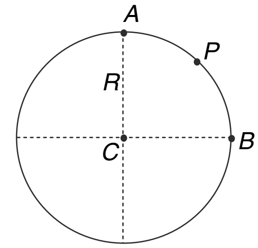
Q. 102. A uniform bar $AB$ of length 6$a$ has been placed on a horizontal smooth table of width 5 $a$ as shown in the figure. Length 2$a$ of the bar is overhanging. Mass of the bar is 4$m$. An insect of mass $m$ is sitting at the end $A$ of the bar. The insect walks along the length of the bar to reach its other end $B$.




(a) Will the bar topple when the insect reaches end $B$ of the bar?

(b) After the insect reaches at $B$, another insect of mass $M$ lands on the end $A$ of the bar. Find the largest value of $M$ which will not topple the bar.

Q. 103. A disc of mass $M$ and radius $R$ lies on a smooth horizontal table. Two men, each of mass $\frac{M}{2}$, are standing on the edges of two perpendicular radii at $A$ and $B$.

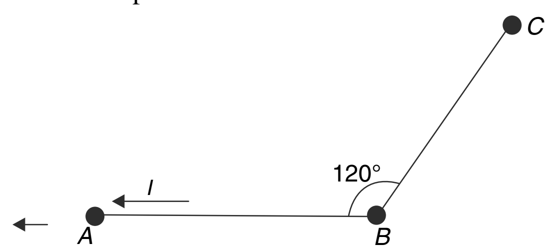



Find the displacement of the centre of the disc if

(a) The two men walk radically relative to the disc so as to meet at the centre $C$.

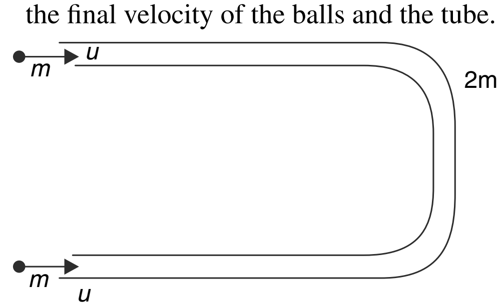
(b) The two men walk along the circumference to meet at the midpoint(P) of the are $AB$.

Q. 104. There particles $A$, $B$ and $C$ have masses $m$, 2$m$ and $m$ respectively. They lie on a smooth horizontal table connected by light inextensible strings $AB$ and $BC$. The string are taut and <$ABC$ = 120°. An impulse is applied to particle $A$ along $BA$ so that it acquires a velocity $u$. Find the initial speeds of $B$ and $C$.

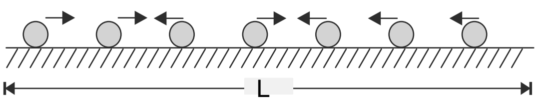



## LEVEL 3

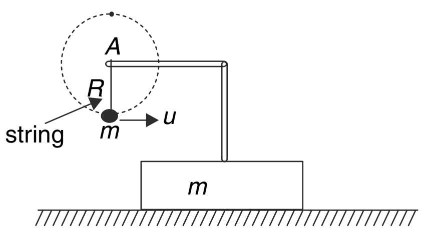
Q. 105. A smooth hollow $U$ shaped tube of mass 2$m$ is lying at rest on a smooth horizontal table. Two small balls of mass $m$, moving with velocity $u$ enter the tube simultaneously in symmetrical fashion. Assume all collisions to be elastic. Find the final velocity of the balls and the tube.




Q. 106. There are 40 identical balls travelling along a straight line on a smooth horizontal table. All balls have equal speed $v$ and each one is travelling to right or left. All collisions between the balls is head on elastic. At some point in time all balls will have fallen off the table. The time at which this happens will definitely depend on initial positions of the balls. Over all possible initial positions of the balls; what is the longest amount of time that you would need to wait to ensure that the table has no more balls? Assume that length of the table is $L$.




Q. 107. A small ball of mass $m$ is suspended from the end $A$ of a $L$ shaped mass less rigid frame which is fixed to a block of mass $m$. The block is placed on a smooth table. The ball is given a horizontal impulse so as to impart it a velocity of $u$. The ball beings to rotate in a circle of radius $R$ about the point $A$, while the block and the frame slide on the table. Find the tension in the string, to which the ball is attached, at the instant the ball is at the top most position. The rod does not interfere with the string during the motion.




Q. 108. A heavy rope of mass $m$ and length $2L$ is hanged on a smooth little peg with equal lengths on two sides of the peg. Right part of the rope is pulled a little longer and released. The rope begins to slide under the action of gravity. There is a smooth cover on the peg (so that the rope passes through the narrow channel formed between the peg and the cover) to prevent the rope from whiplashing.

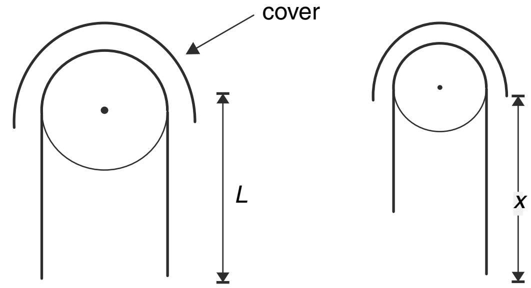
(a) Calculate the speed of the rope as a function of its length ($x$) on the right side.

(b) Differentiate the expression obtained in (a) to find the acceleration of the rope as a function of $x$.

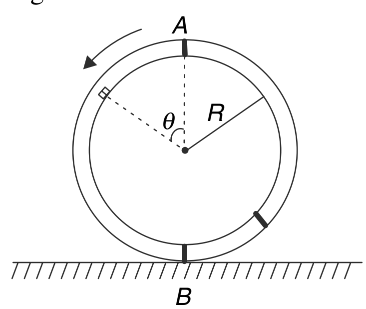
(c) Write the rate of change of momentum of the rope as a function of $x$. Take downward direction as positive

(d) Find the force applied by the rope on the peg as a function of $x$.

(e) For what value of $x$, the force found in (d) becomes zero? What will happen if there is no cover around the peg?




Q. 109. Two thin rings of slightly different radii are joined together to make a wheel (see figure) of radius $R$. There is a very small smooth gap between the two ring. The wheel has a mass $M$ and its centre of mass is at its geometrical centre. The wheel stands on a smooth surface and a small particle of mass $m$ lies at the top ($A$) in the gap between the rings. The system is released and the particle begins to slide down along the gap. Assume that the ring does not lose contact with the surface.

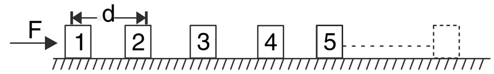



(a) As the particle slides down from top point $A$ to the bottom point $B$, in which direction does the centre of the wheel move?

(b) Find the speed of centre of the wheel when the particle just reaches the bottom point $B$. How much force the particle is exerting on the wheel at this instant?

(c) Find the speed of the centre of the wheel at the moment the position vector of the particle with respect to the centre of the wheel makes an angle $\theta$ with the vertical. Do this calculation assuming that the particle is in contact with the inner ring at desired value of $\theta$.

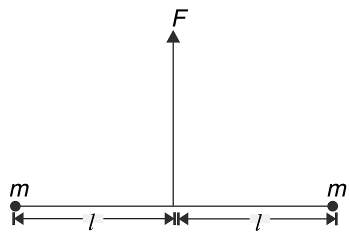
Q. 110. A large number of small identical blocks, each of mass $m$, are placed on a smooth horizontal surface with distance between two successive blocks being $d$. A constant force $F$ is applied on the first block as shown in the figure.



(a) If the collisions are elastic, plot the variation of speed of block 1 with time.

(b) Assuming the collisions to be perfectly inelastic, find the speed of the moving blocks after $n$ collisions. To what value does this speed tend to if $n$ is very large.

Q. 111. Two small balls, each of mass $m$ are placed on a smooth table, connected with a light string of length $2l$, as shown in the figure. The midpoint of the string is pulled along $y$ direction by applying a constant force $F$. Find the relative speed of the two particles when they are about to collide. If the two masses collide and stick to each other, how much kinetic energy is lost.

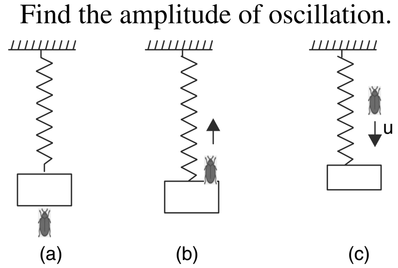



Q. 112. A block of mass $M$ is tied to a spring of force constant $K$ and the system is suspended vertically. Consider three situations shown in fig. (a), (b) and (c).

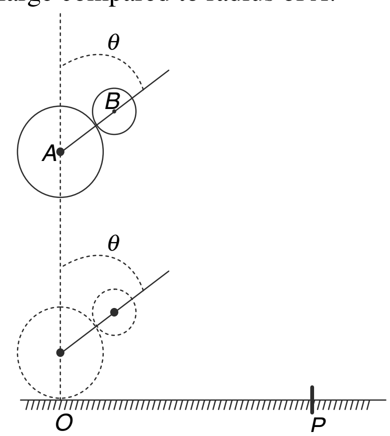
(a) In fig. (a), an insect of mass $M$ is clinging to the block and the system is in equilibrium. The insect leaves the block and falls. Find the amplitude of resulting oscillations.

(b) In fig. (b), an insect of mass $M$ is resting on the top of the block and the system is in equilibrium. The insect suddenly jumps up with a sped $u = g\sqrt{\frac{M}{K}}$ and the block starts oscillating. Find amplitude of oscillation assuming that the insect never falls back on the block.

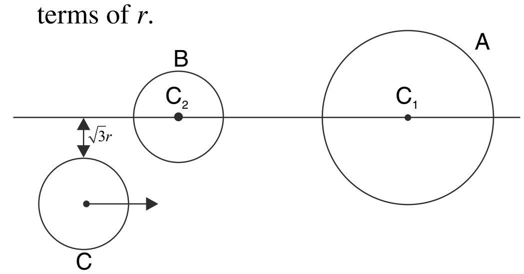
(c) In fig. (c), an insect of mass $M$ falls on the block that is in equilibrium. The insect hits the block with velocity $u = g\sqrt{\frac{M}{K}}$ while moving downwards and sticks to the block. Find the amplitude of oscillation.




Q. 113. A massive ball ($A$) is dropped from height $h$ on a smooth horizontal floor. $A$ smaller ball ($B$) is also dropped simultaneously. Initially ball $B$ was just touching ball $A$ (see fig.). Radii of both balls is much smaller than $h$. Ball $A$ hits the floor, rebounds and immediately hits $B$. Motion of both the balls is vertical before the collision of two balls. All collision are elastic and there is no friction. Ball $B$ lands at point $P$ on the ground after colliding with $A$. Find $OP$, assuming that it is large compared to radius of $A$.




Q. 114. Disc A of radius $R$ is lying flat on a horizontal surface. Disc $B$ is also at rest. Disc $C$, which is identical to $B$ is traveling along the surface with its velocity parallel to the line joining the centre $C_1$ and $C_2$ of the discs $A$ and $B$. The distance between the line $C_1C_2$ and the line of motion of centre of disc $C$ is $\sqrt{3}.r$ , where $r$ is radius of both $B$ and $C$. Impact of $C$ with $B$ is completely elastic. Subsequently it is observed that both $B$ and $C$ just miss hitting the disc $A$. Find the radius ($R$) of $A$ in terms of $r$.




Q. 115. A mass $m$ moving with speed $u$ in $x$ direction collides elastically with a stationary mass $2m$. After the collision, it was found that both masses have equal $x$ components of velocity. What angle does the velocity of mass $2m$ make with the $x$ axis?

Q. 116. A ball of mass $M$ collides elastically with another stationary ball of mass $m$. If $M > m$, find the maximum angle of deflection of $M$.

Q. 117. A tennis ball is lying on a rigid floor. A steel ball is dropped on it from some height. The steel ball bounces vertically after hitting the ball on the floor. Is it possible that the tennis ball will also bounce?

# ANSWERS

**1.** $\Delta p = 10\sqrt{29}$ Ns; $\theta = \cos^{-1}\left(\dfrac{3}{\sqrt{29}}\right)$

**2.** $F_2$

**3.** (a) 400 $J$

(b) Both performed equal work.

**4.** $\vec{a}_{car} = -2\hat{i} + 2\hat{j}$

**5,** $\left(\dfrac{\sqrt{7}-\sqrt{3}}{4}\right)\rho A V^2$

**6.** $\mu_{\min} = \dfrac{mg}{\rho S (V+u)^2}$

**7.** $mu^2$

**8.** (i) $\dfrac{M}{m} = 1$

(ii) 1/3

**9.** $\dfrac{u+V}{2}$

**10.** 3.24 $ms^{-1}$making an angle of 44° with the normal to the wall

**11.** A time $\dfrac{u}{g}$ after first collision

**12.** $\dfrac{M}{m} \le 2$

**13.** $V = \sqrt{\dfrac{K}{2M}}x_0$

**14.** (a) $KE_{\text{loss}} = \dfrac{mu^2}{3}$

(b) No

**15.** (a) 3 $m/s$ towards left

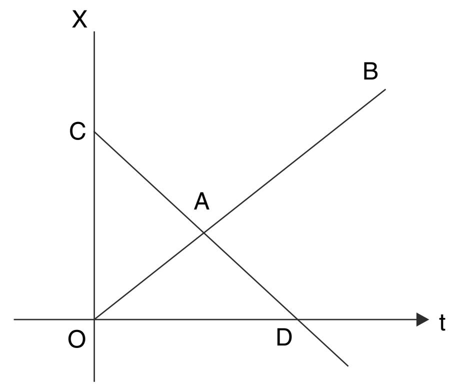
(b) $K_{\min} = 2J$

(c) $K_{\min} = 2J$

**16.** $\dfrac{m^2u^2}{2\mu g\left(M^2 - m^2\right)}$

**17.** $\dfrac{u}{2}$

**18.** (a)




$O - A - D \rightarrow$ *Ball* 1.

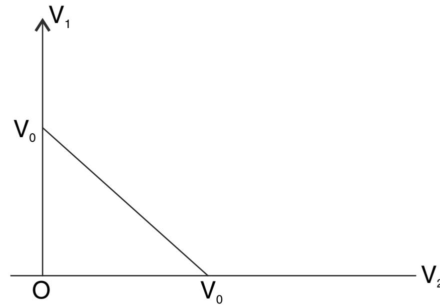
$C - A - B \rightarrow$ *Ball* 2.

(b) 10

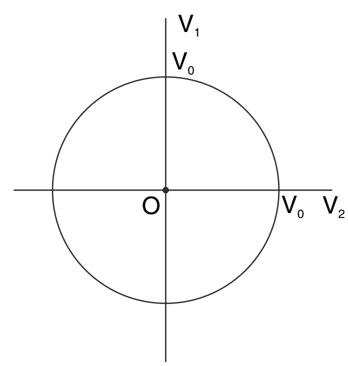
**19.** 2

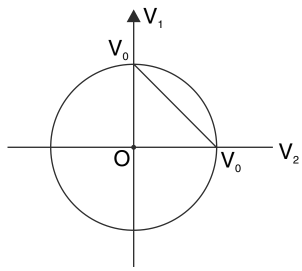
**20.** $\dfrac{u_1 - u_2}{V_2 - V_1}$

**21.** (a)




(b)



(c) 


$V_1 = 0, V_2 = V_0$

(d) $V_1 = 0.5$ *m/s* ; $V_2 = 1.5$ *m/s*; % loss in KE = 37.5%

**22.** (a) The heavier particle moves with a velocity of $\frac{2V_0}{21}$ in the direction of $\vec{V}_0$.

Lighter particle moves with velocity $\frac{19V_0}{21}$ opposite to $\vec{V}_0$.

The incident particle comes to rest. The other particle moves with $\vec{V}_0$.

**23.** 0.1*m*

**24.** less than *H*

**25.** 1/3

**26.** Slightly less than 6 *m/s*

**27.** 13 *m*

**28.** $\frac{n(n+1)}{2}$

**29.** 100 *m*, 92.8%

**30.** (a) $e = 0$ (b) $\frac{1}{3}$

**32.** 12

**33.** (a) *A* will be at rest and *B* will have a velocity *u*

(b) $\frac{mu^2}{4}$

(c) Both will be travelling with velocity $\frac{u}{2}$. Loss in $KE = \frac{mu^2}{4}$

**34.** $\frac{L}{260}$

**35.** $\frac{20}{3}kg$

**36.** (i) $\frac{r}{2015}$

(ii) A particle next to the blank vertex.

**37.** (a) $\frac{1}{2}\sqrt{\frac{k}{m}}x_0$

(b) $\frac{\sqrt{3}kx_0}{4m}$

**38.** 0.2 *m*

**39.** (a) 4 *m/s*

(b) 1560*N*

**40.** (To right)

**41.** 40.56 *m*

**42.** $t = \sqrt{\frac{4ML}{3F}}$

**43.** No

**44.** 7.5*R*

**45.** (a) 10 *m/s*

(b) 100 *m*

**46.** $v = \frac{u}{1 + \frac{M}{2bt}}$

**47.** (i) $T_{s1} = T_{s2} = T_{s3} = \rho A v^2$

(ii) $\frac{\pi M V_0}{mu}$

**48.** (a) 5 *s*

(b) 25 $ms^{-1}$

**49.** $F \simeq 1200\ N$

**50.** (a) $\frac{g}{2}$

(b) $g - \frac{v_0^2}{l_0}$

**51.** (a) Smaller bullet

(b) $\frac{Mu}{M + \rho A u t}$

(c) $\frac{M}{\rho A}\ell n\left[1 + \frac{\rho A u}{M}t\right]$

**52.** (a) $\sqrt{\frac{8}{5}\frac{L}{g}}$

(b) $\frac{mgL}{5}$

**53.** (a) $\frac{2mg}{3}$

(b) $2\sqrt{gL}$

**54.** $\Delta t = \frac{1}{12}s$

**55.** (a) $d = 90\,L \quad u = \sqrt{90gL}$

(b) $m\sqrt{45gL}$

**56.** $V_A = \frac{36}{7} m/s;\ V_B = \frac{30}{7} m/s\ ;\ V_C = \frac{33}{7} m/s$

**57.** (a) $\frac{L}{v}$

(b) $\frac{3}{4}\ mv^2$

**58.** 40 *m/s*

**60.** (i) $\frac{L}{5}$

(ii) 5 $m/s^{-1}$

**61.** $\frac{2}{3}\left[L + \mu\sqrt{\frac{2H}{g}}\right]$

**62.** 5 *m/s*

**63.** (a) $\frac{5}{3} m$

(b) $\frac{96}{9} = 10.67m$

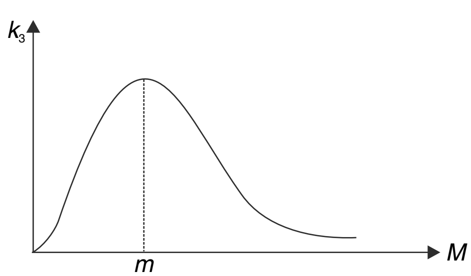
**65.** $T_P = \lambda v^2;\ F_{ext} = \lambda v^2$

**66.** (i) $v = \frac{Mv_0}{M + \lambda x}$

(ii) $T = \frac{M^3 {v^2}_0 \lambda}{(M + \lambda x)^3}$

**67.** (i) (a) $16k_0 \frac{m^2 M^2}{(M+m)^4}$

(b)




(ii) $k_{10} = k_1$

**68.** 4

**69.** (a) Left

(b) Zero

**70.** 2.96 *m*

**71.** (a) 90 *kg*

(b) $\frac{1}{4}$

**73.** (b) $h = \frac{v^2 \cos^2\theta\left(1 - e^2\right)}{2g}$

$x = (e+1)\frac{v^2}{2g}\sin 2\theta$

**74.** (a) $\frac{9}{16}$

(b) 36 %

**75.** 120°

**76.** (a) $\theta_0 = 60°$

(b) $T = \frac{2mg}{\sqrt{3}}$

**77.** (i) $v_m = \left(\frac{m - M}{M + m}\right)u;\ v_M = \left(\frac{2mu}{M + m}\right)$

(ii) $Mg\left(\frac{R + r}{R - r}\right)$

**78.** $\frac{4\sin\theta\sqrt{gR(1 - \sin\theta)}}{1 + \sin^2\theta}$ where $\sin\theta = \frac{1}{\sqrt{5}}$

**79.** (i) $\frac{H^5}{L^4}$

(ii) (a) $\frac{3}{4}mg$

(b) $\frac{5l}{4}$

(c) $2\pi\sqrt{\frac{3l}{4g}}$

(d) $\sqrt{\frac{5l}{2g}}$

**81.** (b) $\sqrt{u^2 + 3u_0^2 + 3uu_0}$

**82.** (a) $2R + 9h$

**83.** 6 *m/s*

**84.** $\frac{mx}{M}$

**85.** $\frac{2u}{3}$, 180°

**86.** $\frac{Ft^2}{4m} + \frac{l}{2}$

**87.** $m_2 = 4\ kg$ ; $l_0 = 6\ cm$

**88.** (c) $\frac{1}{2}(m_1 + m_2)V_0^2$

**89.** $v_r = \sqrt{\frac{3k}{2m}}x$

**90.** $\sqrt{\frac{2m}{3k}}\ v_0$

**91.** (a) A circle of radius $\frac{mR}{M+m}$

(b) $\frac{m}{M} = 1$

**92.** $\frac{2}{\pi} = \frac{MV^2}{R}$

**93.** 1.92 J

**94.** $\frac{11h}{15}$

**95.** (a) Closer to O

(b) 0.03 $R$

**96.** $\frac{mg}{3K}$

**97.** (a) $\frac{R\sin\alpha}{\alpha}$

(b) $V = 2\sqrt{\frac{(\sqrt{2}-1)\text{gR}}{\pi}}$ ; $V_{cm} = \frac{4\sqrt{2}}{\pi}\sqrt{\frac{(\sqrt{2}-1)\text{gR}}{\pi}}$

(c) $\frac{R}{\pi}$

**98.** (a) $t_0 = \frac{2\pi R}{V_0}$

(b) $\frac{V_0}{2}$

**99.** $\frac{2ML}{3(M_0 + M)}$

**100.** (a) $\frac{mu}{M+m}$

(b) $mg.\ u$

**101.** (a) 6 $cm$

(b) 0.18 $m/s$

**102.** (a) No

(b) 85 $m$

**103.** (a) $\frac{R}{2\sqrt{2}}$

(b) $\frac{(\sqrt{2}-1)R}{2\sqrt{2}}$

**104.** $v_B = \frac{2\sqrt{31}}{11}u,\ v_c = \frac{4u}{11}$

**105.** $v_{ball} = 0$; $v_{tube} = u$

**106.** $\frac{L}{v}$

**107.** $T = \frac{mu^2}{R} - 13\ mg$

**108.** (a) $v = \sqrt{\frac{g}{L}}(x - L)$

(b) $a = \frac{g}{L}(x - L)$

(c) $\frac{dp}{dt} = 2mg\left(\frac{x-L}{L}\right)^2$

(d) $F = mg\left[1 - 2\left(\frac{x-L}{L}\right)^2\right]$

(e) $x = L + \frac{L}{\sqrt{2}}$

**109.** (a) First moves to right and then to left

(b) $v_w = 2m\sqrt{\frac{gR}{M(M+m)}}$

(c) $v_w = \sqrt{\frac{2m^2 gR\cos^2\theta(1-\cos\theta)}{(M+m)^2 + Mm\text{co}^2\theta}}$

**110.** (a) See the solution for the graph

(b) $\sqrt{\frac{n}{n+1}\frac{Fd}{m}};\sqrt{\frac{Fd}{m}}$

**111.** $2\sqrt{\frac{Fl}{m}}; Fl$

**112.** (a) $\frac{Mg}{K}$ (b) $\sqrt{2}\frac{Mg}{K}$

(c) $\frac{\sqrt{6}}{2}\frac{Mg}{K}$

**113.** $16h\ \sin 2\theta\left[\frac{1}{2} + \cos 2\theta\right]$

**114.** $R = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)r$

**115.** 45°

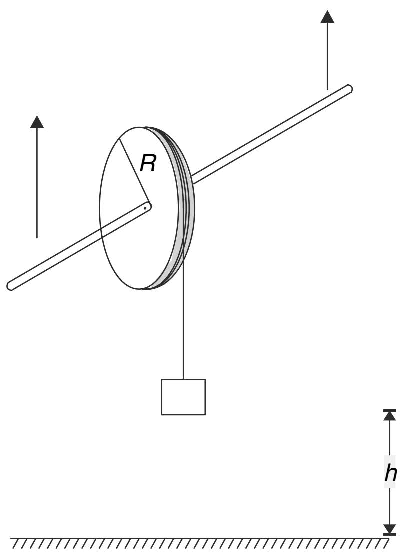
**116.** $\sin^{-1}(m/M)$

**117.** Yes.

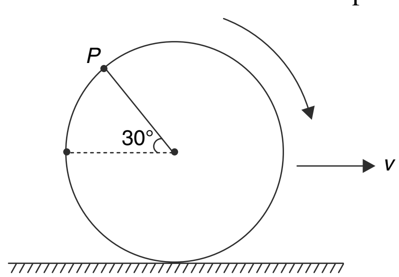
# 06 ROTATIONAL MOTION

## Level 1

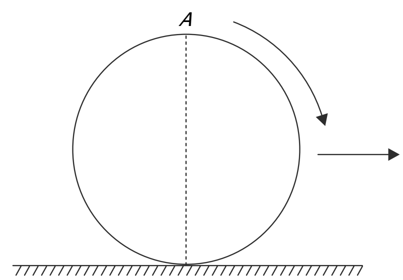
Q. 1. The pulley of radius $R$ can rotate freely about its axle as shown in the figure. A thread is tightly wrapped around the pulley and its free end carries a block of mass $m$. When the block is at a height $h$ above the ground the system is released (i.e., the pulley is made free to rotate & the block is allowed to fall) and at the same instant the axle is moved up keeping it horizontal all the time. When the block hits the floor the axle has gone up by a distance $2h$. Find the angle by which the pulley must have rotated by this time.



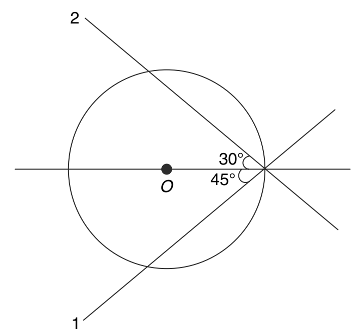

Q. 2. A disc is rolling without sliding on a horizontal surface. Velocity of the centre of the disc is $v$. Find the maximum relative speed of any point on the circumference of the disc with respect to point $P$.



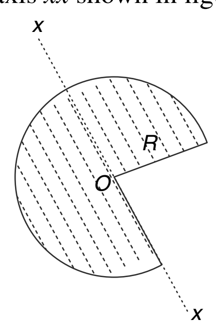

Q. 3. A ring is rolling, without slipping on a horizontal surface with constant velocity. Speed of point $A$ (at the top) is $v_A$. After an interval $T$, the speed of point $A$ again becomes $v_A$. During what fraction of the interval $T$ speed of point $A$ was greater than $\frac{\sqrt{3}}{2}v_A$.



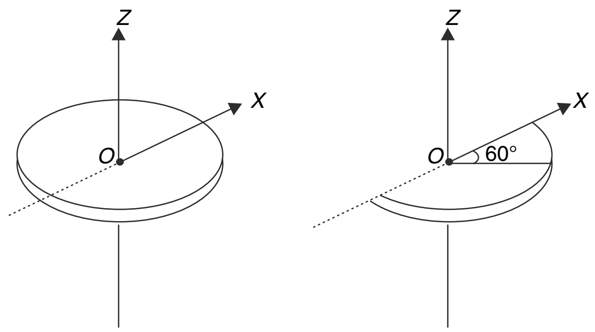

Q. 4. Calculate the ratio of moment of inertia of a thin uniform disc about axis 1 and 2 marked in the figure. $O$ is the centre of the disc.



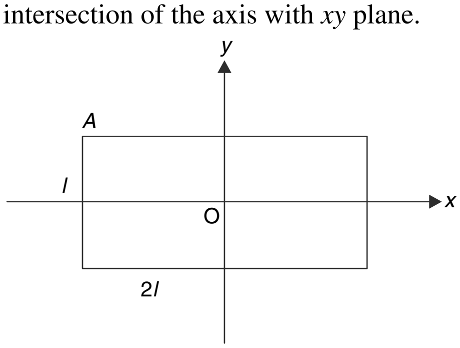

Q. 5. A uniform circular disc has a sector of angle 90° removed from it. Mass of the remaining disc is $M$. Write the moment of inertia of the remaining disc about the axis $xx$ shown in figure (Radius is $R$)



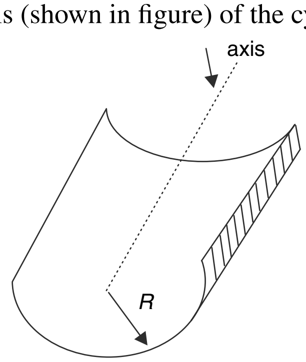
Q. 6. An Indian bread "Roti" is a uniform disc of mass $M$ and radius $R$. Before eating a person usually folds it about its diameter (say about $x$ axis). After folding it a sector of angle 60° is removed from it. Find the moment of inertia of the remaining "Roti" about Z-axis.




Q. 7. A uniform rectangular plate has side length $\ell$ and $2\ell$. The plate is in $x - y$ plane with its centre at origin and sides parallel to $x$ and $y$ axes. The moment of inertia of the plate about an axis passing through a vertex (say A) perpendicular to the plane of the figure is $I_0$. Now the axis is shifted parallel to itself so that moment of inertia about it still remains $I_0$. Write the locus of point of intersection of the axis with $xy$ plane.



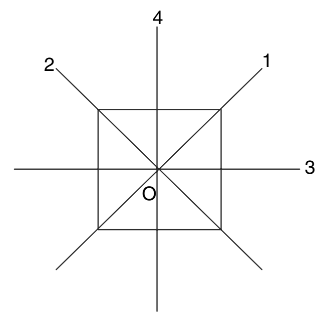

Q. 8. A thin semi circular cylindrical shell has mass $M$ and radius $R$. Find its moment of inertia about a line passing through its centre of mass parallel to the axis (shown in figure) of the cylinder.




Q. 9. Consider a uniform square plate shown in the figure. $I_1$, $I_2$, $I_3$ and $I_4$ are moment of inertia of the plate about the axes 1, 2, 3 and 4 respectively. Axes 1 and 2 are diagonals and 3 and 4 are lines passing through centre parallel to sides of the square. The moment of inertia of the plate about an axis passing through centre and perpendicular to the plane of the figure is equal to which of the followings.

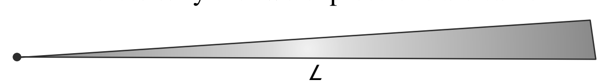
(a) $I_3 + I_4$ (b) $I_1 + I_3$

(c) $I_2 + I_3$ (d) $\frac{1}{2}(I_1 + I_2 + I_3 + I_4)$

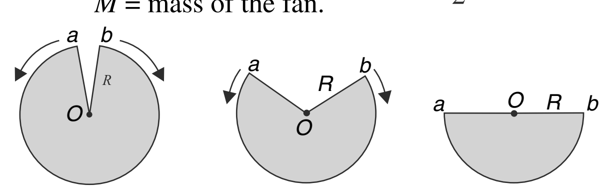



Q. 10. An asteroid in the shape of a uniform sphere encounters cosmic dust. A thin uniform layer of dust gets deposited on it and its mass increases by 2%. Find percentage change in its moment of inertia about diameter.

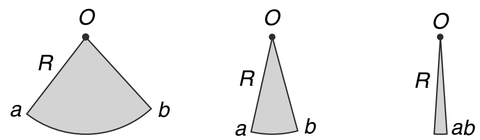
Q. 11. (i) Consider an infinitesimally thin triangular strip having mass $M$ and length $L$. Find the moment of inertia of the strip about on axis passing through its tip and perpendicular to the plane. Compare the result with moment of inertia of a uniform disc of mass $M$ and radius $L$ about an axis passing through its centre and perpendicular to the plane the disc. Why the two expressions are same?




(ii) A circular fan made of paper is in shape of a disc of radius $R$. The fan can be folded (various stages shown in figure ($a$) through ($f$)) to the shape of a thin stick. The moment of inertia of the circular fan about an axis passing through centre $O$ and perpendicular to the plane of the figure is $I_0 = \frac{1}{2}MR^2$ where $M$ = mass of the fan.




(a) (b) (c)




(d) (e) (f)

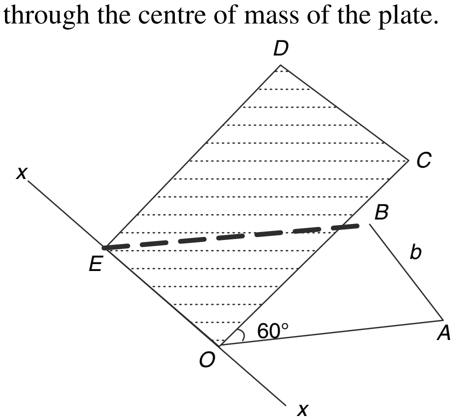
(a) How does the moment of inertia ($I$), about an axis perpendicular to the plane of the figure passing through $O$, change as the fan is folded through stage a to b to c to d to e?

(b) When the fan is completely folded in the shape of a stick (fig. ($f$)), write its moment of inertia about the above mentioned axis.

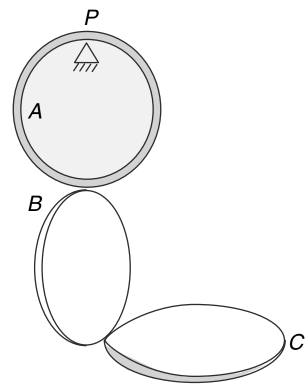
Note : Moment of inertia of a uniform rod about an axis through its end and perpendicular to it is $\frac{ML^2}{3}$.

Q. 12. A uniform rectangular plate has moment of inertia about its longer side, equal to $I$. The moment of inertia of the plate about an axis in its plane, passing through the centre and parallel to the shorter sides is also equal to $I$. Find its moment of inertia about an axis passing through its centre and perpendicular to its plane.

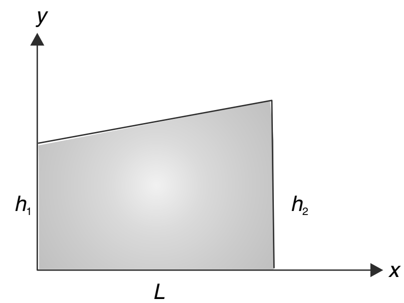
Q. 13. A uniform rectangular plate has been bent as shown in the figure. The two angled parts of the plate are of identical size. The moment of inertia of the bent plate about axis $xx$ is $I$. Find its moment of inertia about an axis parallel to $xx$ and passing through the centre of mass of the plate.



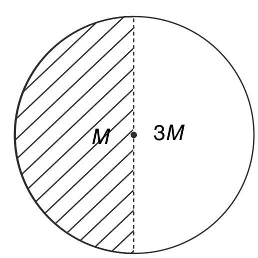

Q. 14. Three identical rings each of mass $M$ and radius $R$ are welded together with their planes mutually perpendicular to each other. Ring $A$ is vertical and $B$ is also vertical in a plane perpendicular to $A$. Ring $C$ is in horizontal plane. Find moment of Inertia of this system about a horizontal axis perpendicular to the plane of the figure passing through point $P$ (top point of ring $A$)



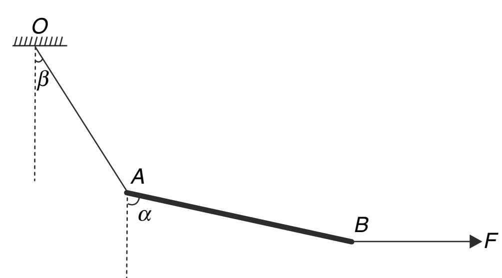

Q. 15. Determine the moment of inertia of the shaded area about $y$ axis. The mass of the shaded area is $M$.




Q. 16. Two uniform semicircular discs, each of radius $R$, are stuck together to form a disc. Masses of the two semicircular parts are $M$ and $3M$. Find the moment of inertia of the circular disc about an axis perpendicular to its plane and passing through its centre of mass.

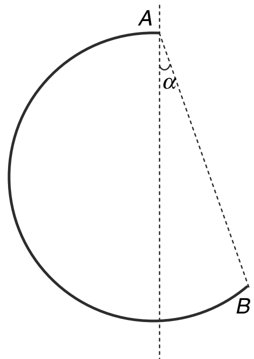



Q. 17. A stick $AB$ of mass $M$ is tied at one end to a light string $OA$. A horizontal force $F = Mg$ is applied at end $B$ of the stick and its remains in equilibrium in position shown. Calculate angles $\alpha$ and $\beta$.

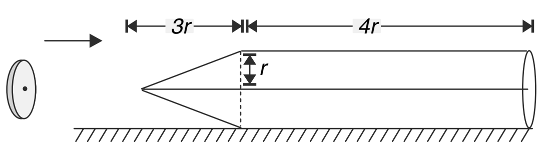


Q. 18. When brakes are applied on a moving car, the car dips to the front. Why ? [That is try to show that front wheels are more pressed as compared to rear ones when the brakes are applied]. Assume that centre of mass of the car is equidistant from the front and rear wheels.

Q. 19. A uniform wire has been bent in shape of a semi circle. The semicircle is suspended about a horizontal axis passing through one of its ends, so that the semicircular wire can swing in vertical plane. Find the angle $\alpha$ that the diameter of the semicircle makes with vertical in equilibrium.

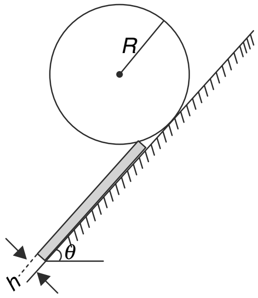



Q. 20. A uniform cylindrical body of radius $r$ has a conical nose. The length of the cylindrical and conical parts are $4r$ and $3r$ respectively. Mass of the conical part is $M$. The body rests on a horizontal surface as shown. A ring of radius $\frac{r}{2}$ is to be tightly fitted on the nose of the body. What is maximum permissible mass of the ring so that the body does not topple?




Q. 21. There is a step of height $h$ on an incline plane. The step prevents a ball of radius $R$ from rolling down.

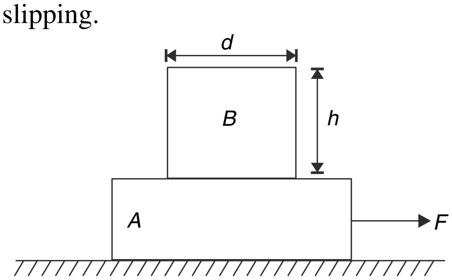
(a) If the inclination ($\theta$) of the incline is increased gradually, at what value of $\theta$ the ball will just manage to climb the step?



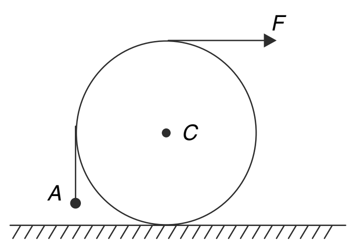

(b) Does the gravitational potential energy of the ball increases or decreases as it climbs the step?

Q. 22. The centre of mass of an inhomogeneous sphere is at a distance of 0.3 $R$ from its geometrical centre. $R$ is the radius of the sphere. Find the maximum inclination ($\theta$) of an incline plane on which this sphere can be placed in equilibrium. Assume that friction is large enough to prevent slipping.

Q. 23. Rectangular block $B$, having height $h$ and width $d$ has been placed on another block $A$ as shown in the figure. Both blocks have equal mass and there is no friction between $A$ and the horizontal surface. A horizontal time dependent force $F = kt$ is applied on the block $A$. At what time will block $B$ topple? Assume that friction between the two blocks is large enough to prevent $B$ from slipping.



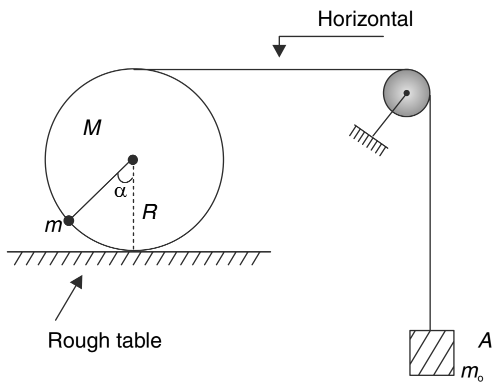

Q. 24. A cylinder $C$ rests on a horizontal surface. A small particle of mass $m$ is held in equilibrium connected to an overhanging string as shown. The other end of the mass less string is being pulled horizontally by a force $F$ as shown. Find $F$.



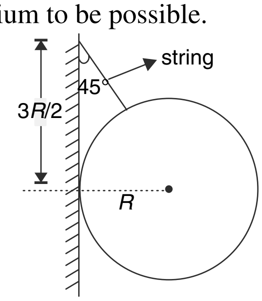

Q. 25. A hollow cylindrical pipe of mass $M$ and radius $R$ has a thin rod of mass $m$ welded inside it, along its length. A light thread is tightly wound on the surface of the pipe. A mass $m_0$ is attached to the end of the thread as shown in figure. The system stays in equilibrium when the cylinder is placed such that $\alpha = 30°$. The pulley shown in figure is a disc of mass $\frac{M}{2}$.

(a) Find the direction and magnitude of friction force acting on the cylinder.

(b) Express mass of the rod '$m$' in terms of $m_0$



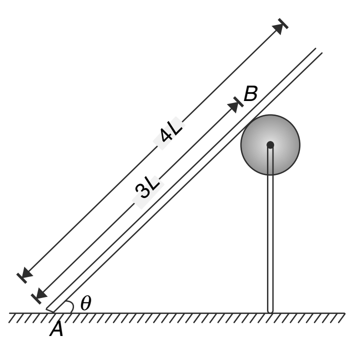

Q. 26. A sphere of radius $R$ is supported by a rope attached to the wall. The rope makes an angle $\theta = 45°$ with respect to the wall. The point where the rope is attached to the wall is at a distance of $\frac{3R}{2}$ from the point where the sphere touches the wall. Find the minimum coefficient of friction ($\mu$) between the wall and the sphere for this equilibrium to be possible.




Q. 27. A uniform rod has mass $M$ and length $4L$. It rests in equilibrium with one end on a rough horizontal surface at $A$. At point $B$, at a distance $3L$ from $A$, it is supported by a fixed smooth roller. The rod just remains in equilibrium when $\theta = 30°$

(a) Find the normal force applied by the horizontal surface on the rod at point $A$.

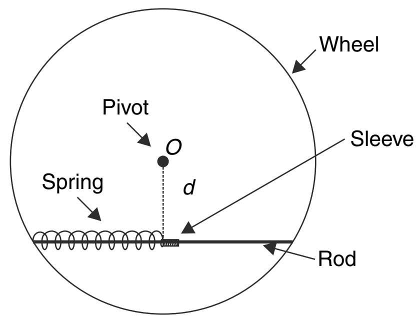
(b) Find the coefficient of friction between the rod and the surface.



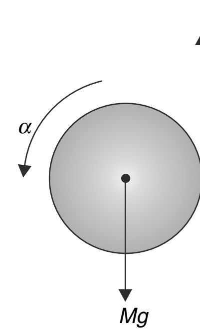

Q. 28. A wheel is mounted on frictionless central pivot and it can rotate freely in the vertical plane. There is a horizontal light rod fixed to the wheel below the pivot. There is a small sleeve of mass $m$ which can slide along the rod without friction. The sleeve is connected to a light spring. The other end of the spring is fixed to the rim as shown. The sleeve is at the centre of the rod and the spring is relaxed. Now the wheel is held at rest and the sleeve is moved towards left so as to compress the sprig by some distance. The sleeve and the wheel are released simultaneously from this position.

(a) Is it possible that the wheel does not rotate as the sleeve perform SHM on the rod ?

(b) Find the value of spring constant $k$ for situation described in (a) to be possible. The distance of rod from centre of the wheel is $d$.

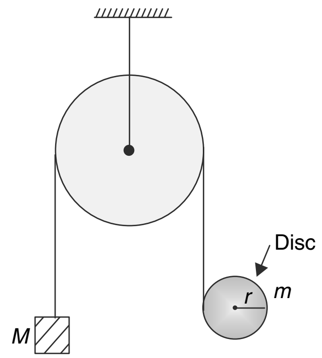



Q. 29. A string is wrapped around a cylinder of mass $M$ and radius $R$. The string is pulled vertically upward to prevent the centre of mass from falling as the string unwinds. Assume that the cylinder remains horizontal throughout and the thread does not slip. Find the length of the string unwound when the cylinder has reached an angular speed $\omega$.




Q. 30. A mass less string is wrapped around a uniform disc of mass $m$ and radius $r$. The string passes over a mass less pulley and is tied to a block of mass $M$ at its other end (see figure). The system is released from rest. Assume that the string does not slip with respect to the disc.

(a) Find the acceleration of the block for the case $M = m$

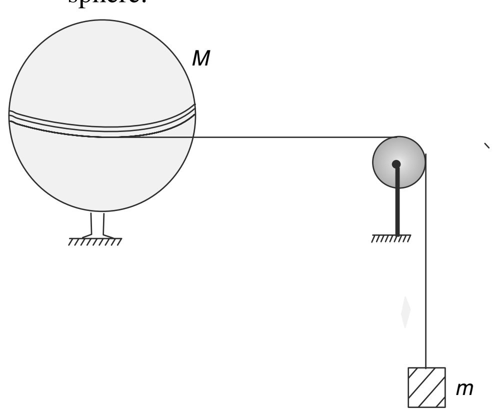



(b) Find $\frac{M}{m}$ for which the block can accelerate upwards.

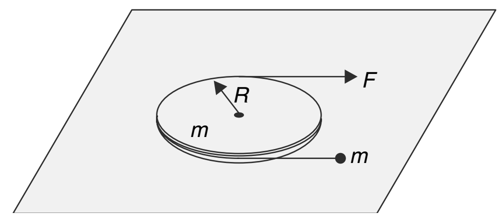
Q. 31. A solid uniform sphere of mass $M$ and radius $R$ can rotate about a fixed vertical axis. There is no frictional torque acting at the axis of rotation. A light string is wrapped around the equator of the sphere. The string has exactly 6 turns on the sphere. The string passes over a light pulley and carries a small mass $m$ at its end (see figure). The string between the sphere and the pulley is always horizontal. The system is released from rest and the small mass falls down vertically. The string does not slip on the sphere till 5 turns get unwound. As soon as $5^{th}$ turn gets unwound completely, the friction between the sphere and the string vanishes all of a sudden.

(a) Find the angular speed of the sphere as the string leaves it.

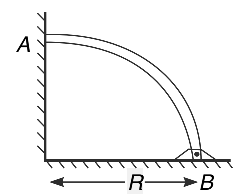
(b) Find the change in acceleration of the small mass $m$ after 5 turns get unwound from the sphere.




Q. 32. A disc of mass $m$ and radius $R$ lies flat on a smooth horizontal table. A mass less string runs halfway around it as shown in figure. One end of the string is attached to a small body of mass $m$ and the other end is being pulled with a force $F$. The circumference of the disc is sufficiently rough so that the string does not slip over it. Find acceleration of the small body.



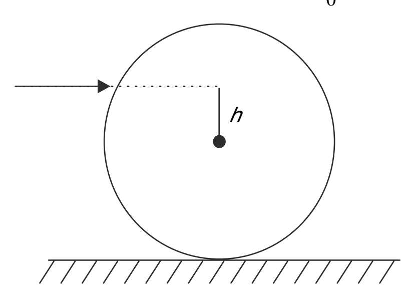

Q. 33. A uniform quarter circular thin rod of mass $M$ and radius $R$ is pivoted at a point $B$ on the floor. It can rotate freely in the vertical plane about $B$. It is supported by a smooth vertical wall at its other free end $A$ so that it remains at rest. Find the reaction force of wall on the rod.




Q. 34. A ball is rolling without sliding down an incline. Is the force applied by the ball on the incline larger than or less than its (ball's) own weight ?

Q. 35. A solid sphere of mass $M$ and radius $R$ is covered with a thin shell of mass $M$. There is no friction between the inner wall of the shell and the sphere. The ball is released from rest, and then it rolls without slipping down an incline that is inclined at an angle $\theta$ to the horizontal. Find the acceleration of the ball.

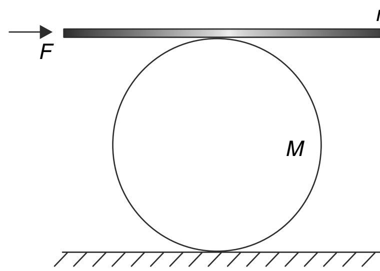
Q. 36. A homogeneous solid sphere of radius $R$ is resting on a horizontal surface. It is set in motion by a horizontal impulse imparted to it at a height $h$ above the centre. If $h$ is greater than $h_0$, the velocity of the sphere increases in the direction of its motion after the start. If $h < h_0$, the velocity decreases after the start. Find $h_0$



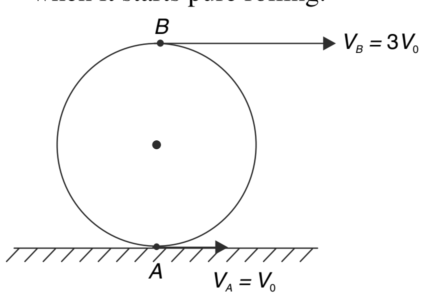
Q. 37. A boy pushes a cylinder of mass $M$ with the help of a plank of mass $m$ as shown in figure. There is no slipping at any contact. The horizontal component of the force applied by the boy on the plank is $F$. Find

(a) The acceleration of the centre of the cylinder

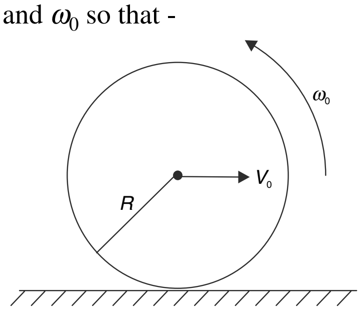
(b) The friction force between the plank and the cylinder




Q. 38. (i) A solid sphere of radius $R$ is released on a rough horizontal surface with its top point having thrice the velocity of its bottom point A ($V_A = V_0$) as shown in figure. Calculate the linear velocity of the centre of the sphere when it starts pure rolling.




(ii) Solid sphere of radius $R$ is placed on a rough horizontal surface with its centre having velocity $V_0$ towards right and its angular velocity being $\omega_0$ (in anticlockwise sense). Find the required relationship between $V_0$ and $\omega_0$ so that -




(a) the slipping ceases before the sphere loses all its linear momentum.

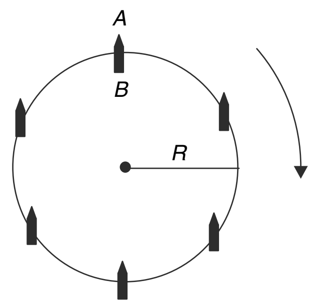
(b) the sphere comes to a permanent rest after some time.

(c) the velocity of centre becomes zero before the spinning ceases.

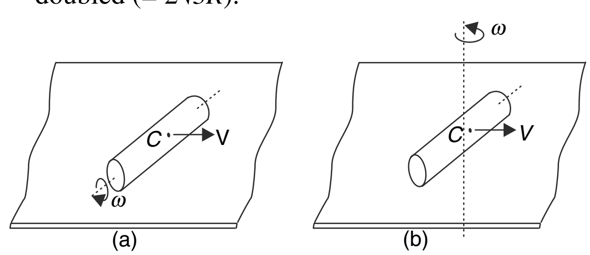
Q. 39. A thin pencil of mass $M$ and length $L$ is being moved in a plane so that its centre (i.e. centre of mass) goes in a circular path of radius $R$ at a constant angular speed $\omega$. However, the orientation of the pencil does not change in space. Its tip (A) always remains above the other end (B) in the figure shown

(a) Write the kinetic energy of the pencil.

(b) Find the magnitude of net force acting on the pencil.

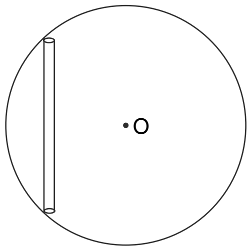



Q. 40. In figure (a) there is a uniform cylinder of mass $M$ and radius $R$. Length of the cylinder is $L = \sqrt{3}R$. The cylinder is rolling without sliding on a horizontal surface with its centre moving at speed $V$. In figure (b) the same cylinder is moving on a horizontal surface with its centre moving at speed $V$ and the cylinder rotating about a vertical axis passing through its centre. [Place your pencil on the table and give a sharp blow at its end. Look at the motion of the pencil. This is how the cylinder is moving]. The angular speed is $\omega = \dfrac{V}{R}$. Write the kinetic energy of the cylinder in two cases. In which case, the kinetic energy would have been higher if length of the cylinder were doubled ($= 2\sqrt{3}R$).




Q. 41. There is a fixed hollow cylinder having smooth inner surface. Radius of the cylinder is $R = 4m$. A uniform rod of $M = 4kg$ and length $L = 4m$ is released from vertical position inside the cylinder as shown in the figure. Convince yourself that the rod will perform pure rotation about the axis of the cylinder passing through $O$.

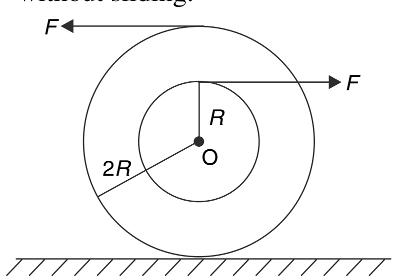
Fid the angular speed of the rod when its becomes horizontal.




Q. 42. A disc shaped body has two tight windings of light threads - one on the inner rim of radius $R = 1m$ and the other on outer rim of radius $2R$ (see figure). It is kept on a horizontal surface and the ends of the two threads are pulled horizontally in opposite directions with force of equal magnitude $F = 20N$. Mass of the body and its moment of inertia about an axis through centre $O$ and perpendicular to the plane of the figure are $M = 4kg$ and $I = 8kg - m^2$ respectively. Find the kinetic energy of the body 2 seconds after the forces begin to act, if

(i) the surface is smooth,

(ii) the surface is rough enough to ensure rolling without sliding.




Q. 43. A uniform square plate has mass $M$ and side length $a$. It is made to oscillate in vertical plane in two different ways shown in figure (A) and (B). In figure (A), the plate is hinged at its upper corners with the help of two mass less rigid rods each of length $a$. The rods can rotate freely about both ends.

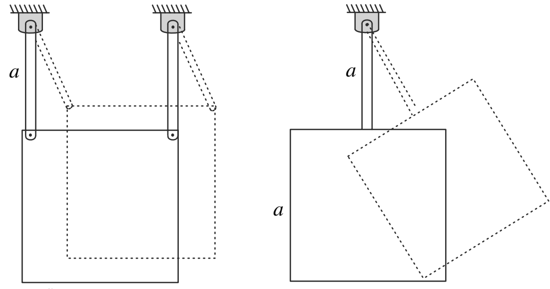
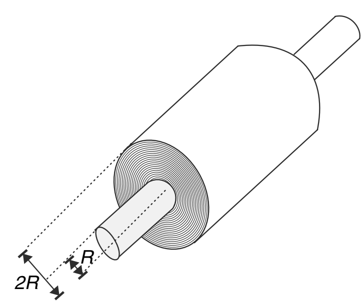



fig. (a) fig. (b)

In figure (B) the plate is rigidly connected at the centre of its top edge to a mass less rod of length $a$. The rod can rotate about its upper end only. In both cases the plate is pushed from its equilibrium position so that centre of mass of the plate acquires a speed $V$. In which case will the centre of mass of the plate rise to a greater height. There is no friction

Q. 44. A thin carpet of mass $2m$ is rolled over a hollow cylinder of mass $m$. The cylinder wall is thin and radius of the cylinder is $R$. The carpet rolled over it has outer radius $2R$ (see figure). This roll is placed on a rough horizontal surface and given gentle push so that the carpet begins to roll and unwind. Friction is large enough to prevent any slipping of the carpet on the floor. Also assume that the carpet does not slip on the surface of the cylinder. The entire carpet is laid out on the floor and the hollow cylinder rolls out with speed $V$. Find $V$.

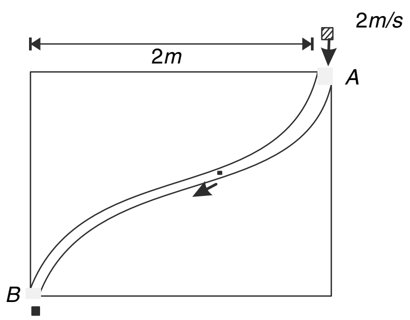



Q. 45. A uniform rod of mass $M$ is moving in a plane and has a kinetic energy of $\frac{4}{3}MV^2$ where $V$ is speed of its centre of mass. Find the maximum and minimum possible speed of the end point of the rod.

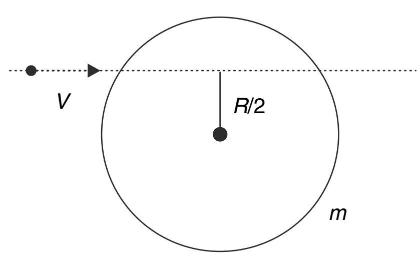
Q. 46. The propeller of a small airplane is mounted in the front. The propeller rotates clockwise if seen from behind by the pilot. The plane is flying horizontally and the pilot suddenly turns it to the right. Will the body of the plane have a tendency to get inclined to the horizontal? If yes, does the nose of the plane veer upward or downward? Why?

Q. 47. A massive star is spinning about its diameter with an angular speed $\omega_0 = \frac{\pi}{1000}$ rad/day. After its fuel is exhausted, the star collapses under its own gravity to form a neutron star. Assume that the volume of the star decreases to $10^{-12}$ times the original volume and its shape remains spherical. Assuming that density of the star is uniform, find the angular speed of the neutron star.

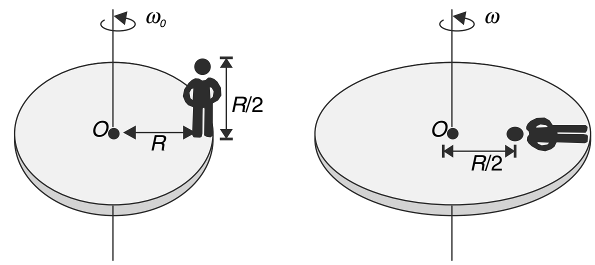
Q. 48. A square plate of side length $2m$ has a groove made in the shape of two quarter circles joining at the centre of the plate. The plate is free to rotate about vertical axis passing through its centre. The moment of inertia of the plate about this axis is $4\ kg - m^2$. A small block of mass $1\ kg$ enters the groove at end A travelling with a velocity of $2m/s$ parallel to the side of the square plate. The block move along the frictionless groove of the horizontal plate and comes out at the other end $B$ with speed $V$. Find $V$ assuming that width of the groove is negligible.




Q. 49. A disc of mass $m$ and radius $R$ lies flat on a smooth horizontal table. A particle of mass $m$, moving horizontally along the table, strikes the disc with velocity $V$ while moving along a line at a distance $\frac{R}{2}$ from the centre. Find the angular velocity acquired by the disc if the particle comes to rest after the impact.



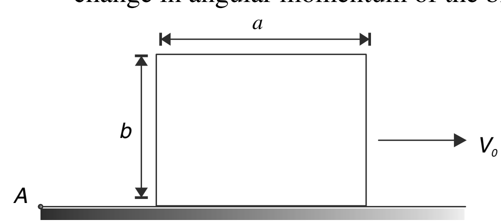

Q. 50. A disc of mass $M$ and radius $R$ is rotating with angular velocity $\omega_0$ about a vertical axis passing through its centre (O). A man of mass $\frac{M}{2}$ and height $\frac{R}{2}$ is standing on the periphery. The man gradually lies down on the disc such that his head is at a distance $\frac{R}{2}$ from the centre and his feet touching the edge of the disc. For simplicity assume that the man can be modelled as a thin rod of length $\frac{R}{2}$. Calculate the angular speed ($\omega$) of the platform after the man lies down.



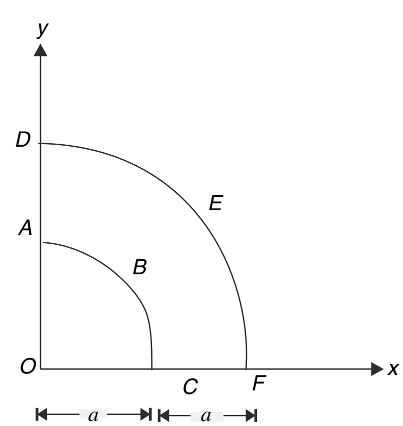

Q. 51. A uniform block of mass $M$ and dimensions as shown in the figure is placed on a rough horizontal surface and given a velocity $V_0$ to the right. $A$ is a point on the surface to the left of the block.

(a) Write the angular momentum of the block about point $A$ just after it begins to move

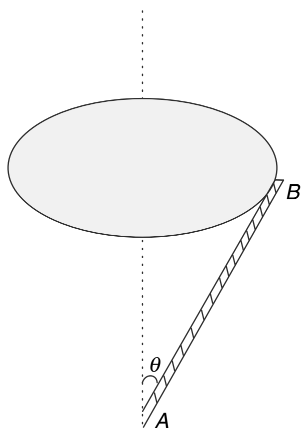
(b) Due to friction the block stops. What happened to its angular momentum about point $A$? Which torque is responsible for change in angular momentum of the block?



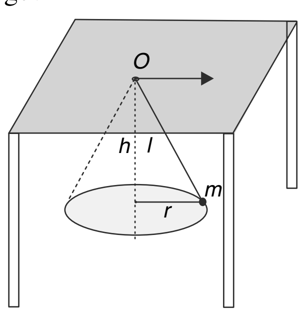

Q. 52. ABCFED is a uniform plate (shown in figure). $ABC$ and $DEF$ are circular arcs with common centre at $O$ and having radii $a$ and $2a$ respectively. This plate is lying on a smooth horizontal table. A particle of mass half the mass of the plate strikes the plate at point $A$ while travelling horizontally along the $x$ direction with velocity $u$. The particle hits the plate and rebounds along negative $x$ with velocity $\frac{u}{2}$. Find the velocity of point $D$ of the plate immediately after the impact. [Take $\frac{28}{9\pi} \simeq 1$]



Q. 53. A uniform rod of mass $m$ and length $L$ is fixed to an axis, making an angle $\theta$ with it as shown in the figure. The rod is rotated about this axis so that the free end of the rod moves with a uniform speed '$v$'. Find the angular momentum of the rod about the axis. Is the angular momentum of the rod about point A constant?




Q. 54. A mass $m$ in attached to a mass less string and swings in a horizontal circle, forming a conical pendulum, as shown in the figure. The other end of the string passes through a hole in the table and is dragged slowly so as to reduce the length $l$. The string is slowly drawn up so that the depth $h$ shown in the figure becomes half. By what factor does the radius ($r$) of the circular path of the mass $m$ change?

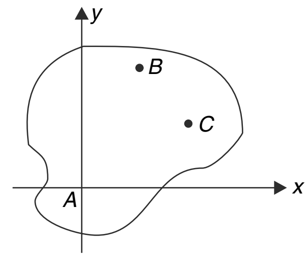



## LEVEL 2

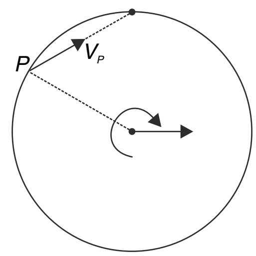
Q. 55. A flat rigid body is moving in $x - y$ plane on a table. The plane of the body lies in the $x - y$ plane. At an instant it was found that some of the velocity components of its three particles $A$, $B$ and $C$ were $V_{Ax} = 4m/s$, $V_{Bx} = 3m/s$ and $V_{cy} = -2m/s$, respectively. At the instant the three particles $A$, $B$ and $C$ were located at (0,0) (3,4) , (4,3) (all in meter ) respectively in a co-ordinate system attached to the table.

(a) Find the velocity of $A$, $B$ and $C$

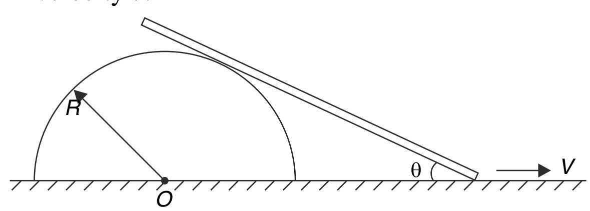
(b) Find the angular velocity of the body.




Q. 56. A wheel is rolling without sliding on a horizontal surface. Prove that velocities of all points on the circumference of the wheel are directed towards the top most point of the wheel.



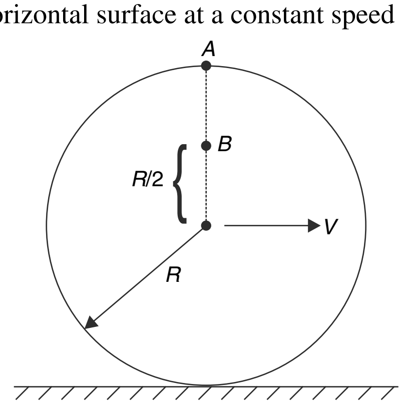

Q. 57. There is a fixed half cylinder of radius $R$ on a horizontal table. A uniform rod of length $2R$ leans against it as shown. At the instant shown, $\theta = 30°$ and the right end of the rod is sliding with velocity $v$.




(a) Calculate the angular speed of the rod at this instant.

(b) Find the vertical component of the velocity of the centre of the rod at this instant.

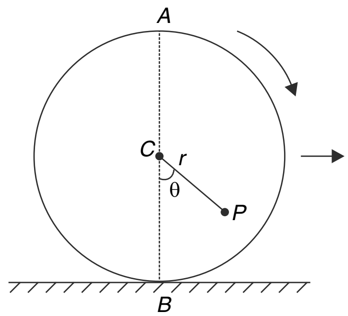
Q. 58. A disc of radius $R$ is rolling without sliding on a horizontal surface at a constant speed of $v$



(a) What is speed of points $A$ and $B$ on the vertical diameter of the disc ? Given $AB = \frac{R}{2}$

(b) After what time, for the first time, speed of point $A$ becomes equal to present speed (i.e., the speed at the instant shown in the figure) of point $B$?

Q. 59. A uniform disc of radius $R = 2\sqrt{3}\ m$ is moving on a horizontal surface without slipping. At some instant its angular velocity is $\omega = 1$ rad/s and angular acceleration is $\alpha = \sqrt{3}\ rad/s^2$.

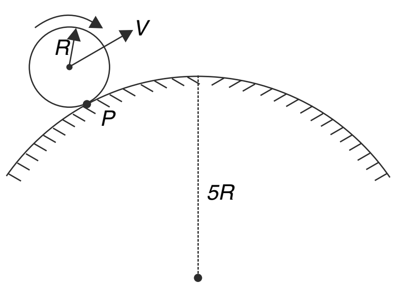



(a) Find acceleration of the top point $A$.

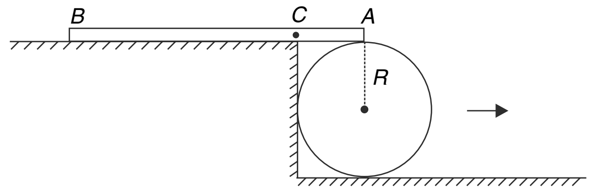
(b) Find acceleration of contact point $B$.

(c) Find co - oridnates $(r, \theta)$ for a point $P$ which has zero acceleration.

Q. 60. A convex surface has a uniform radius of curvature equal to $5R$. A wheel of radius $R$ is rolling without sliding on it with a constant speed $v$. Find the acceleration of the point ($P$) of the wheel which is in contact with the convex surface.



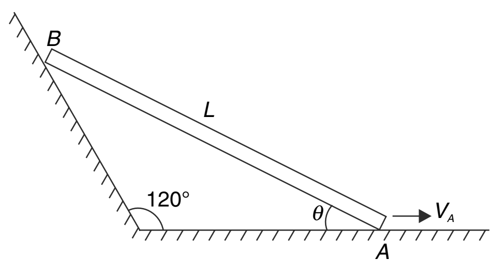

Q. 61. $AB$ is a non uniform plank of length $L = 4R$ with its centre of mass at $C$ such that $AC = R$. It is placed on a step with its one end $A$ supported by a cylinder of radius $R$ as shown in figure. The centre of mass of the plank is just outside the edge of the step. The cylinder is slowly rolled on the lower step such that there is no slipping at any of its contacts. Calculate the distance through which the centre of the cylinder moves before the plank loses contact with the horizontal surface of the upper step.




Q. 62. A wheel of radius $R$ is rolling without sliding uniformly on a horizontal surface. Find the radius of curvature of the path of a point on its circumference when it is at highest point in its path.

Q. 63. A wall is inclined to a horizontal surface at an angle of 120° as shown. A rod $AB$ of length $L = 0.75\ m$ is sliding with its two ends $A$ and $B$ on the horizontal surface and on the wall respectively. At the moment angle $\theta = 20°$ (see figure), the velocity of end $A$ is $v_A = 1.5\ m/s$ towards right. Calculate the angular speed of the rod at this instant.

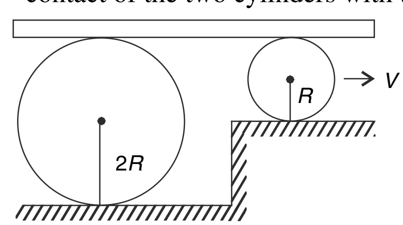
[Take cos 40° = 0.766]



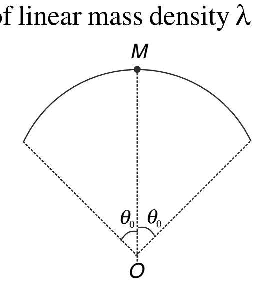

Q. 64. In the figure the plank resting on two cylinders is horizontal. The plank is pulled to the right such that the centre of smaller cylinder moves with a constant velocity $v$. Friction is large enough to prevent slipping at all surfaces. Find-

(a) The velocity of the centre of larger cylinder.

(b) The ratio of accelerations of the points of contact of the two cylinders with the plank.




Q. 65. A wire of linear mass density λ ($kg$/m) is bent into



an arc of a circle of radius $R$ subtending an angle $2\theta_0$ at the centre. Calculate the moment o inertia of this circular arc about an axis passing through its midpoint ($M$) and perpendicular to its plane.

Q. 66. A metallic plate has been fabricated by welding two semicircular discs - $D_1$ and $D_2$ of radii $R$ and $\frac{R}{2}$ respectively (fig. a). $O$ and $O'$ are the centre of curvature of the two discs and each disc has a mass $6m$. The plate is in $xy$ plane. Now the plate is folded along the $y$ axis so as to bring the part $OAB$ in $yz$ plane. (fig. b). The plate is now set free to be able to rotate freely about the $z$ – axis. A particle of mass $m$, moving with a velocity $v$ in the $xy$ plane along the line $x = d$ hits the plate and sticks to it ($d < R$). Just before collision speed of the particle was $v$.

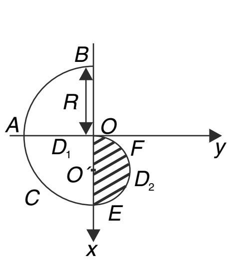


Fig (a)

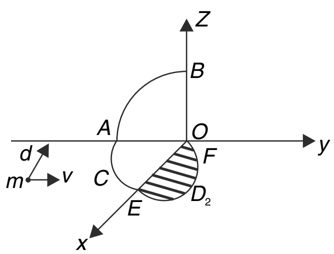


Fig (b)

(a) Find the moment of inertia of inertia of the plate about $z$ axis.

(b) Find the angular speed of the plate after collision.

Q. 67. There is a square plate of side length $a$. It is divided into nine identical squares each of side $\frac{a}{3}$ and the central square is removed (see fig. (i)). Now each of the remaining eight squares of side length $\frac{a}{3}$ are divided into nine identical squares and central square is removed from each of them (see fig. (ii)).

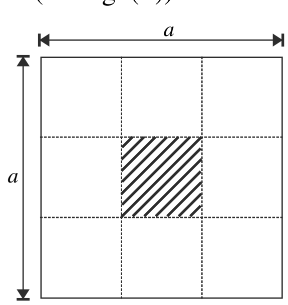


Fig. (i)

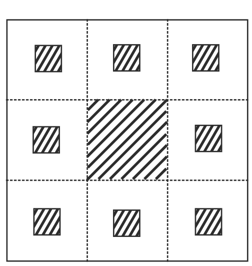
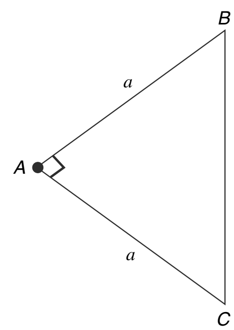


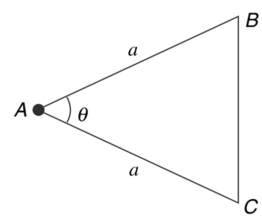
Fig. (ii)

Mass of the plate with one big and eight small holes is $M$. Find its moment of inertia about an axis passing through its centre and perpendicular to its plane.

Q. 68. $ABC$ is an isosceles triangle right angled at $A$. Mass of the triangular plate is $M$ and its equal sides are of length $a$. Find the moment of inertia of this plate about an axis through $A$ perpendicular to the plane of the plate. Use the expression of moment of inertia for a square plate that you might have studied.



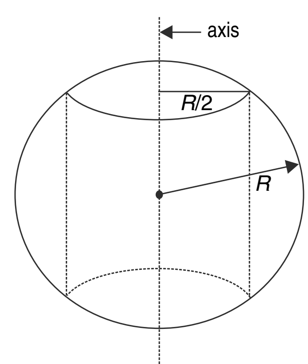

Q. 69. The triangular plate described in the last question has angle $\angle A = \theta$. Now find its moment of inertia about an axis through $A$ perpendicular to the plane of the plate.



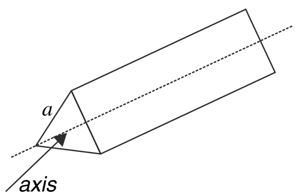

Q. 70. A thin uniform spherical shell of radius $R$ is bored such that the axis of the boring rod passes through the centre of the sphere. The boring rod is a cylinder of radius $\frac{R}{2}$. Take the mass of the sphere before boring to be $M$.

(a) Find the mass of the leftover part

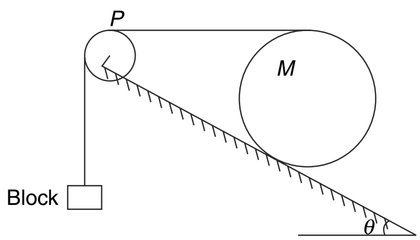
(b) Find the moment of inertia of the leftover part about the axis shown.



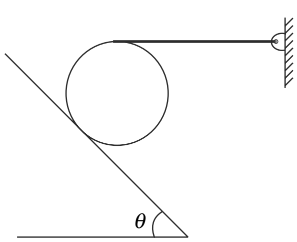
Q. 71. Consider an equilateral prism as shown in the figure. The mass of the prism is $M$ and length of each side of its cross section is $a$. Find the moment of inertia of such a prism about the central axis shown.



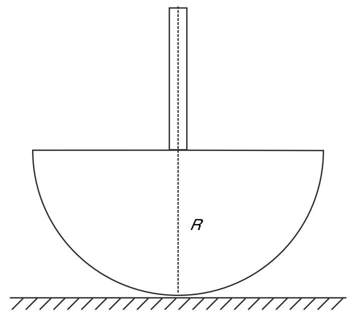

Q. 72. In the arrangement shown in figure the cylinder of mass $M$ is at rest on an incline. The string between the cylinder and the pulley (P) is horizontal. Find the minimum coefficient of friction between the incline and the cylinder which can keep the system in equilibrium. Also find the mass of the block. Assume no friction between the pulley (P) and the string.



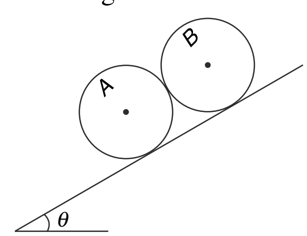

Q. 73. A horizontal stick of mass $m$ has its right end attached to a pivot on a wall, while its left end rests on the top of a cylinder of mass $m$ which in turn rests on an incline plane inclined at an angle $\theta$. The stick remains horizontal. The coefficient of friction between the cylinder and both the plane and the stick is $\mu$. Find the minimum value of $\mu$ as function of $\theta$ for which the system stays in equilibrium.



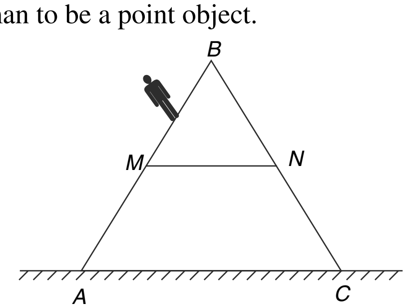

Q. 74. Consider the object shown in the figure. It consist of a solid hemisphere of mass $M$ and radius $R$. There is a solid rod welded at its centre. The object is placed on a flat surface so that the rod is vertical. Mass of the rod per unit length is $\frac{M}{2R}$. What is the maximum length of the rod that can be welded so that the object can perform oscillations about the position shown in diagram? Note : Centre of mass of a solid hemisphere is at a distance of $\frac{3R}{8}$ from its base.



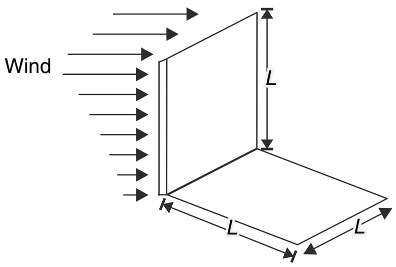

Q. 75. Two cylinders $A$ and $B$ have been placed in contact on an incline. They remain in equilibrium. The dimensions of the two cylinders are same. Which cylinder has larger mass?



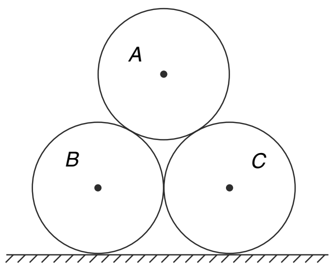

Q. 76. The ladder shown in the figure is light and stands on a frictionless horizontal surface. Arms $AB$ and $BC$ are of equal length and $M$ and $N$ are their mid points. Length of $MN$ is half that of $AB$. A man of mass $M$ is standing at the midpoint of $BM$. Find the tension in the mass less rod $MN$. Consider the man to be a point object.




Q. 77. A uniform metal sheet of mass $M$ has been folded to give it $L$ shape and it is placed on a rough floor as shown in figure. Wind is blowing horizontally and hits the vertical face of the sheet as shown. The speed of air varies linearly from zero at floor level to $v_0$ at height $L$ from the floor. Density of air is $\rho$. Find maximum value of $v_0$ for which the sheet will not topple. Assume that air particles striking the sheet come to rest after collision, and that the friction is large enough to prevent the sheet from sliding.

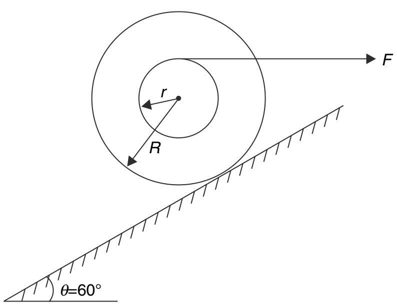



Q. 78. Three identical cylinders have mass $M$ each and are placed as shown in the figure. The system is in equilibrium and there is no contact between $B$ and $C$. Find the normal contact force between $A$ and $B$.




Q. 79. A spool is kept in equilibrium on an incline plane as shown in figure. The inner and outer radii of the spool are in ratio $\frac{r}{R} = \frac{1}{2}$. The force applied on the thread (wrapped on part of radius $r$) is horizontal. Find the angle that the force applied by the incline on the spool makes with the vertical.

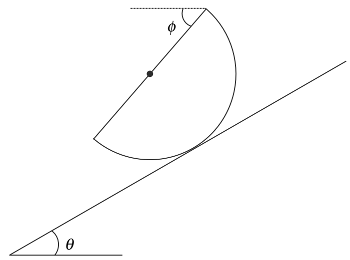
[Take $\tan^{-1}\left(\frac{\sqrt{3}}{5}\right) \simeq 19°$]



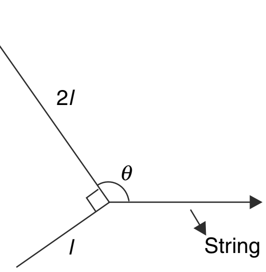

Q. 80. A uniform hemisphere placed on an incline is on verge of sliding. The coefficient of friction between the hemisphere and the incline is $\mu = 0.3$.

Find the angle $\phi$ that the circular base of the hemisphere makes with the horizontal.

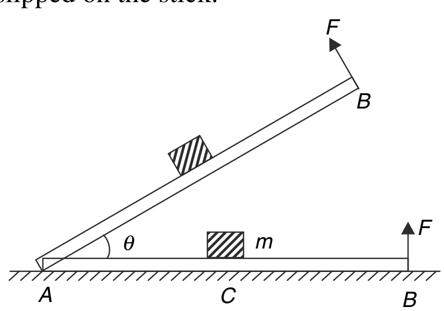
Given $\sin(\tan^{-1} 0.3) \simeq 0.29$ and $\sin^{-1}(0.77) \simeq 50°$




Q. 81. A $L$ shaped, uniform rod has its two arms of length $l$ and $2l$. It is placed on a horizontal table and a string is tied at the bend. The string is pulled horizontally so that the rod slides with constant speed. Find the angle $\theta$ that the longer side makes with the string. Assume that the rod exerts uniform pressure at all points on the table.




Q. 82. A uniform meter stick $AB$ of mass $M$ is lying in state of rest on a rough horizontal plane. A small block of mass $m$ is placed on it at its centre $C$. A variable force $F$ is applied at the end $B$ of the stick so as to rotate the stick slowly about $A$ in vertical plane. The force $F$ always remains perpendicular to the length of the stick. The stick is raised to $\theta = 60°$ and it was observed that neither the end $A$ slipped on the ground nor the block of mass $m$ slipped on the stick.




(a) $F_1$ is force applied by the stick on the block. Plot the variation of $F_1$ with $\theta$ $(0 \le \theta \le 60°)$.
(b) What must be the minimum coefficient of friction between the block and the stick.
(c) $f$ is the friction force acting at end $A$ of the stick. Plot variation of $f$ vs $\theta$ $(0° \le \theta \le 60°)$.

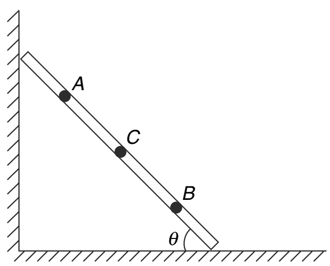
Q. 83. A ladder of mass $M$ and length $L$ stays at rest against a smooth wall. The coefficient of friction between the ground and the ladder is $\mu$.

(a) Let $F_{wall}$, $W$ and $F_g$ be the force applied by wall, weight of the ladder and force applied by ground on the ladder. Argue to show that the line of action of these three forces must intersect.

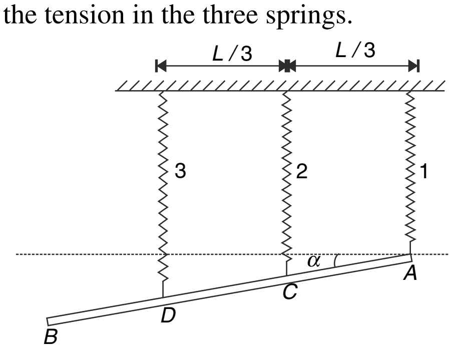
(b) Using the result obtained in (a) show that line of action of $F_g$ makes an angle $\tan^{-1}(2\tan\theta)$ with the horizontal ground where $\theta$ is the angle made by the ladder with the ground.

(c) Find the smallest angle that the ladder can make with the ground and not slip.

(d) You climb up the ladder, your presence makes the ladder more likely to slip. Where are you at *A* or *B*? *C* is the centre of mass of the ladder.



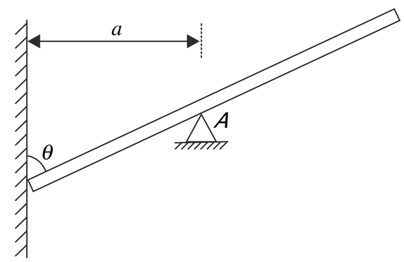

Q. 84. A uniform rod *AB* has mass *M* and length *L*. It is in equilibrium supported in vertical plane by three identical springs as shown in figure. The springs are connected at *A*, *C* and *D* such that $AC = CD = \frac{L}{3}$. Assume that the springs are very stiff and the angle $\alpha$ made by the rod with the horizontal in equilibrium position is very small. (All springs are nearly vertical). Calculate the tension in the three springs.



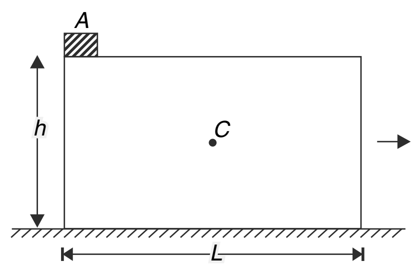

Q. 85. A uniform rod of length *b* can be balanced as shown in figure. The lower end of the rod is resting against a vertical wall. The coefficient of friction between the rod and the wall and that between the rod and the support at A is $\mu$. Distance of support from the wall is *a*.

(a) Find the ratio $\frac{a}{b}$ if the maximum value of $\theta$ is $\theta_1$.

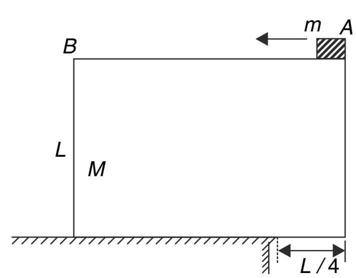
(b) Find the ratio $\frac{a}{b}$ if the minimum value of $\theta$ is $\theta_2$.




Q. 86. A uniform rectangular block is moving to the right on a rough horizontal floor (the block is retarding due to friction). The length of the block is *L* and its height is *h*. A small particle (*A*) of mass equal to that of the block is stuck at the upper left edge. Coefficient of friction between the block and the floor is $\mu = \frac{2}{3}$. Find the value of *h* (in terms of *L*) if the normal reaction of the floor on the block effectively passes through the geometrical centre (*C*) of the block.



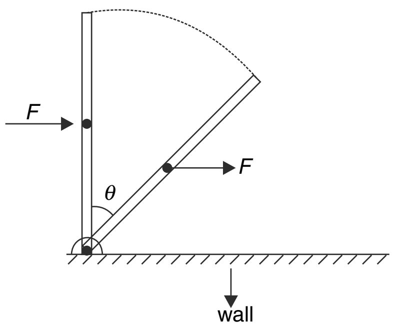

Q. 87. A uniform cubical block of mass *M* and side length *L* is lying on the edge of a rough table with $\frac{1}{4}th$ of its edge overhanging. When a small block of mass *m* is placed on its top surface at the right edge (see fig.), the cube is on verge of toppling. The block of mass *m* is given a sharp horizontal impulse so that it acquires a velocity towards *B*. The small block moves on the top surface and falls on the other side. What is maximum coefficient of friction between the small block and the cube so that the cube does not rotate as the block moves over it. Assume that the friction between the cube and the table is large enough to prevent sliding of the cube on the table.



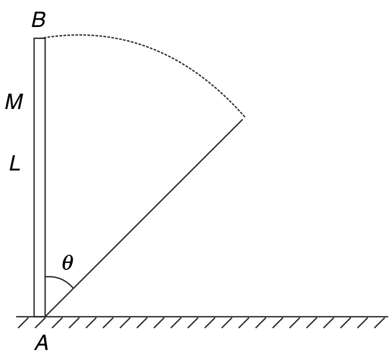
Q. 88. A uniform rod of mass $M$ and length $L$ is hinged at its end to a wall so that it can rotate freely in a horizontal plane. When the rod is perpendicular to the wall a constant force $F$ starts acting at the centre of the rod in a horizontal direction perpendicular to the rod. The force remains parallel to its original direction and acts at the centre of the rod as the rod rotates. (Neglect gravity).

(a) With what angular speed will the rod hit the wall ?

(b) At what angle $\theta$ (see figure) the hinge force will make a 45° angle with the rod ?




Q. 89. A rod of mass $M = 5 kg$ and length $L = 1.5\ m$ is held vertical on a table as shown. A gentle push is given to it and it starts falling. Friction is large enough to prevent end $A$ from slipping on the table.




(a) Find the sum of linear momentum of all the particles of the rod when it rotates through an angle $\theta = 37°$.

(b) Find the friction force and normal reaction force by the table on the rod, when $\theta = 37°$.

(c) Find value of angle $\theta$ when the friction force becomes zero.

$$[\tan 37° = \frac{3}{4} \text{ and } g = 10\ m/s]$$

Q. 90. (a) In the system shown in figure 1, the uniform rod of length $L$ and mass $m$ is free to rotate in vertical plane about point $O$. The string an pulley are mass less. The block has mass equal to that of the rod. Find the acceleration of the block immediately after the system is released with rod in horizontal position.

(b) System shown in figure 2 is similar to that in figure 1 apart from the fact that rod is mass less and a block of mass $m$ is attached to the centre of the rod with the help of a thread. Find the acceleration of both the blocks immediately after the system is released with rod in horizontal position.

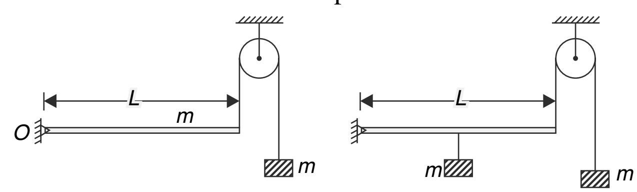
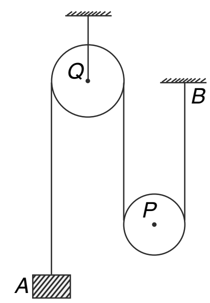



Fig.(1) Fig.(2)

Q. 91.

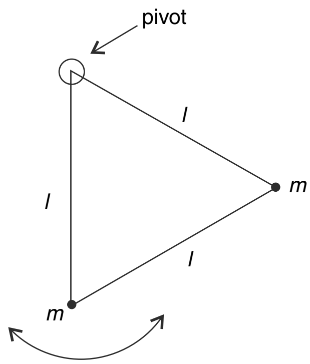



A light thread is wrapped tightly a few turns around a disc $P$ of mass $M$. One end of the thread is fixed to the ceiling at $B$. The other end of the thread is passed over a mass less pulley ($Q$) and carries a block of mass $M$. All segment of the thread (apart from that on the pulley and disc) are vertical when the system is released. Find the acceleration of block $A$. On which object – the block A or the ceiling at $B$ – does the thread exert more force ?

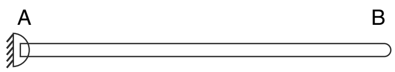
Q. 92. An equilateral triangle is made from three mass less rods, each of length $l$. Two point masses m are attached to two vertices. The third vertex is hinged and triangle can swing freely in a vertical plane as shown. It is released the position shown with one of the rods vertical. Immediately after the system is released, find –

(a) tensions in all three rods (specify tension or compression),

(b) accelerations of the two masses




Q. 93.

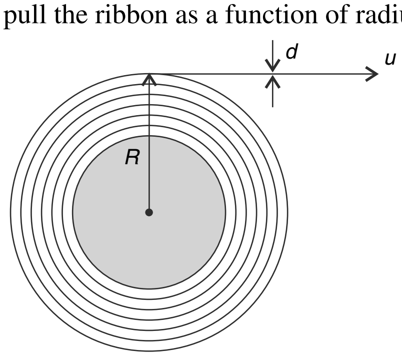



A rod of mass $M$ and length $L$ is hinged about its end $A$ so that it can rotate in vertical plane. When the rod is released from horizontal position it takes $t_0$ time for it to become vertical.

(a) A particle of mass $M$ is stuck at the end $B$ of the rod and the rod is once again released from its horizontal position. Will it take more time or less time (than $t_0$) for the rod to become vertical from its horizontal position.

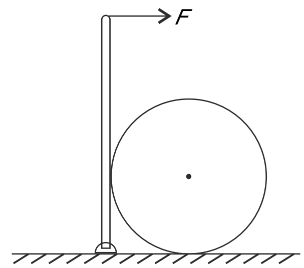
(b) At what distance $x$ from end $A$ shall the particle of mass $M$ be stuck so that it takes minimum time for the rod to become vertical from its horizontal position.

Q. 94. A disc is free to rotate about an axis passing through its centre and perpendicular to its plane. The moment of inertia of the disc about its rotation axis is $I$. A light ribbon is tightly wrapped over it in multiple layers. The end of the ribbon is pulled out at a constant speed of u. Let the radius of the ribboned disc be $R$ at any time and thickness of the ribbon be $d$ ($<< R$). Find the force ($F$) required to pull the ribbon as a function of radius $R$.



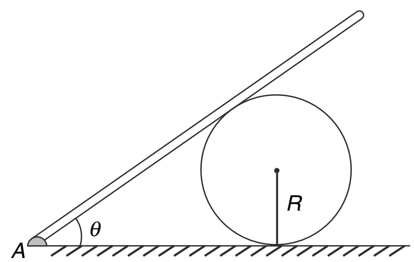

Q. 95. A uniform rod of mass $M$ and length $L$ is hinged at its lower end on a table. The rod can rotate freely in vertical plane and there is no friction at the hinge. A ball of mass $M$ and radius $R = \frac{L}{3}$ is placed in contact with the vertical rod and a horizontal force $F$ is applied at the upper end of the rod.

(a) Find the acceleration of the ball immediately after the force starts acting.



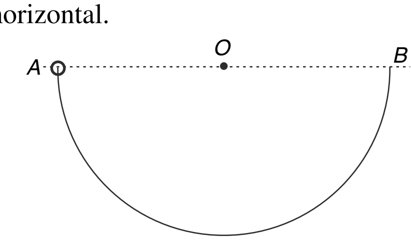

(b) Find the horizontal component of hinge force acting on the rod immediately after force $F$ starts acting.

Q. 96.



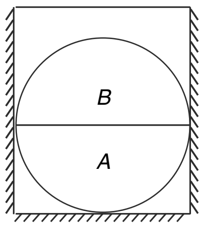

A ring of mass $M$ and radius $R$ is held at rest on a rough horizontal surface. A rod of mass $M$ and length $L = 2\sqrt{3}R$ is pivoted at its end A on the horizontal surface and is supported by the ring. There is no friction between the ring and the rod. The ring is released from this position. Find the acceleration of the ring immediately after the release if $\theta = 60°$. Assume that friction between the ring on the horizontal surface is large enough to prevent slipping of the ring.

Q. 97. A uniform semicircular wire is hinged at '$A$' so that it can rotate freely in vertical plane about a horizontal axis through '$A$'. The semicircle is released from rest when its diameter $AB$ is horizontal.

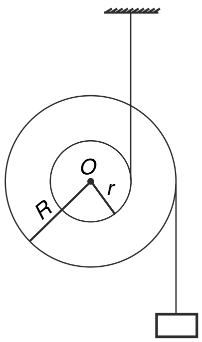



Find the hinge force at '$A$' immediately after the wire is released.

Q. 98. A uniform solid hemisphere $A$ of mass $M$ radius $R$ is joined with a thin uniform hemispherical shell $B$ of mass $M$ and radius $R$ (see fig.). The sphere thus formed is placed inside a fixed box as shown. The floor , as well as walls of the box are smooth. On slight disturbance, the sphere begins to rotate. Find its maximum angular speed ($\omega_0$) and maximum angular acceleration ($\alpha_0$) during the subsequent motion. Do the walls of the box apply any force on the sphere while it rotates?




Q. 99.



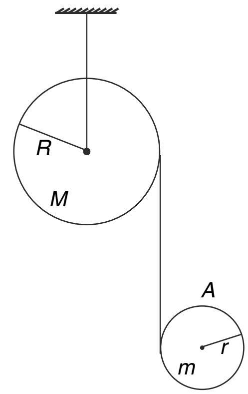

In the arrangement shown, the double pulley has a mass $M$ and the two mass less threads have been tightly wound on the inner (radius = $r$) and outer circumference (radius $R$ = $2r$). The block shown has a mass 4 $M$. The moment of inertia of the double pulley system about a horizontal axis passing through its centre and perpendicular to the plane of the figure is $I = \frac{Mr^2}{2}$.

(a) Find the acceleration of the center of the pulley after the system is released.

(b) Two seconds after the start of the motion the string holding the block breaks. How long after this the pulley will stop ascending?

Q. 100. A thread is tightly wrapped on two pulleys as shown in figure. Both the pulleys are uniform disc with upper one having mass $M$ and radius $R$ being free to rotate about its central horizontal axis. The lower pulley has mass $m$ and radius $r$ and it is released from rest. It spins and falls down. At the instant of release a small mark ($A$) was at the top point of the lower pulley.

(a) After what minimum time ($t_0$) the mark will again be at the top of the lower pulley?




(b) Find acceleration of the mark at time $t_0$.

(c) Is there any difference in magnitude of acceleration of the mark and that of a point located on the circumference at diametrically opposite end of the pulley.

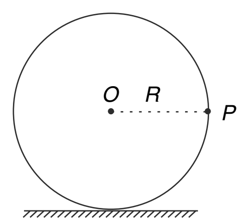
Q. 101. A point mass $m$ = 1 $kg$ is attached to a point P on the circumference of a uniform ring of mass $M$ = 3$kg$ and radius $R$ = 2.0 $m$. The ring is placed on a horizontal surface and is released from rest with line OP in horizontal position (O is centre of the ring). Friction is large enough to prevent sliding. Calculate the following quantities immediately after the ring is released-

(a) angular acceleration ($\alpha$) of the ring,

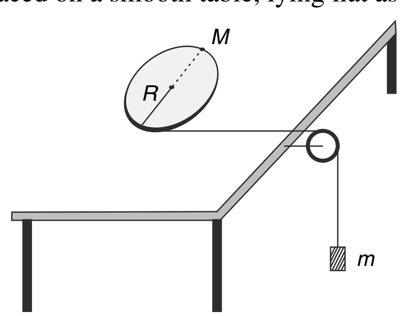
(b) normal reaction of the horizontal surface on the ring and

(c) the friction force applied by the surface on the ring.

[Take g = 10 $m/s^2$]




Q. 102. A light thread has been tightly wrapped around a disc of mass $M$ and radius $R$. The disc has been placed on a smooth table, lying flat as shown.

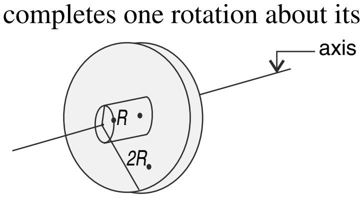


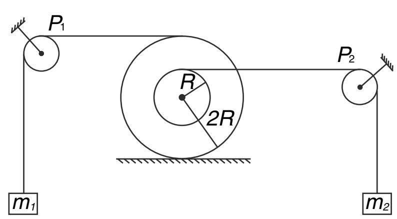
The other end of the string has been attached to a mass $m$ as shown. The system is released from rest. If $m = M$, which point of the disc will have zero acceleration, immediately after the system is released?

Q. 103. A spool has the shape shown in figure. Radii of inner and outer cylinders are $R$ and $2R$ respectively. Mass of the spool is $3m$ and its moment of inertia about the shown axis is $2mR^2$. Light threads are tightly wrapped on both the cylindrical parts. The spool is placed on a rough surface with two masses $m_1 = m$ and $m_2 = 2m$ connected to the strings as shown. The string segment between spool and the pulleys $P_1$ and $P_2$ are horizontal. The centre of mass of the spool is at its geometrical centre. System is released from rest.

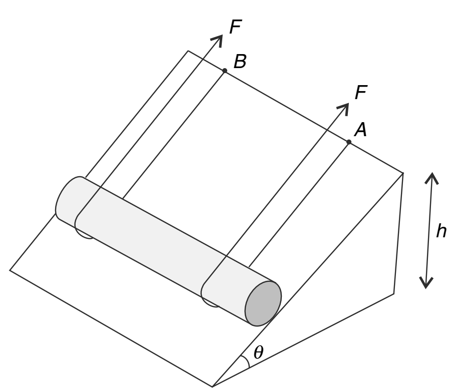
(a) What is minimum value of coefficient of friction between the spool and the table so that it does not slip?

(b) Find the speed of $m_1$ when the spool completes one rotation about its centre.




Q. 104.




A heavy uniform log of mass $M$ is pulled up an incline surface with the help of two parallel ropes as shown in figure. The ropes are secured at point $A$ and $B$. The height of the incline is h and its inclination is $\theta$.

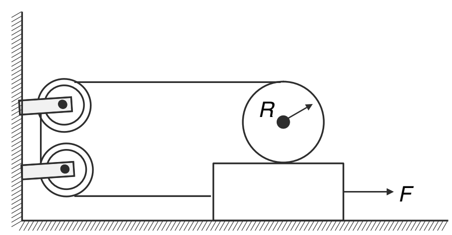
(a) Find the minimum force $F_0$ needed to roll the log up the incline.

(b) Find the work done by the force in moving the log from the bottom to the top of the incline if the applied force is $F = 2F_0$

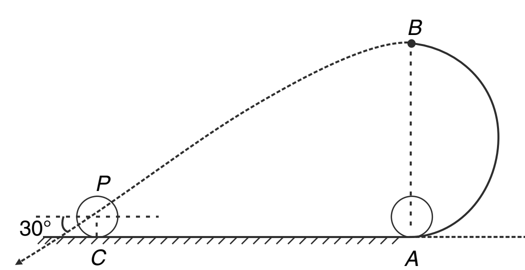
Q. 105. In the figure shown, the light thread is tightly wrapped on the cylinder and masses of plank and cylinder are same each equal to $m$. An external agent begins to pull the plank to the right with a constant force $F$. The friction between the plank and the cylinder is large enough to prevent slipping. Assume that the length of the plank is quite large and the cylinder does not fall off it for the time duration concerned.

(a) Find the acceleration of the cylinder. (Hint : don't write any equations)

(b) Find the kinetic energy of the system after time $t$.




Q. 106. A disc of radius $r = 0.1$ $m$ is rolled from a point $A$ on a track as shown in the figure. The part $AB$ of the track is a semi-circle of radius $R$ in a vertical plane. The disc rolls without sliding and leaves contact with the track at its highest point $B$. Flying through the air it strikes the ground at point $C$. The velocity of the center of mass of the disc makes an angle of 30° below the horizontal at the time of striking the ground. At the same instant, velocity of the topmost point $P$ of the disc is found to be 6 $m/s$. (Take $g = 10$ $m/s^2$).

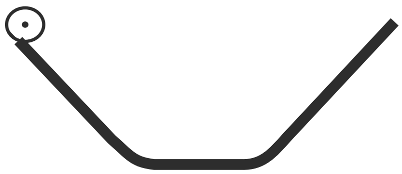



(a) Find the value of $R$.

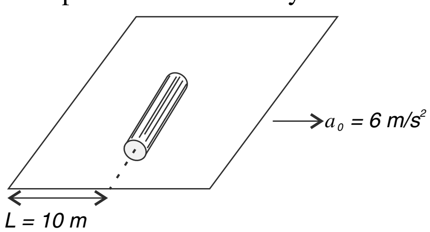
(b) Find the velocity of the center of mass of the disc when it strikes the ground.

(c) Find distance $AC$.

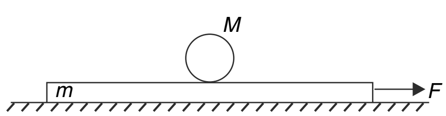
Q. 107. A trough has two identical inclined segments and a horizontal segment. A ball is released on the top of one inclined part and it oscillates inside the trough. Friction is large enough to prevent slipping of the ball. Time period of oscillation is $T$. Now the liner dimension of each part of the trough is enlarged four times. Find the new time period of oscillation of the ball.



Q. 108. A uniform cylinder is lying on a rough sheet of paper as shown in fig. The strip is pulled horizontally to the right with a constant acceleration of $a_0 = 6\ m/s^2$. Initially the cylinder is located at a distance of $L = 10\ m$ from the left end of the strip. Find the velocity of the centre of the cylinder at the instant it moves off the edge of the strip. Assume that the cylinder does not slip.

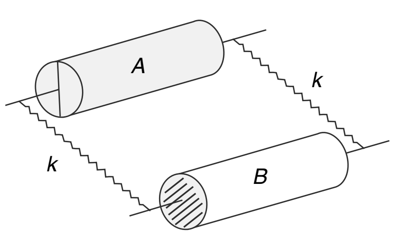



Q. 109.




A hollow pipe of mass $M = 6\ kg$ rests on a plate of mass $m = 1.5\ kg$. The thickness of the pipe is negligible. The coefficient of friction at all contacts is $\mu = 0.2$. The system is initially at rest. A horizontal force F of magnitude $25N$ is applied on the plate as shown in figure. Will the cylinder slide on the plate? Find the acceleration of the centre of the cylinder.

Q. 110.

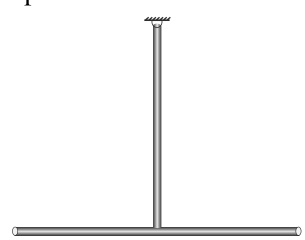



A hollow cylindrical pipe A has mass $M$ and radius $R$. With the help of two identical springs (each of force constant k) it is connected to solid cylinder B having mass $M$ and radius $R$. The springs are connected symmetrically to the axle of the cylinders. Moment of inertia of the two Bodies A and B about their axles are $I_A = MR^2$ and $I_B = \frac{1}{2}MR^2$ respectively. Cylinders are pulled apart so as to stretch the springs by $x_0$ and released. During subsequent motion the cylinders do not slip.

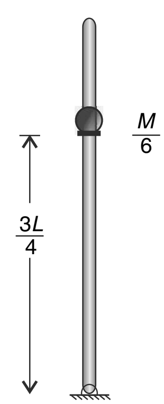
(a) Find acceleration of the centre of mass of the system immediately after it is released.

(b) Find the distance travelled by cylinder A by the time it comes to rest for the first time after being released.

Q. 111. Two identical uniform thin rods have been connected at right angles to form a '$T$' shape. One end of a rod is connected to the centre of the other rod. Length of each rod is $L$. The upside down '$T$' can swing like a pendulum about a horizontal axis passing through the top end (see fig.). Axis is perpendicular to plane of the fig. The speed of the meeting point of the two rods is $u = 2\sqrt{gL}$ when it is at its lowest position. Calculate the angular acceleration of the '$T$' shaped object when it is at extreme position of its oscillation.



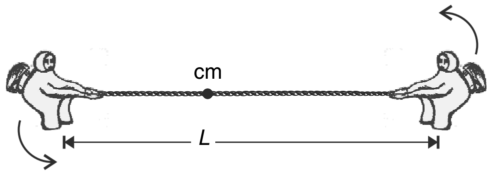
Q. 112. A uniform rod of mass $M$ and length $L$ is hinged at its lower end so as to rotate freely in the vertical plane of the fig. There is a small tight fitting bead of mass $\frac{M}{6}$ on the rod at a distance $\frac{3L}{4}$ from the hinged end. A small mass less pin welded to the rod supports the bead. The system is released from the vertical position shown. It was observed that the bead just begins to slide on the rod when the rod becomes horizontal.




(a) Find the normal contact force between the rod and the bead when the rod gets horizontal. What is the direction of this force?

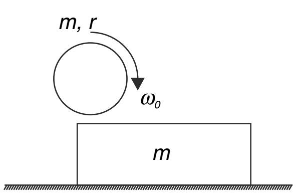
(b) Find the coefficient of friction between the bead and rod.

Q. 113




Two astronauts having mass of 75 *kg* and 50 *kg* are connected by a rope of length $L = 10m$ and negligible mass. They are in space, orbiting their centre of mass at an angular speed of $\omega_0 = 5$ *rad/s*. The centre of mass itself is moving uniformly in space at a velocity of 10 *m/s*. By pulling on the rope, the astronauts shorten the distance between them to $\frac{L}{2} = 5m$. How much work is done by the astronauts in shortening the distance between them ? Assuming that the astronauts are athletic and each of them can generate a power of 500 watt, is it possible for the two astronauts to reduce the distance between them to 5 *m*, within a minute?

Q. 114. In the figure shown a plank of mass *m* is lying at rest on a smooth horizontal surface. A cylinder of same mass *m* and radius *r* is rotated to an angular speed $w_0$ and then gently placed on the plank. It is found that by the time the slipping between the plank and the cylinder cease, 50% of total kinetic energy of the cylinder and plank system is lost. Assume that plank is long enough and $\mu$ is the coefficient of friction between the cylinder and the plank.

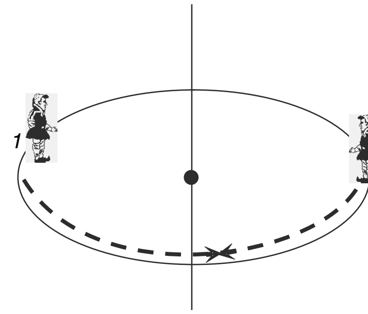



(a) Find the final velocity of the plank.

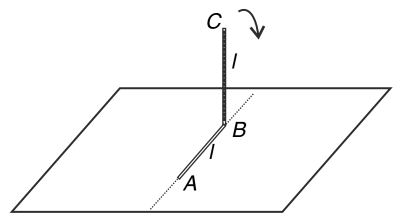
(b) Calculate the magnitude of the change in angular momentum of the cylinder about its centre of mass.

(c) Distance moved by the plank by the time slipping ceases between cylinder and plank.

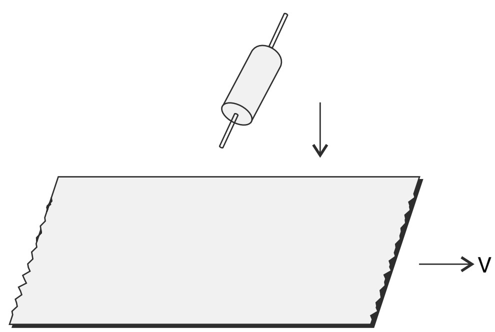
Q. 115. A horizontal turn table of mass 90*kg* is free to rotate about a vertical axis passing through its centre. Two men – 1 and 2 of mass 50*kg* and 60*kg* respectively are standing at diametrically opposite point on the table. The two men start moving towards each other with same speed (relative to the table) along the circumference. Find the angle rotated by table by the time the two men meet. Treat the men as point masses.




Q. 116. A *L* shaped uniform rod has both its sides of length *l*. Mass of each side is *m*. The rod is placed on a smooth horizontal surface with its side AB horizontal and side BC vertical. It tumbles down from this unstable position and falls on the surface. Find the speed with which end C of the rod hits the surface.




Q.117.

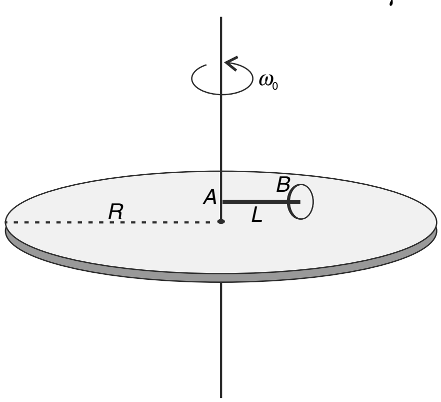



A flat horizontal belt is running at a constant speed *V*. There is a uniform solid cylinder of mass *M* which can rotate freely about an axle passing through its centre and parallel to its length. Holding the axle parallel to the width of the belt, the cylinder is lowered on to the belt. The cylinder begins to rotate about its axle and eventually stops slipping. The cylinder is, however, not allowed to move forward by keeping its axle fixed. Assume that the moment of inertia of the cylinder about its axle is $\frac{1}{2}MR^2$ where *M* is its mass and *R* its radius and also assume that the belt continues to move at constant speed. No vertical force is applied on the axle of the cylinder while holding it.

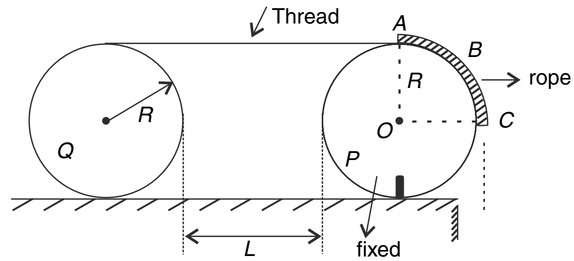
(a) Calculate the extra power that the motor driving the belt has to spend while the cylinder gains rotational speed. Assume coefficient of friction = $\mu$.

(b) Prove that 50% of the extra work done by the motor after the cylinder is placed over it, is dissipated as heat due to friction between the belt and the cylinder.

Q.118. A uniform disc of mass $M$ and radius $R$ is rotating freely about its central vertical axis with angular speed $\omega_0$. Another disc of mass $m$ and radius $r$ is free to rotate about a horizontal rod $AB$. Length of the rod $AB$ is $L$ (< $R$) and its end A is rigidly attached to the vertical axis of the first disc. The disc of mass m, initially at rest, is placed gently on the disc of mass $M$ as shown in figure. Find the time after which the slipping between the two discs will cease. Assume that normal reaction between the two discs is equal to mg. Coefficient of friction between the two discs is $\mu$.




Q.119. P is a fixed smooth cylinder of radius $R$ and $Q$ is a disc of mass $M$ and radius $R$. A light thread is tightly wound on $Q$ and its end is connected to a rope ABC. The rope has a mass m and length $\frac{\pi R}{2}$ and is initially placed on the cylinder with its end A at the top. The system is released from rest. The rope slides down the cylinder as the disc rolls without slipping. The initial separation between the disc and the cylinder was $L = \frac{\pi R}{2}$ (see fig). Find the speed with which the disc will hit the cylinder. Assume that the rope either remains on the cylinder or remains vertical; it does not fly off the cylinder.

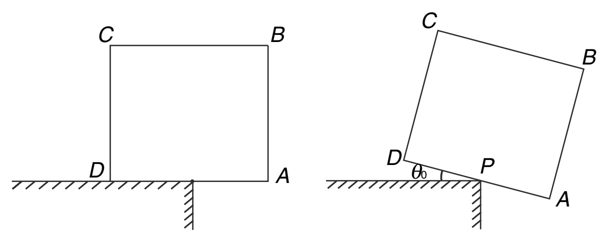



Q.120. A uniform cube of mass $M$ and side length $a$ is placed at rest at the edge of a table. With half of the cube overhanging from the table, the cube begins to roll off the edge. There is sufficient friction at the edge so that the cube does not slip at the edge of the table. Find -

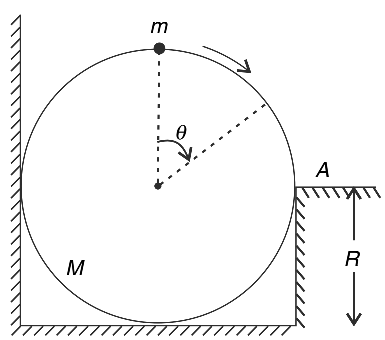
(a) the angle $\theta_0$ through which the cube rotates before it leaves contact with the table.

(b) the speed of the centre of the cube at the instant it breaks off the table.

(c) the rotational kinetic energy of the cube at the instant its face AB becomes horizontal.




Q.121. A uniform frictionless ring of mass $M$ and radius $R$, stands vertically on the ground. $A$ wall touches the ring on the left and another wall of height $R$ touches the ring on right (see figure).There is a small bead of mass m positioned at the top of the ring. The bead is given a gentle push and it being to slide down the ring as shown. All surfaces are frictionless.

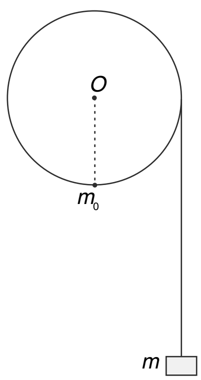



(a) As the bead slides, up to what value of angle $\theta$ the force applied by the ground on the ring is larger than $Mg$?

(b) Write the torque of force applied by the bead on the ring about point A as function of $\theta$.

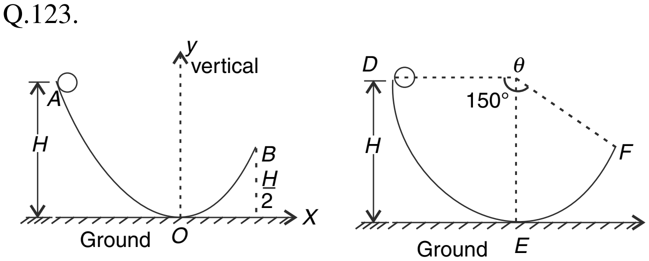
(c) What is the maximum possible value of torque calculated in (b)? Using this result tell what is the largest value of $\frac{m}{M}$ for which the ring never rises off the ground ?

Q.122.




A uniform disc shaped pulley is free to rotate about a horizontal axis passing through the centre of the pulley. A light thread is tightly wrapped over it and supports a mass m at one of its end. A small particle of mass $m_0 = \sqrt{2}m$ is stuck at the lowest point of the disc and the system is released from rest. Will the particle of mass $m_0$ climb to the top of the pulley?

Q.123.



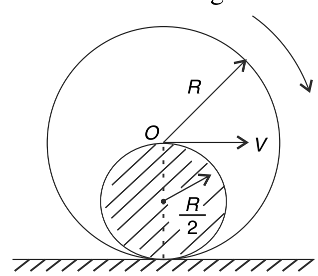

AOB is a frictionless parabolic track in vertical plane. The equation of parabolic track can be expressed as $y = \frac{3}{2H}x^2$ for co-ordinate system shown in the figure. The end B of the track lies at $y = \frac{H}{2}$. When a uniform small ring is released on the track at A it was found to attain a maximum height of $h_1$, above the ground after leaving the track at B. There is another track DEF which is in form of an arc of a circle of radius $H$ subtending an angle an angle of 150° at the centre. The radius of the track at D is horizontal. The same ring is released on this track at point D and it rolls without sliding. The ring leaves the track at F and attains a maximum height of $h_2$ above the ground. Find the ratio $\frac{h_1}{h_2}$.

Q.124. A uniform sphere of radius $R$ has a spherical cavity of radius $\frac{R}{2}$ (see figure). Mass of the sphere with cavity is $M$. The sphere is rolling without sliding on a rough horizontal floor [the line joining the centre of sphere to the centre of the cavity remains in vertical plane]. When the centre of the cavity is at lowest position, the centre of the sphere has horizontal velocity $V$. Find:

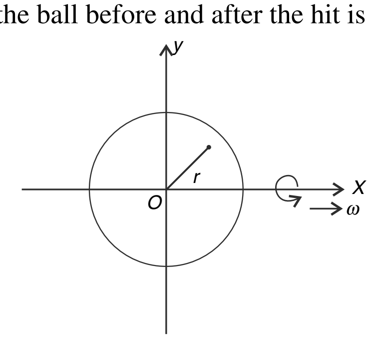
(a) The kinetic energy of the sphere at this moment.

(b) The velocity of the centre of mass at this moment.

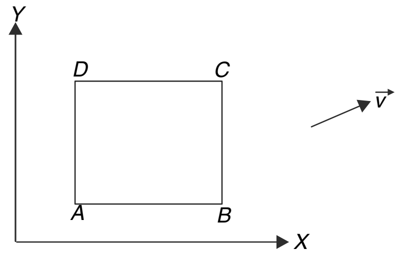
(c) The maximum permissible value of $V$ ( in the position shown ) which allows the sphere to roll without bouncing




Q. 125. A uniform ball of mass $M$ and radius $R$ can rotate freely about any axis through its centre. Its angular velocity vector is directed along positive $x$ axis. A bullet is fired along negative $Z$ direction and it pierces through the ball along a line that is at a perpendicular distance $r$ ($\leq R$) from the centre of the ball. The bullet passes quickly and its net effect is that it applies an impulse on the ball. Mass of the bullet is m and its velocity charges from u to $v$ ($\leq u$) as it passes through the ball. As a result the ball stops rotating about $X$ axis and begins to rotate about y axis. The angular speed of the ball before and after the hit is $\omega$. Find $r$.



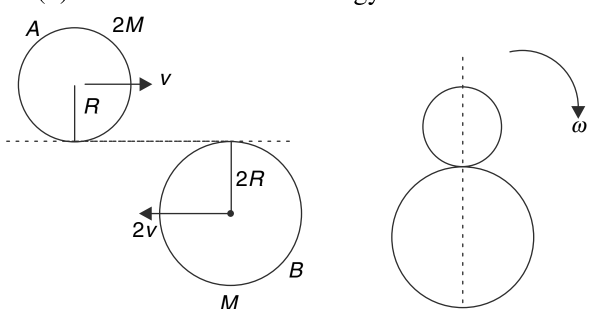
Q. 126. A uniform square plate *ABCD* has mass *M* side length *a*. It is sliding on a horizontal smooth surface with a velocity of $\vec{v} = v_0\left(4\hat{i} + 2\hat{j}\right)$. There is no rotation. Vertex A of the plate is suddenly fixed by a nail. Calculate the velocity of centre of the plate immediately after this.



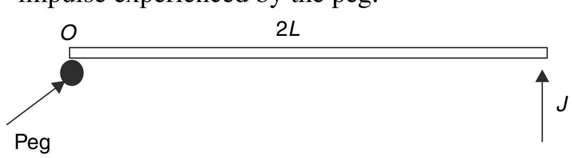

Q. 127. Two discs *A* and *B* are moving with their flat circular surface on a smooth horizontal surface. Mass, radius and velocity of the two discs are – $m_A = 2M$, $m_B = M$, $r_A = R$, $r_B = 2R$, $v_A = v$, and $v_B = 2v$. The velocities of the two discs are oppositely directed so that they just cannot avoid collision and stick to each other (see figure)

(a) Find the angular speed of the composite system after collision

(b) Find loss in kinetic energy due to collision



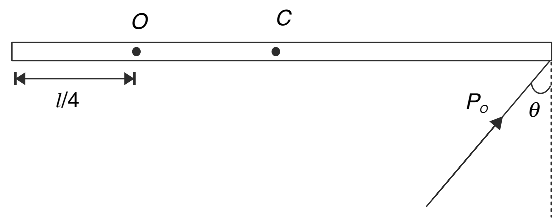

Q. 128. A uniform rod of mass *M* and length 2*L* lies on a smooth horizontal table. There is a smooth peg *O* fixed on the table. One end of the rod is kept touching the peg as shown in the figure. An impulse *J* is imparted to the rod at its other end. The impulse is horizontal and perpendicular to the length of the rod. Find the magnitude of impulse experienced by the peg.



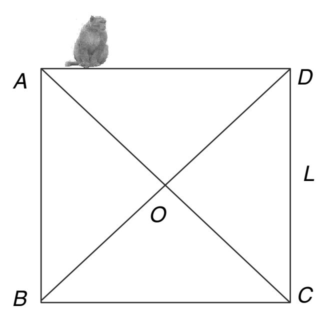

Q. 129. A uniform rod of mass *M* and length $\ell$ is hinged at point *O* and is free to rotate on a horizontal smooth surface. Point *O* is at a distance of $\frac{\ell}{4}$ from one end of the rod. A sharp impulse $P_0 = 2\sqrt{130}\ kg\ m/s$ is applied along the surface at one end of the rod as shown in figure $\left[\tan\theta = \frac{9}{7}\right]$

(a) Find the angular speed of the rod immediately after the hit

(b) Find the impulse on the rod due to the hinge.




Q. 130. Four thin rods of length $L = 1.0\ m$ each are joined to form a square *ABCD*. The opposite vertices of the square are joined by mass less rods *AC* and *BD*. This square frame is mounted on a horizontal axis through its centre so that the frame can rotate freely in the vertical plane. Masses of rods *AB* and BC are $m = 2kg$ each and the rod *AD* and *DC* have mass $M = 4kg$ each. A monkey of mass $m_o$ =12 kg is at rest on the horizontal rod *AD* and keeps the system in equilibrium. The monkey takes a sudden jump and rises to a height *H* from its initial position. Calculate minimum value of *H* so that the square frame is able to complete a rotation about its central axis. Assume no further contact between monkey and the frame.

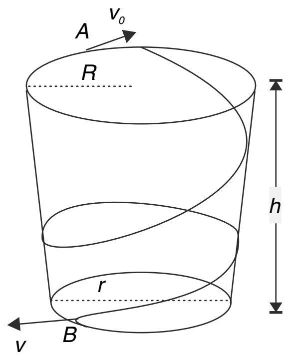



Q. 131. A frustum has been mounted with its axis vertical. It has a height *h* and radii of its upper and lower cross sections are *R* and *r* respectively. A particle is projected with horizontal velocity $v_0$ along its upper brim. The particle spirals down the inner surface and leaves the lower face at point *B*. The inner wall of the frustum is smooth.

(a) Find the vertical component of velocity of the particle as it leaves the frustum at *B*.

(b) Find minimum value of *h* for which the particle will never come out of the frustum.

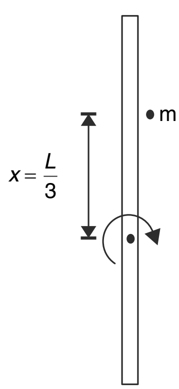
Take $r = \frac{R}{2}$ for solving this part of the problem.




Q. 132. A uniform thin stick of mass $M = 24kg$ and length $L$ rotates on a friction less horizontal plane, with its centre of mass stationary. A particle of mass $m$ is placed on the plane at a distance $x = \frac{L}{3}$ from the centre of the stick . This stick hits the particle elastically

(a) Find the value of $m$ so that after the collision, there is no rotational motion of the stick

(b) For what minimum of $x$ can we get a value of '$m$' so that the rod has no rotational motion after elastic collision?



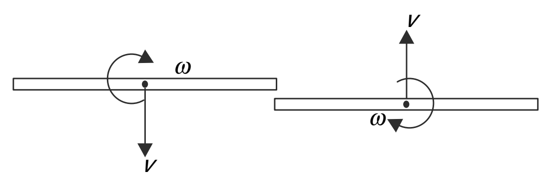

Q. 133. A uniform rod of length $L$ is rotating in a horizontal plane about a vertical axis passing through one of its ends. At a distance $x$ ($< L$) from the axis there is a fixed vertical pole. The rod hits the pole and its direction of motion is reversed. Find $x$ if it is known that during the impact the axis of rotation imparts no impulse to the rod. Does your answer depend on coefficient of restitution?

[NOTE : If you hit a lamp post with a rod, the hand holding the rod gets hurt as long as the impact misses the so called sweet spot of the rod (and hits either above or below the sweet spot). After solving the above problem you know where the sweet spot is ! You may assume that during the impact the rod is rotating about its holding hand. And if you play cricket, you know that there is a sweet spot in your bat too ! If the ball hits way above or below the spot you get stung.]

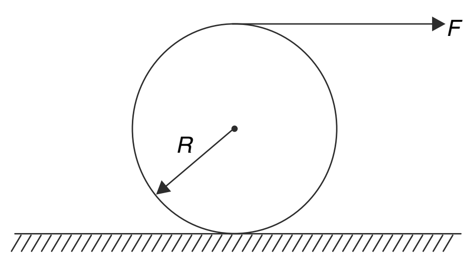
Q. 134. Two identical thin rods are moving on a smooth table, as shown. Both of them are rotating with angular speed $\omega$, in clockwise sense about their centres. Their centres have velocity $V$ in opposite directions. The rods collide at their edge and stick together. Length of each rod is $L$.

(a) For what value of $\frac{V}{\omega L}$ there will be no motion after collision ?

(b) If the ratio $\frac{V}{\omega L}$ is half the value found in (a) above, what fraction of kinetic energy is lost in the collision?




Q. 135. Light thread is tightly wound on a uniform solid cylinder of radius $R$. The cylinder is placed on a smooth horizontal table and the thread is pulled horizontally as shown, by applying a constant force $F$. How much length of the thread is unwound from the cylinder by the time its kinetic energy becomes equal to $K$.




Q. 136. A horizontal disc of radius $R$ and mas 20 $M$ is pivoted to rotate freely about a vertical axis through its centre. A small insect $A$ of mass $M$ and another small insect $B$ of mass $m = \frac{M}{4}$ are initially at diametrically opposite points on the periphery of the disc. The whole system is imparted an angular speed $\omega_0$. Insect $A$ walks along the diameter with constant velocity $v$ relative to the disc unit it reaches $B$ which remains at rest on the disc. $A$ then eats $B$ and returns to its starting point along the original path with same speed $v$ relative to the disc.

(a) Find the angular speed of the disc when $A$ reaches the centre after eating $B$.

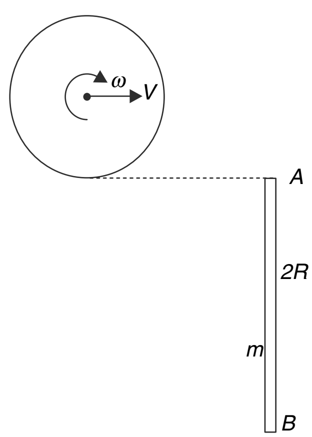
(b) Plot approximately, the variation of angular speed of the disc with time for the entire journey of the insect $A$.

**Q.137.** A disc of mass $m$ and radius $R$ is moving on a smooth horizontal surface with the flat circular face on the surface. It is spinning about its centre with angular peed $\omega$ and has a velocity $V$ (see figure). It just manages to hit a stick $AB$ at its end A. The stick was lying free on the surface and stick to the disc. [The combined object becomes like a badminton racket]. Mass and length of the stick are $m$ and $2R$ respectively.

(a) Calculate the angular speed of the combined object assuming $V = R\omega$

(b) Calculate loss in kinetic energy. Why is energy lost?

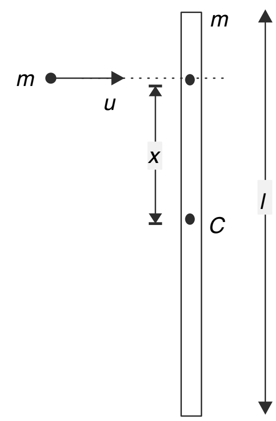
(c) If $V = \eta(R\omega)$, loss in kinetic energy is minimum. Find $\eta$. [Assume $\omega$ is given]




Q. 138. A uniform rod of mass $m$ and length $\ell$ has been placed on a smooth table A particle of mass $m$, travelling perpendicular to the rod, hits it at a distance $x = \frac{\ell}{\sqrt{6}}$ from the centre $C$ of the rod. Collision is elastic.

(a) Find the speed of the centre of the rod and the particle after the collision.

(b) Do you think there is a chance of second Collision? If yes, how is the system of particle and stick moving after the second collision?

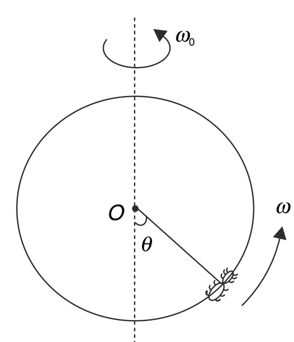



Q. 139. A ring is made to rotate about its diameter at a constant angular speed of $\omega_0$. A small insect of mass $m$ walks along the ring with a uniform angular speed $\omega$ relative to the ring (see figure). Radius of the ring is $R$.

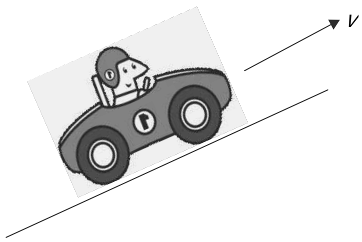
(a) Find the external torque needed to keep the ring rotating at constant speed as the insect walks. Express your answer as a function of $\theta$. For what value of $\theta$ is this torque maximum?

[given your answer for $0 \le \theta \le 90°$]

(b) Find the component of force perpendicular to the plane of the ring, that is applied by the ring on the insect. For what value of $\theta$ is this force maximum? Argue quantitatively to show that indeed the force should be maximum for this value of $\theta$. [Give your answer for $0° \le \theta \le 90°$]



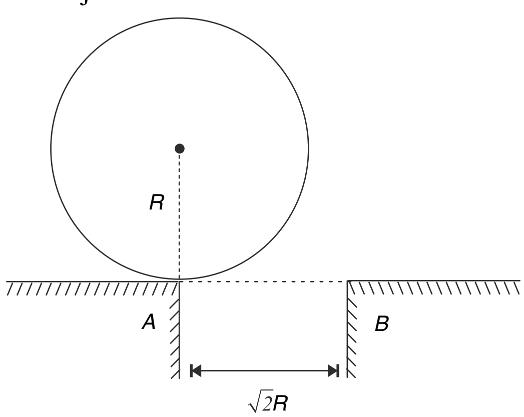

Q. 140. A small car took off a ramp at a speed of 30 $m/s$. Immediately after leaving the ramp, the driver applied brakes on all the wheels. The brakes retarded the wheels uniformly to bring them to rest in 2 second. Calculate the angle by which the car will rotate about its centre of mass in the 2 second interval after leaving the ramp. Radius of each wheel is $r = 0.30\ m$ . Moment of inertia of the car along with the driver, about the relevant axis through its centre of mass is $I_M = 80 kgm^2$ and the moment of inertia of each pair of wheels about their respective axles is $0.3 kg\ m^2$. Assume that the car remained in air for more than 2 second . Also assume that before take- off the wheels rolled without sliding.




Q. 141. A disc of radius $R$ stands at the edge of a table. If the given a gentle push and it beings to fall. Assume that the disc does not slip at A and it rotates about the point as it falls. The falling disc hits the edge of another table placed at same height as the first one at a horizontal distance of $\sqrt{2}\,R$. Imagine that the disc hits the edge $B$ and rotates (up) about the edge

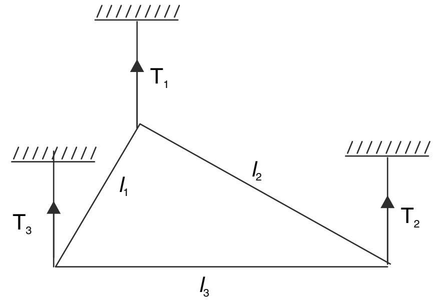
(a) Find the speed of the centre of the disc at the instant just before it hits the edge $B$.

(b) Find the angular speed of the disc about $B$ just after the hit.

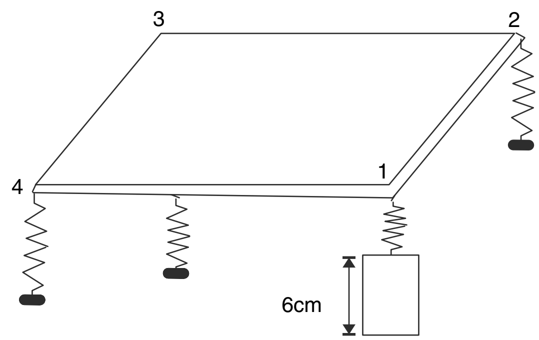



## LEVEL 3

Q. 142. A uniform triangular plate is kept horizontal suspended with the help of three vertical threads as shown. The sides of the plate have length $l_1$, $l_2$ and $l_3$. Find tension $T_1$, $T_2$ and $T_3$ in the three threads. Mass of the plate is $M$.



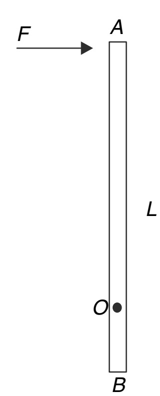

Q. 143. A rigid large uniform square platform is resting on a flat horizontal ground supported at its vertices by four identical spring. At vertex 1 a wooden block, 6 $cm$ high, is inserted below the spring. Calculate the change in height of the centre of the platform. Assume change in height to be small compared to dimension of the platform.




Q. 144. A uniform rod of mass $M$ and length $L$ is placed freely on a horizontal table. A horizontal force $F$ is applied perpendicular to the rod at one of its ends. The force $F$ is increased gradually from zero and it is observed that when its value becomes $F_0$, the rod just begins to rotate about point $O$

(i) Find length $AO$

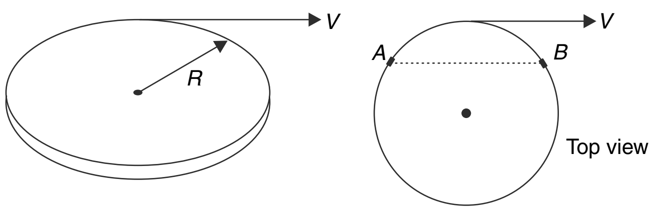
(ii) Find $F_0$




Q. 145. A ring of mass $M$ and radius $R$ lies flat on a horizontal table. A light thread is wound around it and its free end is pulled with a constant velocity $v$.

(a) Two small segment $A$ and $B$ (see fig.) in the ring are rough and have a coefficient of friction $\mu$ with the table. Rest of the ring is smooth. Find the speed with which the ring moves.

(b) Find the speed of the ring if coefficient of friction is $\mu$ everywhere; for all points on the ring.

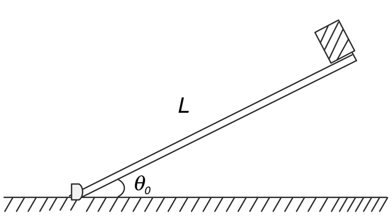



Q. 146. A uniform stick of length $L$ is pivoted at one end on a horizontal table. The stick is held forming an angle $\theta_0$ with the table. A small block of mass m is placed at the other end of the stick and it remains at rest. The system is released from rest

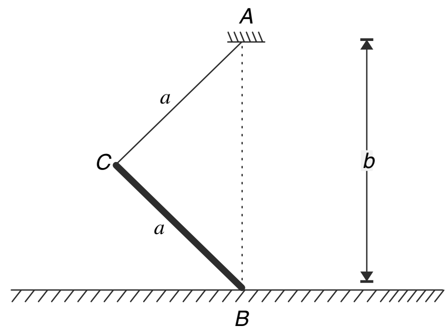
(a) Prove that the stick will hit the table before the block if $\cos\theta_0 \geq \sqrt{\frac{2}{3}}$

(b) Find the contact force between the block and the stick immediately before the system is released. Take $\theta_0 = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

(c) Find the contact force between the block and the stick immediately after the system is released if $\theta_0 = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$.



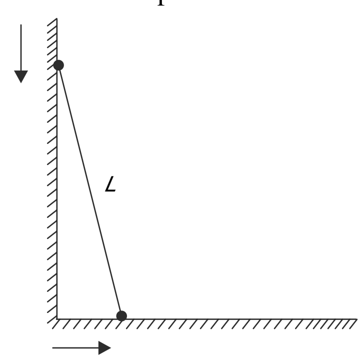

Q. 147. A uniform rod $BC$ with length $a$ is attached to a light string $AC$. End $A$ of the string is fixed to the ceiling and the end $B$ of the rod is on a smooth horizontal surface. $B$ is exactly below point A and length $AB$ is $b$ ($a < b < 2a$). The system is released from rest and the rod begins to slide. Find the speed of the centre of the rod when the string becomes vertical.




Q. 148. A dumb – bell has a rigid mass less stick and two point masses at its ends. Each mass is $m$ and length of the stick is $L$. The dumb- bell leans against a frictionless wall, standing on a frictionless ground. It is initially held motionless, with its bottom end an infinitesimal distance from the wall. It is released from this position and its bottom end slides away from the wall where as the top end slides down along the wall.

(a) Show that centre of mass of the dumb-bell moves along a circle.

(b) When the dumb-bell loses contact with the wall what is speed of the centre of mass?



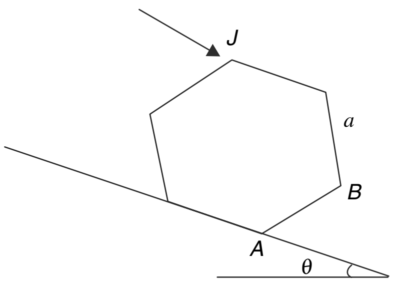

Q. 149. A hexagonal pencil of mass $M$ and sides length $a$ has been placed on a rough incline having inclination $\theta$. Friction is large enough to prevent sliding. If at all the pencil moves, during one full rotation each of its 6 edges, in turn, serve as instantaneous axis of rotation.

(a) Show that for $\theta > 30°$ the pencil cannot remain at rest.

(b) For inclination of incline $\theta$ (< 30°) the pencil will not roll on its own. A sharp impulse $J$ is given to the pencil parallel to the incline at its upper edge (see figure). Friction does not allow the pencil to slide but it begins to rotate about the edge through A with initial angular speed $\omega_0$. Find $\omega_0$. Moment of inertia of the pencil about its edge is $I$.

(c) Find minimum value of $J$ so that the pencil will turn about $A$; and $B$ will land on the incline.

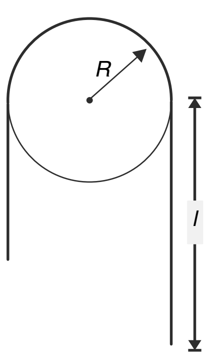
(d) If kinetic energy acquired by the pencil just after the impulse is $K_0$, find its kinetic energy just before edge $B$ lands on the incline



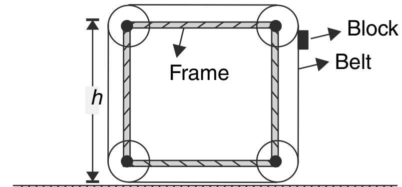

Q. 150. A rope of length $L$ and mass per unit length $\lambda$ passes over a disc shaped pulley of mass $M$ and radius $R$. The rope hangs on both sides of the pulley and the length of larger hanging part is $l$. The pulley can rotate about a horizontal axis passing through its centre. The system is released from rest and it begins to move. The pulley has no friction at its axle and the rope has large enough friction to prevent it from slipping on the pulley.

(a) Find the acceleration of the rope immediately after it is released.

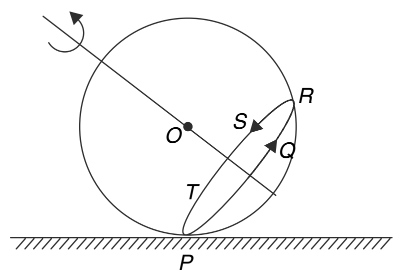
(b) Find the horizontal component of the force applied by the axle on the pulley immediately after the system is released



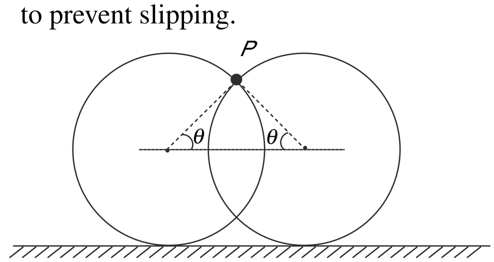

Q. 151. A toy is made of a rectangular wooden frame with four small wheels at its vertices. A tight fitting belt of negligible mass runs around the frame passing over the wheels. Mass of the complete toy is $M$. Now a small block of mass $m$ is stuck at the top of the right vertical segment of the belt and the system is released. Height of the toy is $h$. Find the speed of the block when it is about to hit the ground. Assume no slipping anywhere and neglect the dimension of the wheels.



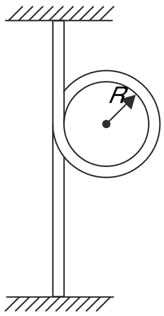

Q. 152. Consider an idealized case of rolling of a solid ball in which the point $P$ does not rotate in a vertical plane. But it rotates on circular path $PQRSTP$ when observed from the centre of the ball. The radius of circular path $PQRSTP$ is half the radius of the ball. The ball rolls without sliding with its centre moving with speed $v_0$ in direction perpendicular to the plane of the figure calculate the kinetic energy of the rolling ball. Mass of the ball and its radius are $M$ and $R$ respectively.




Q. 153. Two identical rings, each of mass $M$ and radius $R$, are standing on a rough horizontal surface. The rings overlap such that the horizontal line passing through their centre makes an angle of $\theta = 45°$with the radius through their intersection point $P$. A small object of mass $m$ is placed symmetrically on the rings at point $P$ and released. Calculate the acceleration of the centre of the ring immediately after the release. There is no friction between the small object and the rings. The friction between the small object and the rings, and the friction between the rings and the ground is large enough to prevent slipping.



Q. 154. A uniform rope tightly wraps around a uniform thin ring the mass $M$ and radius $R$. The mass of the segment of the rope around the ring (i.e. , mass of the length $2\pi R$ of the rope) is also $M$. The ends of the rope are fixed one above the other and it is taut. The ring is let go. Find its acceleration. Assume no slipping and thickness of the rope to be negligible.

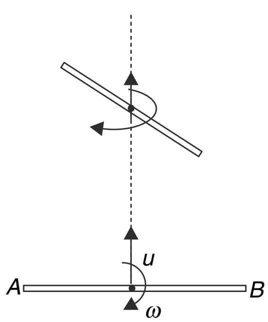



Q. 155. A uniform stick $AB$ has length $L$. It is tossed up from horizontal position such that its centre receives a velocity $u = \pi\sqrt{gL}$ in vertically upward direction and the stick gets an angular velocity. The stick lands back to its point of projection in horizontal position. During its course of flight its angular velocity remained constant and the stick made one complete rotation. Stick rotates in vertical plane.

(a) Calculate the angular velocity ($\omega$) imparted to the stick.

(b) Calculate the maximum height, above the point of projection, to which the end $B$ of the stick rises.

[Take solution of equation $\cos x = 2x$ to be $x = 0.45$ and $\sin(0.45) = 0.43$]

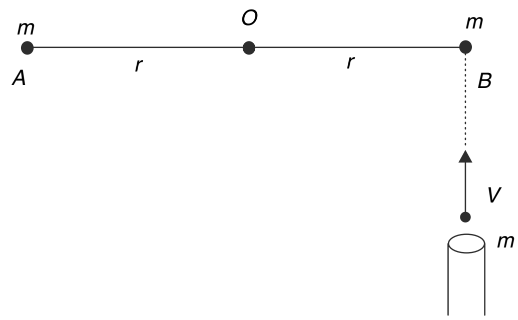



Q. 156. A uniform rod of mass $4m$ and length $2r$ can rotate in horizontal plane about a vertical axis passing through its centre $O$. Two small balls each of mass $m$ are attached to its ends. A fixed gun fires identical balls with speed $v$ in horizontal direction. The firing is being done at suitable intervals so that the fired balls either hit the ball at end $A$ or $B$ while moving in the direction of velocity of $A$ or $B$. All collisions are elastic.

(i) Initial angular velocity of the rod is zero and its angular velocity after $n^{th}$ collision is $\omega_n$. Write $\omega_{n+1}$ in terms of $\omega_n$

(ii) Solve the above equation to get $\omega_n$

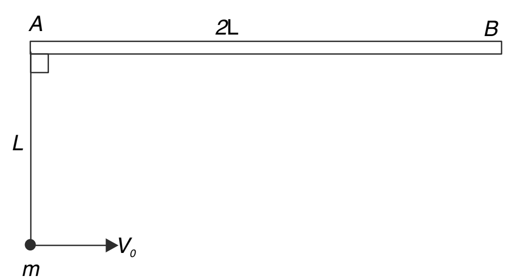
(iii) Find the limiting value of $\omega$.




Q. 157. A uniform rod of mass $m$ and length $2L$ on a smooth horizontal surface. A particle of mass $m$ is connected to a string of length $L$ whose other end is connected to the end '$A$' of the rod. Initially the string is held taut perpendicular to the rod and the particle is given a velocity $v_0$ parallel to the initial position of the rod.

(a) Calculate the acceleration of the centre of the rod immediately after the particle is projected.

(b) The particle strikes the centre of the rod and sticks to it. Calculate the angular speed of the rod after this.




Q. 158. Two boys support by the ends a uniform rod of mass $M$ and length $2L$. The rod is horizontal. The two boys decided to change the ends of the rod by throwing the rod into air and catching it. The boys do not move from their position and the rod remained horizontal throughout its flight. Find the minimum impulse applied by each boy on the rod when it was thrown.

Q. 159 A uniform rod of mass m and length $l$ pivoted at one of its top end is hanging freely in vertical plane. Another identical rod moving horizontally with velocity $v$ along a line passing through its lower end hits it and sticks to it. The two rods were perpendicular during the hit and later also they remain perpendicularly connected to each other. Find the maximum angle turned by the two-rod system after collision.

## ANSWERS

**1.** $\frac{3h}{R}$

**2.** $2v$

**3.** $\frac{1}{3}$

**4.** $\frac{3}{2}$

**5.** $\frac{1}{4}MR^2$

**6.** $\frac{MR^2}{3}$

**7.** $x^2 + y^2 = \frac{5\ell^2}{4}$

**8.** $MR^2\left(1-\frac{4}{\pi^2}\right)$

**9.** $a, b, c, d$

**10.** $\frac{10}{3}\%$

**11.** (i) $\frac{ML^2}{2}$

(ii) (a) $MOI$ does not change

(b) $I = I_0 = \frac{1}{2}MR^2$

**12.** $5\frac{I}{4}$

**13.** $\frac{7I}{16}$

**14.** $29\ MR^2$

**15.** $\frac{ML^2}{6}\left(\frac{h_1+3h_2}{h_1+h_2}\right)$

**16.** $2MR^2\left(1-\frac{8}{9\pi^2}\right)$

**17.** $\alpha = \tan^{-1}(2)$ ; $\beta = 45°$

**19.** $\alpha = \tan^{-1}\left(\frac{2}{\pi}\right)$

**20.** $\frac{29}{6}M$

**21.** (a) $\cos\theta = \frac{R-h}{R}$

(b) Decreases

**22.** $\sin^{-1}(0.3)$

**23.** $t = \frac{2d.g.m}{kh}$

**24.** $F = \frac{mg}{2}$

**25.** (a) $f = m_0 g$ towards left

(b) $m = 4m_0$

**26.** $\mu_{\min} = \frac{1}{2}$

**27.** (a) $\frac{Mg}{2}$

(b) $\frac{1}{\sqrt{3}}$

**28.** (a) Yes

(b) $k = \frac{mg}{d}$

**29.** $\frac{R^2\omega^2}{4g}$

**30.** (a) $\frac{g}{2}$

(b) $\frac{M}{m} < \frac{1}{3}$

**31.** (a) $\omega = \sqrt{\frac{100\pi mg}{R(5m+2M)}}$

(b) $g\left(\frac{2M}{5m+2M}\right)$

**32.** $a = \frac{F}{4m}$

**33.** $N = \left(1 - \frac{2}{\pi}\right)\text{Mg}$

**34.** Less than weight

**35.** $a = \frac{3g\sin\theta}{4}$

**36.** $h_0 = \frac{2R}{5}$

**37.** (a) $\frac{4F}{3M+8m}$

(b) $\frac{3MF}{3M+8m}$

**38.** (i) $\frac{12V_0}{7}$

(ii) (a) $V_0 > \frac{2}{5}\omega_0 R$

(b) $V_0 = \frac{2}{5}\omega_0 R$

(c) $V_0 < \frac{2}{5}\omega_0 R$

**39.** (a) $\frac{1}{2}M(\omega R)^2$

(b) $M\omega^2 R$

**40.** $\frac{3}{4}MV^2$ in both cases. In case (b) the Kinetic energy will be higher if length was doubled.

**42.** (i) 100 $J$
(ii) 33.33 $J$

**43.** Case (b)

**44.** $V = \sqrt{5gR}$

**45.** $V_{max} = (\sqrt{5} + 1)\ V$; $V_{min} = (\sqrt{5} - 1)\ V$

**46.** The nose of the plane tends to veer downward.

**47.** $\pi \times 10^5$ rad/s

**48.** $\frac{6}{5} m/s$

**49.** $\frac{V}{R}$

**50.** $\omega = \frac{24}{19}\omega_0$

**51.** (a) $\frac{MV_0 b}{2}$

(b) Normal reaction produces a torque

**52.** $\frac{3u}{4}$

**53.** $\frac{1}{3} mvL\sin\theta$, No

**54.** $\left(\frac{1}{2}\right)^4$

**55.** (a) $\vec{V}_A = 4i - 3j\,; \vec{V}_B = 3i - \frac{9}{4}j;$

$\vec{V}_C = \frac{13}{4}i - 2j$ all in unit of m/s

(b) $\omega = \frac{1}{4}$ rad/s

**57.** (a) $\omega = \frac{v}{2\sqrt{3}R}$

(b) $v_y = \frac{v}{4}(\downarrow)$

**58.** (a) $V_A = 2V$, $V_B = \frac{3V}{2}$

(b) $t = \frac{2R}{V}\cos^{-1}\left(\frac{3}{4}\right)$

**59.** (a) $\sqrt{156}\ ms^{-2}$

(b) $2\sqrt{3}\ ms^{-2}$

(c) $(3, \pi/6)$

**60.** $\dfrac{5v^2}{6R}$

**61.** $R$

**62.** $4R$

**63.** 2.26 rad/s

**64.** (a) $v$

(b) $\dfrac{1}{2}$

**65.** $I = 4\lambda R^3\ (\theta_0 - \sin\theta_0)$

**66.** (a) $\dfrac{9}{2}mR^2$

(b) $\omega = \dfrac{2vd}{11R^2}$

**67.** $\dfrac{91}{486}Ma^2$

**68.** $\dfrac{Ma^2}{3}$

**69** $\dfrac{1}{2}Ma^2\left(1-\dfrac{2}{3}\sin^2\dfrac{\theta}{2}\right)$

**70.** (a) $\dfrac{\sqrt{3}}{2}M$

(b) $\dfrac{3\sqrt{3}}{8}MR^2$

**71.** $\dfrac{Ma^2}{12}$

**72.** $\mu_{\text{min}} = \dfrac{\sin\theta}{1+\cos\theta}$ ; $m_{\text{block}} = \dfrac{M\sin\theta}{1+\cos\theta}$

**73.** $\dfrac{3\sin\theta}{1+\cos\theta}$

**74.** Less than $\sqrt{\dfrac{3}{2}}R$

**75.** $M_A < M_B$

**76.** $\dfrac{\sqrt{3}}{4}Mg$

**77.** $(v_0)_{\max} = \dfrac{1}{L}\sqrt{\dfrac{3Mg}{\rho}}$

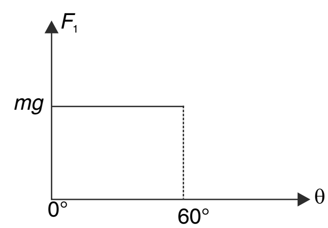
**78.** $\dfrac{Mg}{2}$

**79.** 41°

**80.** 50°

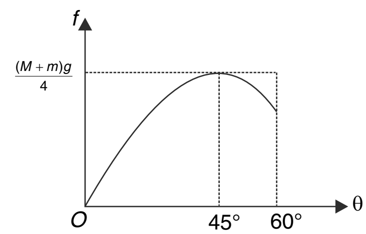
**81.** $\theta = \pi - \tan^{-1}\left(\dfrac{1}{4}\right)$

**82.** (a)




(b) $\mu_{\min} = \sqrt{3}$

(c)




**83.** (b) $\alpha = \tan^{-1}(2\tan\theta)$

(c) $\theta_{\min} = \tan^{-1}\left(\dfrac{1}{2\mu}\right)$

(d) Slippage is more likely when at *A*.

**84.** $T_1 = \dfrac{Mg}{12}; T_2 = \dfrac{Mg}{3}; T_3 = \dfrac{7Mg}{12}$

**85.**

(a) $\dfrac{a}{b} = \dfrac{1}{2}\sin^2\theta_1\left[\left(1-\mu^2\right)\sin\theta_1 - 2\mu\cos\theta_1\right]$

(b) $\dfrac{a}{b} = \dfrac{1}{2}\sin^2\theta_2\left[\left(1-\mu^2\right)\sin\theta_2 + 2\mu\cos\theta_2\right]$

**86.** $h = \dfrac{L}{2}$

**87.** $\mu_{\max} = \dfrac{1}{2}$

**88.** (a) $\sqrt{\frac{3F}{ML}}$

(b) $\theta = \tan^{-1}\left(\frac{1}{10}\right)$

**89.** (a) 7.5 kg m/s

(b) $f = 9\ N$, $N = 24.5\ N$

(c) $\cos^{-1}\left(\frac{2}{3}\right)$

**90.** (a) $\frac{3g}{8}$

(b) $\frac{2g}{5}$ and $\frac{g}{5}$

**91.** $\frac{4g}{11}$ ; Thread exerts more force on $A$

**92.** (a) $\frac{5}{4}mg, \frac{mg}{2}, \frac{3}{4}mg$

(b) $\frac{\sqrt{3}}{4}g$

**93.** (a) More time

(b) $\frac{\sqrt{21}-3}{6}$

**94.** $F = \frac{Idu^2}{2\pi R^4}$

**95.** (a) $\frac{3F}{4M}$

(b) $\frac{7F}{8}$

**96.** $a = \frac{g}{4\sqrt{3}}$

**97.** $Mg\sqrt{\frac{1}{4}+\frac{1}{\pi^2}}$

**98.** $\omega_0 = \sqrt{\frac{15}{32}\frac{g}{R}}$ ; $\alpha_0 = \frac{15}{128}\frac{g}{R}$; Yes

**99.** (a) $\frac{6g}{11}$

(b) $\frac{18}{11}s$

**100.** (a) $t_0 = \sqrt{\frac{2\pi(2m+3M)r}{Mg}}$

(b) $a = \frac{2g}{2m+3M}\sqrt{M^2 + (4\pi M + M + m)^2}$

(c) Yes.

**101.** (a) $\frac{5}{8} rad/s^2$

(b) $\frac{155}{4}N$

(c) 5 $N$

**102.** A point at a distance $\frac{R}{2}$ from centre.

**103.** $\mu_{min} = \frac{1}{g}$

**104.** (a) $F_0 = \frac{Mg\sin\theta}{4}$

(b) 2 $Mgh$

**105.** (a) Zero

(b) $\frac{F^2t^2}{3m}$

**106.** (a) 1 m

(b) 12 $m/s$

(c) 3.6 $\sqrt{3}$ m

**107.** 2T

**108.** $t = \sqrt{5}\ s$.

**109.** No; $\frac{10}{9} m/s^2$

**110.** (a) $\frac{kx_0}{6M}$

(b) $\frac{6x_0}{7}$

**111.** $\frac{\sqrt{935}}{34}\frac{g}{L}$

**112.** (a) $\frac{2Mg}{123}$ Vertically down

(b) $\mu = 22.5$

**113.** 112.5 $KJ$ ; No.

**114.** (a) $\frac{r\omega_0}{4}$

(b) $\frac{1}{4}mr^2\omega_0$

(c) $\frac{r^2\omega_0^2}{32\mu g}$

**115.** 5.8º

**116.** $\sqrt{3gl}$

**117.** (a) $P = \mu mg.V$

**118.** $\frac{MR^2L\omega_0}{2\mu g\left[MR^2 + mL^2\right]}$

**119.** $V = \sqrt{\left(\frac{8+3\pi^2}{\pi}\right)\left(\frac{mgR}{3M+8m}\right)}$

**120.** (a) $\cos^{-1}\left(\frac{6}{11}\right)$

(b) $\sqrt{\frac{3ga}{11}}$

(c) $\frac{Mga}{11}$

**121.** (a) $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

(b) $mgR\,(2\cos\theta - 3\cos^2\theta)$

(c) $\tau_{\max} = \frac{mgR}{3}; \left(\frac{m}{M}\right)_{\max} = 3$

**122.** No.

**123.** $\frac{h_1}{h_2} = \frac{14}{11}$

**124.** (a) $\frac{31}{40}MV^2$

(b) $\frac{15V}{14}$

(c) $V \le \sqrt{14gR}$

**125.** $\frac{2\sqrt{2}}{5}\frac{MR^2\omega}{m(u-v)}$

**126.** $v_c = \frac{3v_0}{4}\left(\hat{i} - \hat{j}\right)$

**127.** (a) $\omega = \frac{2v}{3R}$

(b) $kE_{\text{loss}} = Mv^2$

**128.** $\frac{J_0}{2}$

**129.** (a) $\omega = \frac{36P_0\cos\theta}{7m\ell} = \frac{36}{m\ell}$

(b) $P = P_0\sqrt{\sin^2\theta + \left(\frac{2}{7}\cos\theta\right)^2} = \sqrt{85}N - s$

**130.** $4(\sqrt{2} - 1)m$

**131.** (a) $\sqrt{2gh - v_0^2\left(\frac{R^2}{r^2} - 1\right)}$

(b) $h_{\min} = \sqrt{\frac{3}{2g}}v_0$

**132.** (a) $m = 72\ kg$

(b) $x > \frac{L}{2\sqrt{3}}$

**133.** $\frac{2\ell}{3}$

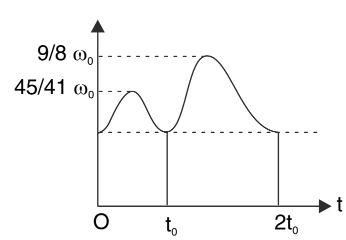
**134.** (a) $\frac{1}{6}$

(b) $\frac{49}{52}$

**135.** $\frac{2K}{3F}$

**136.** (a) $\frac{9}{8}\omega_0$

(b)



**137.** (a) $\omega_0 = \dfrac{9}{17}\omega$

(b) $\Delta E = \dfrac{5}{68}mV^2 + \dfrac{7}{34}mR^2\omega^2 - \dfrac{3}{17}mV R\omega$

(c) $\eta = \dfrac{6}{5}$

**138.** (a) Both have speed $\dfrac{u}{2}$

(b) The stick is at rest. Particle moves in original direction with speed

**139.** (a) $\tau = mR^2 \omega_0 \omega \sin 2\theta$ ; $\tau$ is maximum for $\theta = 45°$

(b) $F_{\perp} = 2mR\omega_0 \omega\cos\theta$ ; $\theta = 0°$

**140.** $\dfrac{3}{4}$ rad $\simeq 43°$

**141.** (a) $v = \sqrt{\dfrac{2\sqrt{2}\left(\sqrt{2}-1\right)}{3}gR}$

(b) $\omega = \dfrac{1}{3}\sqrt{\dfrac{2\sqrt{2}\left(\sqrt{2}-1\right)}{3}\dfrac{g}{R}}$

**142.** $T_1 = T_2 = T_3 = \dfrac{Mg}{3}$

**143.** 1.5 *cm*

**144.** (i) $\dfrac{L}{\sqrt{2}}$

(ii) $F_0 = \mu Mg\,(\sqrt{2}-1)$

**145.** (a) $\dfrac{v}{2}$

(b) $\dfrac{v}{2}$

**146.** (b) $mg$

(c) zero

**147.** $v = \sqrt{g\left(a - \dfrac{b}{2}\right)}$

**148.** (b) $\sqrt{\dfrac{gL}{6}}$

**149.** (b) $\omega_0 = \dfrac{\sqrt{3}Ja}{I}$

(c) $\omega = \sqrt{\dfrac{2Mga}{I}\left(1 - \cos\left(30° - \theta\right)\right)}$

(d) $K = K_0 + Mga\sin\theta$

**150.** (a) $\dfrac{\lambda g\left(2\ell + \pi R - L\right)}{\left(\lambda L + \dfrac{M}{2}\right)}$

(b) $\dfrac{2\lambda^2 Rg\left(2\ell + \pi R - L\right)}{\left(\lambda L + \dfrac{M}{2}\right)}$

**151.** $2\sqrt{\dfrac{mgh}{2m+M}}$

**152.** $\dfrac{13}{10}Mv_0^2$

**153.** $\dfrac{mg}{4M+m}$

**154.** $\dfrac{g}{2}$

**155.** (a) $\omega = \sqrt{\dfrac{g}{L}}$

(b) $h_B = 5.04L$

**156.** (i) $\omega_{n+1} = \dfrac{6v}{13r} + \dfrac{7}{13}\omega_n$

(ii) $\omega_n = \dfrac{v}{r}\left[1 - \left(\dfrac{7}{13}\right)^{n-1}\right]$

(iii) $\dfrac{v}{r}$

**157.** (a) $\dfrac{v_0^2}{5L}$

(b) $\dfrac{3}{2}\dfrac{v_0}{L}$

**158.** $\dfrac{M}{2}\sqrt{\dfrac{\pi Lg}{3}}$

**159.** $\cos^{-1}\left\{\dfrac{3}{\sqrt{10}}\left(1 - \dfrac{v^2}{5g\ell}\right)\right\} + \cos^{-1}\left(\dfrac{3}{\sqrt{10}}\right)$

# 07 GRAVITATION

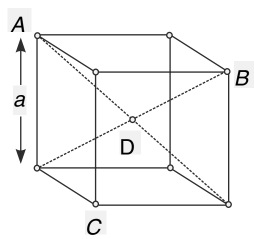
## Level 1

Q.1. Two lead balls of mass $m$ and 2 $m$ are placed at a separation $d$. A third ball of mass $m$ is placed at an unknown location on the line joining the first two balls such that the net gravitational force experienced by the first ball is $\frac{6Gm^2}{d^2}$. What is the location of the third ball?

Q.2. Four identical point masses m each are kept at the vertices $A$, $B$, and $C$ of a cube having side length 'a' (see figure). Another identical mass is placed at the center point $D$ of the cube.

(a) Where will you place a fifth identical mass so that the net gravitational force acting on mass at $D$ becomes zero?

(b) Calculate the net gravitational force acting on the mass at $D$.




Q.3. Two point masses $m$ and $M$ are held at rest at a large distance from each other. When released, they begin moving under their mutual gravitational pull.

(a) Find their relative acceleration ($a$) when separation between them becomes $x$.

(b) Integrate the expression of $a$ obtained above to calculate the relative velocity of the two masses when their separation is $x$.

(c) Write the velocity of centre of mass of the system when separation between them is $x$.

Q.4. Find the height above the surface of the earth where the acceleration due to gravity reduces by (a) 36% (b) 0.36% of its value on the surface of the earth. Radius of the earth $R = 6400\ km$.

Q.5. An astronaut landed on a planet and found that his weight at the pole of the planet was one third of his weight at the pole of the earth. He also found himself to be weightless at the equator of the planet. The planet is a homogeneous sphere of radius half that of the earth. Find the duration of a day on the planet. Given density of the earth = $d_0$.

Q.6. A gravity meter can detect change in acceleration due to gravity ($g$) of the order of $10^{-9}$ %. Calculate the smallest change in altitude near the surface of the earth that results in a detectable change in $g$. Radius of the earth $R = 6.4 \times 10^6\ m$.

Q.7. The earth is a homogeneous sphere of mass $M$ and radius $R$. There is another spherical planet of mass $M$ and radius $R$ whose density changes with distance r from the centre as $\rho = \rho_0 r$.

(a) Find the ratio of acceleration due to gravity on the surface of the earth and that on the surface of the planet.

(b) Find $\rho_0$.

Q.8. A planet having mass equal to that of the earth ($M = 6 \times 10^{24}\ kg$) has radius $R$ such that a particle projected from its surface at the speed of light ($c = 3 \times 10^8\ ms^{-1}$) just fails to escape.

Assuming Newton's Law of gravitation to be valid calculate the radius and mass density of such a planet. Are the numbers realistic?

Note: The radius that you calculated is known as Schwarzschild radius. Actually we need to use theory of general relativity for solving this problem.

Q.9. Angular speed of rotation of the earth is $\omega_0$. A train is running along the equator at a speed $v$ from west to east. A very sensitive balance inside the train shows the weight of an object as $W_1$. During the return journey when the train is running at same speed from east to west the balance shows the weight of the object to be $W_2$. Weight of the object when the train is at rest was shown to be $W_0$ by the balance. Calculate $W_2 - W_1$.

Q.10. If a planet rotates too fast, rocks from its surface will start flying off its surface. If density of a homogeneous planet is $\rho$ and material is not flying off its surface then show that its time period of rotation must be greater then $\sqrt{\frac{3\pi}{G\rho}}$.

Q.11. (a) The angular speed of rotation of the earth is $\omega = 7.27 \times 10^{-5}$ $rad\ s^{-1}$ and its radius is $R = 6.37 \times 10^6$ $m$. Calculate the acceleration of a man standing at a place at 40° latitude. [cos 40° = 0.77]

(b) If the earth suddenly stops rotating, the acceleration due to gravity on its surface will become $g_0 = 9.82$ $ms^{-2}$. Find the effective value of acceleration due to gravity ($g$) at 40° latitude taking into account the rotation of the earth.

Q.12. A planet has radius $\left(\frac{1}{36}\right)$ th of the radius of the earth. The escape velocity on the surface of the planet was found to be $\frac{1}{\sqrt{6}}$ times the escape velocity from the surface of the earth. The planet is surrounded by a thin layer of atmosphere having thickness $h$ (<< radius of the planet). The average density of the atmosphere on the planet is $d$ and acceleration due to gravity on the surface of the earth is $g_e$. Find the value of atmospheric pressure on the surface of the planet.

Q.13. Using a telescope for several nights, you found a celestial body at a distance of $2 \times 10^{11}$ $m$ from the sun travelling at a speed of 60 $km\ s^{-1}$. Knowing that mass of the sun is $2 \times 10^{30}$ $kg$, calculate after how many years you expect to see the body again at the same location.

Q.14. A man can jump up to a height of $h_0 = 1$ $m$ on the surface of the earth. What should be the radius of a spherical planet so that the man makes a jump on its surface and escapes out of its gravity? Assume that the man jumps with same speed as on earth and the density of planet is same as that of earth. Take escape speed on the surface of the earth to be 11.2 $km/s$ and radius of earth to be 6400 $km$.

Q.15. It is known that if the length of the day were $T_0$ hour, a man standing on the equator of the earth would have felt weightlessness. Assume that a person is located inside a deep hole at the equator at a distance of $\frac{R}{2}$ from the centre of the earth. What should be the time period of rotation of the earth for such a person to feel weightlessness? [$R$ = radius of the earth]

Q.16. A small satellite of mass $m$ is going around a planet in a circular orbit of radius $r$. Write the kinetic energy of the satellite if its angular momentum about the centre of the planet is $J$.

Q.17. Suppose that the gravitational attraction between a star of mass $M$ and a planet of mass $m$ is given by the expression $F = K\frac{Mm}{r^n}$ where $K$ and $n$ are constants. If the orbital speed of the planets were found to be independent of their distance ($r$) from the star, calculate the time period ($T_0$) of a planet going around the star in a circular orbit of radius $r_0$.

Q.18. A near surface earth's satellite is rotating in equatorial plane from west to east. The satellite is exactly above a town at 6:00 A.M today. Exactly how many times will it cross over the town by 6:00 A.M tomorrow. [Don't count its appearance today at 6:00 A.M above the town].

Q.19. Imagine an astronaut inside a satellite going around the earth in a circular orbit at a speed of $\sqrt{\frac{gR}{2}}$ where $R$ is radius of the earth and $g$ is acceleration due to gravity on the surface of the earth.

(a) What is weight experienced by the astronaut inside the satellite?

(b) Assume that an alien demon stops the satellite and holds it at rest. What is weight experienced by the astronaut now?

(c) The demon now releases the satellite (from rest). What is weight experienced by the astronaut now?

Q.20. The height of geostationary orbit above the surface of the earth is $h$. Radius of the earth is $R$. The earth shrinks to half its present radius (mass remaining unchanged). Now what will be height of a geostationary satellite above the surface of the earth?

Q.21. (a) Estimate the average orbital speed of the earth going around the sun. The average Earth-sun distance is $1.5 \times 10^{11}$ *m*.

(b) An asteroid going around the sun has an average orbital speed of 15 *km/s*. Is the asteroid farther from the sun or closer to the sun as compared to the Earth? Explain your answer.

Q.22. Assume that the earth is not rotating about its axis and that Scientists have developed an engine which can propel vehicles to very high speed on the surface of the earth. What is the maximum possible speed for any such vehicle running on surface of the earth. Earth is a sphere of radius $R$ = 6400 *km* and acceleration due to gravity on the surface is $g = 10\ m/s^2$.

Q.23. A satellite of Earth is going around in an elliptical orbit. The smallest distance of the satellite from the centre of the earth happens to be $2R$ (where $R$ = radius of the earth). Find the upper limit of the maximum speed of such a satellite.

Q.24. Haley's Comet is going around the Sun in a highly elliptical orbit with a period of 76 *y*. It was closest to the sun in the year 1987 (I was 13 year old then and heard a lot about it on radio). In which year of 21st century do you expect it to have least kinetic energy?

Q.25. A planet goes around the sun in an elliptical orbit. The minimum distance of the planet from the Sun is $2 \times 10^{12}$ *m* and the maximum speed of the planet in its path is 40 $km\ s^{-1}$. Find the rate at which its position vector relative to the sun sweeps area, when the planet is at a distance $2.2 \times 10^{12}$ *m* from the sun.

Q.26. To launch a satellite at a height $h$ above the surface of the earth (radius $R$) a two stage rocket is used . The first stage is used to lift the satellite to the desired height and the second stage is used to impart it a tangential velocity so as to put it in a circular orbital. Assume (incorrectly) that the mass of rocket is negligible and that there is no atmospheric resistance.

(a) If $E_1$ and $E_2$ are the energies delivered by the first and the second stage of the rocket.

Calculate the ratio $\frac{E_1}{E_2}$.

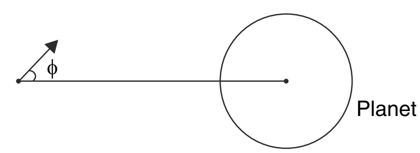
(b) Calculate the time period of the satellite if it is given that $\frac{E_1}{E_2} = 1$. Take mass of the earth to be $M$.

Q.27. A satellite of mass $m$ is going around the earth in a circular orbital at a height $\frac{R}{2}$ from the surface of the earth. The satellite has lived its life and a rocket, on board, is fired to make it leave the gravity of the earth. The rocket remains active for a very small interval of time and imparts an impulse in the direction of motion of the satellite. Neglect any change in mass due to firing of the rocket.

(a) Find the minimum impulse imparted by the rocket to the satellite.

(b) Find the minimum work done by the rocket engine. Mass of the earth = $M$, Radius of the earth = $R$

Q.28. A small asteroid is at a large distance from a planet and its velocity makes an angle $\phi\ (\neq 0)$ with line joining the asteroid to the centre of the planet. Prove that such an asteroid can never fall normally on the surface of the planet.




## LEVEL 2

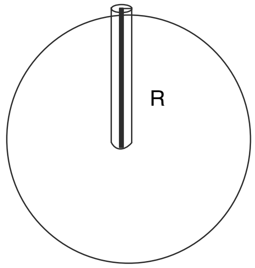
Q.29. Three identical particles, each of mass $m$, are located in space at the vertices of an equilateral triangle of side length $a$. They are revolving in a circular orbital under mutual gravitational attraction.

(a) Find the speed of each particle.

(b) Find the acceleration of the centre of mass of a system comprising of any two particles.

(c) Assume that one of the particles suddenly loses its ability to exert gravitational force. Find the velocity of the centre of mass of the system of other two particles after this.

Q.30. Imagine a hole drilled along the radius of the earth. A uniform rod of length equal to the radius ($R$) of the earth is inserted into this hole. Find the distance of centre of gravity of the rod from the centre .

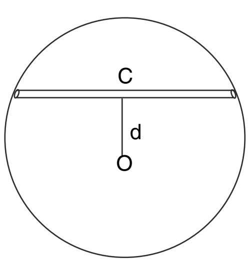



Q.31. A large non rotating star of mass $M$ and radius $R$ begins to collapse under its own gravity and ultimately becomes very small (nearly a point mass). Assume that the density remains uniform inside the sphere in any stage. Plot the variation of gravitational field intensity (well, you can call it acceleration due to gravity) at a distance $\frac{R}{2}$ from the centre vs the radius ($r$) of the star.

Q.32. At a depth $h_1 = \frac{R}{2}$ from the surface of the earth acceleration due to gravity is $g_1$. It's value changes by $\Delta g_1$ when one moves down further by 1 km. At a height $h_2$ above the surface of the earth acceleration due to gravity is $g_2$. It's value changes by $\Delta g_2$ when one moves up further by 1 $km$. If $\Delta g_1 = \Delta g_2$ find $h_2$. Assume the earth to be a uniform sphere of radius $R$.

Q.33. Due to rotation of the earth the direction of vertical at a place is not along the radius of the earth and actually makes a small angle $\phi$ with the true vertical (i.e. with radius). At what latitude ($\theta$) is this angle $\phi$ maximum ?

Q.34. A tunnel is dug along a chord of non rotating earth at a distance $d = \frac{R}{2}$ [$R$ = radius of the earth] from its centre. A small block is released in the tunnel from the surface of the earth. The block comes to rest at the centre (C) of the tunnel. Assume that the friction coefficient between the block and the tunnel wall remains constant at $\mu$.




(a) Calculate work done by the friction on the block.

(b) Calculate $\mu$.

Q.35. Diameter of a planet is $10d_\circ$, its mean density is $\frac{\rho_\circ}{4}$ and mass of its atmosphere is $10m_\circ$ where $d_\circ$, $\rho_\circ$ and $m_\circ$ are diameter, mean density and mass of atmosphere respectively for the earth. Assume that mean density of atmosphere is same on the planet and the earth and height of atmosphere on both the planets is very small compared to their radius.

(a) Find the ratio of atmospheric pressure on the surface of the planet to that on the earth.

(b) If a mercury barometer reads 76 $cm$ on the surface of the earth, find its reading on the surface of the planet.

Q.36. A particle of mass $m$ is projected upwards from the surface of the earth with a velocity equal to half the escape velocity. ($R$ is radius of earth and $M$ is mass of earth)

(a) Calculate the potential energy of the particle at its maximum.

(b) Write the kinetic energy of the particle when it was at half the maximum height.

Q.37. A uniform spherical planet is rotating about its axis. The speed of a point on its equator is $v$ and the effective acceleration due to gravity on the equator is one third its value at the poles. Calculate the escape velocity for a particle at the pole of the planet. Give your answer in multiple of $v$.

Q.38. A planet is a homogeneous ball of radius $R$ having mass $M$. It is surrounded by a dense atmosphere having density $\rho = \frac{\sigma_\circ}{r}$ where $\sigma_\circ$ is a constant and $r$ is distance from the centre of the planet. It is found that acceleration due to gravity is constant throughout the atmosphere of the planet. Find $\sigma_\circ$ in terms of $M$ and $R$.

Q.39. A projectile is to be launched from the surface of the earth so as to escape the solar system. Consider the gravitational force on the projectile due to the earth and the sun only. The projectile is projected perpendicular to the radius vector of the earth relative to the centre of the sun in the direction of motion of the earth. Find the minimum speed of projection relative to the earth so that the projectile escapes out of the solar system. Neglect rotation of the earth.

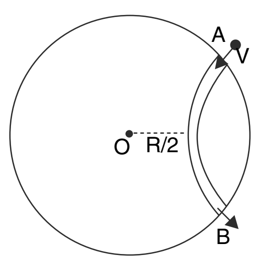
Mass of the sun $M_s = 2 \times 10^{30}\ kg$; Mass of the earth $M_e = 6 \times 10^{24}\ kg$

Radius of the earth $R_e = 6.4 \times 10^6\ m$; Earth-Sun distance $r = 1.5 \times 10^{11}\ m$

Q.40. Assume that there is a tunnel in the shape of a circular arc through the earth. Wall of the tunnel is smooth. A ball of mass $m$ is projected into the tunnel at $A$ with speed $v$. The ball comes out of the tunnel at $B$ and escapes out of the gravity of the earth. Mass and radius of the earth are $M$ and $R$ respectively and radius of the circle shaped tunnel is also $R$.

(a) Find minimum possible value of $v$ (call it $v_0$)

(b) If the ball is projected into the tunnel with speed $v_0$, calculate the normal force applied by the tunnel wall on the ball when it is closest to the centre of the earth. It is given that the closest distance between the ball and the centre of the earth is $\frac{R}{2}$.




Q.41. A celestial body, not bound to sun, will only pass by the sun once. Calculate the minimum speed of such a body when it is at a distance of $1.5 \times 10^{11}\ m$ from the sun (this is average distance between the sun & the earth and is known as astronomical unit- $A.U.$)

The mass of the sun is $M \simeq 2 \times 10^{30}\ kg$.

$$G = 6.67 \times 10^{-11}\ \frac{N-m^2}{kg^2}$$

Show that this speed is $\sqrt{2}$ times greater than speed of earth around the sun, assuming circular trajectory

Q.42. A body is projected vertically upward from the surface of the earth with escape velocity. Calculate the time in which it will be at a height (measured from the surface of the earth) 8 time the radius of the earth ($R$). Acceleration due to gravity on the surface of the earth is $g$.

Q.43. An astronaut on the surface of the moon throws a piece of lunar rock (mass $m$) directly towards the earth at a great speed such that the rock reaches the earth.

Mass of the earth = $M$, Mass of the moon $= \frac{M}{81}$

Radius of the earth = $R$, Distance between the centre of the earth and the moon = $60R$

(a) In the course of its journey calculate the maximum gravitational potential energy of the rock

(b) Find the minimum possible speed of the rock when it enters the atmosphere of the earth.

Q.44. The radius of the circular path of a geostationary satellite was inadvertently made $\Delta r = 1\ km$ larger than the correct radius $r = 42000\ km$

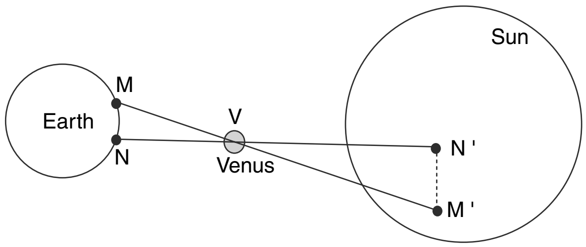
(a) Calculate the difference in angular speed of the satellite and the earth.

(b) If the satellite was exactly above a house on the equator on a particular day, what will be angular separation between the house and the satellite a year later?

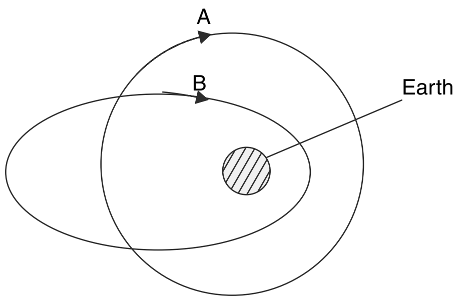
Q.45. A spy satellite $S_1$, travelling above the equator is taking pictures at quick intervals. The satellite is travelling from west to east and is ready with picture around the whole equator in 8 hours. Another similar satellite $S_2$, travelling in the same plane is travelling from east to west and is able to take pictures around the whole equator in 6 hours. Find the ratio of radii of the circular paths of the satellite $S_1$ and $S_2$.

Q.46. A comet is going around the sun in an elliptical orbit with a period of 64 year. The closest approach of the comet to the sun is 0.8 $AU$ [$AU$ = astronomical unit]. Calculate the greatest distance of the comet form the sun.

Q.47. The astronomical phenomenon when the planet Venus passes directly between the Sun and the earth is known as Venus transit. For two separate persons standing on the earth at points $M$ and $N$, the Venus appears as black dots at points $M'$ and $N'$ on the Sun. The orbital period of Venus is close to 220 days. Assuming that both earth and Venus revolve on circular paths and taking distance $MN$ = 1000 $km$, calculate the distance $M'N'$ on the surface of the Sun. [Take $(2.75)^{1/3} = 1.4$]



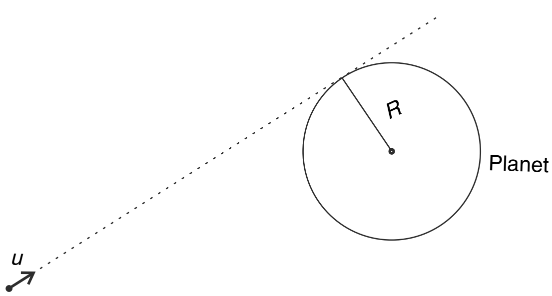

Q.48. Satellite $A$ is following a circular path of radius $a$ around the earth another satellite $B$ follows an elliptical path around the earth. The two satellites have same mechanical energy and their orbits intersect. Find the speed of satellite $B$ at the point where its path intersects with the circular orbit of $A$. Take mass of earth to be $M$.




Q.49. A satellite of mass $m$ is orbiting around the earth (mass $M$, radius $R$) in a circular orbital of radius $4R$. It starts losing energy slowly at a constant rate $-\frac{dE}{dt} = \eta$ due to friction. Find the time ($t$) in which the satellite will spiral down to the surface of the earth.

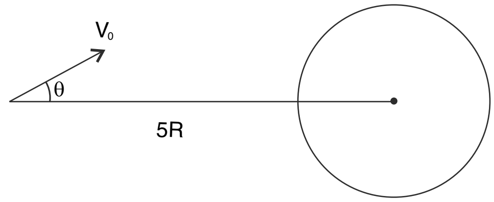
Q.50. Energy of a satellite going around the earth in an elliptical orbit is given by $-\frac{GMm}{2a}$ where $M$ and $m$ are masses of the earth and the satellite respectively and $2a$ is the major axis of the elliptical path. A satellite is launched tangentially with a speed $= \sqrt{\frac{3GM}{5R}}$ from a height $h = R$ above the surface of the earth. Calculate its maximum distance from the centre of the earth

Q.51.




A small asteroid is approaching a planet of mass $M$ and radius $R$ from a large distance. Initially its velocity ($u$) is along a tangent to the surface of the planet. It fall on the surface making an angle of 30° with the vertical. Calculate $u$.

Q.52. An asteroid was fast approaching the earth. Scientists fired a rocket which hit the asteroid at a distance of 5 $R$ from the centre of the earth ($R$ = radius of the earth). Immediately after the hit the asteroid's velocity ($V_0$) was making an angle of $\theta = 30°$ with the line joining the centre of the earth to the asteroid. The asteroid just grazed past the surface of the earth. Find $V_0$ [Mass of the earth = $M$]




Q.53. A satellite is orbiting around the earth in a circular orbit. Its orbital speed is $V_0$. A rocket on board is fired from the satellite which imparts a thrust to the satellite directed radially away from the centre of the earth. The duration of the engine burn is negligible so that it can be considered instantaneous. Due to this thrust a velocity variation $\Delta V$ is imparted to the satellite. Find the minimum value of the ratio $\frac{\Delta V}{V_0}$ for which the satellite will escape out of the gravitational field of the earth.

Q.54. In last question assume that circular orbit of the satellite has radius $r_0$. Find $\frac{\Delta V}{V_0}$ for which the maximum distance of the satellite from the centre of the earth become $2r_0$ after the rocket is fired.

Q.55. A satellite is at a distance $r_1$ from the centre of the earth at its apogee. The distance is $r_2$ when it is at perigee. Mass of the earth is $M$.

(a) Calculate the maximum speed of the satellite in its orbit around the earth.

(b) Estimate the maximum speed of the moon going around the earth. For moon

$r_1 \simeq 400,000\ km$ and $r_2 \simeq 360,000\ km$

mass of the earth $M = 6 \times 10^{24}\ kg$

Q.56. A satellite is going around the earth in an elliptical orbit and has maximum and minimum distance from the centre of earth equal to $10r$ and $r$ respectively. It was planned to fire on board rocket so as to increase the energy of the satellite by maximum amount. Assume that the rocket is fired for a small time (almost instantaneous) and gives an impulse $J$ to the satellite in forward direction. Take $J$ to be small compared to overall momentum of the satellite.

(a) Show that firing the rocket when the satellite is at perigee (nearest to earth) will result in maximum gain in energy of the satellite.

[The orbit of mass orbiter mission, informally called Mangalyaan, was raised in five steps using this principle; before it was given the escape speed]

(b) Find the impulse $J$ that the rocket must impart to the satellite at perigee so that its maximum distance from earth's centre, during its course of motion in elliptical path, becomes $12r$. Take mass of satellite as m and mass of earth as $M$. Assume that there is negligible change in mass of the satellite due to firing of the rocket.

Q.58. Imagine a smooth tunnel along a chord of non-rotating earth at a distance $\frac{R}{2}$ from the centre. $R$ is the radius of the earth. A projectile is fired along the tunnel from the centre of the tunnel at a speed $V_o = \sqrt{gR}$ [$g$ is acceleration due to gravity at the surface of the earth].

(a) Is the angular momentum [about the centre of the earth] of the projectile conserved as it moves along the tunnel?

(b) Calculate the maximum distance of the projectile from the centre of the earth during its course of motion.

Q.59. A geostationary satellite is nearly at a height of $h = 6\,R$ from the surface of the earth where $R$ is the radius of the earth. Calculate the area on the surface of the earth in which the communication can be made using this satellite.

## LEVEL 3

Q.60. (a) There is an infinite thin flat sheet with mass density $\sigma$ per unit area. Find the gravitational force, due to sheet, on a point mass $m$ located at a distance $x$ from the sheet.

(b) Consider a large flat horizontal sheet of material density $\rho$ and thickness $t$, placed on the surface of the earth. The density of the earth is $\rho_0$. If it is found that gravitational field intensity just between the sheet is larger than field just above it, prove that $\rho_o > \frac{3}{2}\rho$. Assume $t << R$

Q.61. A spaceship is orbiting the earth in a circular orbit at a height equal to radius of the earth ($R_c = 6400$ $km$) from the surface of the earth. An astronaut is on a space walk outside the spaceship. He is at a distance of $l = 200$ $m$ from the ship and is connected to it with a simple cable which can sustain a maximum tension of10 $N$. Assume that the centre of the earth, the spaceship and the astronaut are in a line. Mass of astronaut along with all his accessories is 100 $kg$.

(a) Do you think that a weak cable that can only take a load of 10 $N$, can prevent him from drifting in space ? Make a guess.

(b) Estimate the tension in the cable.
[Acceleration due to gravity on the surface of Earth = 9.8 $m/s^2$]

Q.62. Earth is rotating about its axis with angular speed $\omega_0$ and average density of earth is $\rho$. It is proposed to make a space elevator by placing a long rod with uniform mass density extending from just above the surface for the earth out to a radius $nR$ ($R$ is radius of the earth). Prove that the rod can remain above the same point on the equator all time if, $n^2 + n = \frac{8\pi G\rho}{3\omega^2}$, where $\rho$ is density of the earth

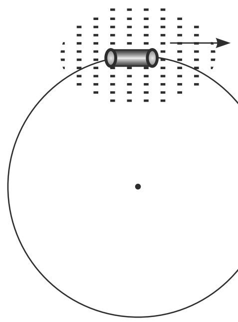
Q.63. A body is projected up from the surface of the earth with a velocity half the escape velocity at an angle of 30° with the horizontal. Neglecting air resistance and earth's rotation, find

(a) the maximum height above the earth's surface to which the body will rise.

(b) will the body move around the earth as a satellite?

Q.64. A near surface earth satellite has cylindrical shape with cross sectional area of $S = 0.5$ $m^2$ and mass of $M = 10$ $kg$. It encounters dust which has density of $d = 1.6 \times 10^{-11}$ $kg/m^3$. Assume that the dust particles are at rest and they stick to the satellite's front face on collision. Take mean density of earth to be $\rho = 5500$ $kg/m^3$

(a) Find the drag force experienced by the satellite



(b) If the dust extends throughout the orbit, find the change in velocity and radius of the circular path of the satellite in one revolution.

# ANSWERS

**1.** Exactly midway between the two balls *OR* at a distance of $\frac{d}{2\sqrt{2}}$ from the ball of mass *m*.

**2.** (a) At the diagonally opposite corner of C on the floor of the cube

(b) $\frac{4Gm^2}{3a^2}$

**3.** (a) $\frac{G(M+m)}{x^2}$

(b) $\sqrt{\frac{2G(M+m)}{x}}$

(c) zero

**4.** (a) 1600 *km*

(b) 11.52 *km*

**5.** $T = \sqrt{\frac{9\pi}{2Gd_0}}$

**6.** 32 $\mu m$

**7.** (a) 1

(b) $\frac{M}{\pi R^4}$

**8.** $R = 8.9\ mm$; $d = 2 \times 10^{30}\ kg\ m^{-3}$

**9.** $\frac{4W_0\omega_0 v}{g}$

**11.** (a) 0.026 $ms^{-2}$
(b) 9.80 $ms^{-2}$

**12.** 6 *dgh*

**13.** The body will never return to the same location.

**14.** 2.5 *km*

**15.** $T_0$

**16.** $\frac{J^2}{2mr^2}$

**17.** $T_0 = \frac{2\pi r_0}{\sqrt{KM}}$

**18.** 16

**19.** (a) Zero

(b) $\frac{mg}{4}$

(c) zero

**20.** $h + \frac{R}{2}$

**21.** (a) 30 *km/s*
(b) Farther

**22.** $V_0 = \sqrt{gR} \simeq 7.9\ km/s$

**23.** $\sqrt{gR} \simeq 7.9\, km/s$

**24.** 2025

**25.** $4 \times 10^{16}\ m^2 s^{-1}$

**26.** (a) $\frac{2h}{R}$

(b) $\sqrt{\frac{27\pi^2 R^3}{2GM}}$

**27.** (a) $(\sqrt{2}-1)m\sqrt{\frac{2GM}{3R}}$

(b) $\frac{GMm}{3R}$

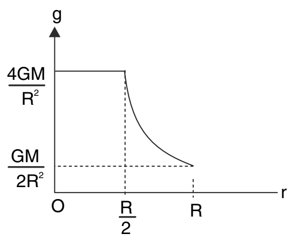
**29.** (a) $\sqrt{\frac{Gm}{a}}$

(b) $\frac{\sqrt{3}}{2}\frac{Gm}{a^2}$

(c) $\frac{1}{2}\sqrt{\frac{Gm}{a}}$

**30.** $\frac{2R}{3}$

**31.**




**32.** $h_2 = R\left(2^{\frac{1}{3}} - 1\right)$

**33.** 45°

**34.** (a) $-\frac{3GMm}{8R}$

(b) $\frac{\sqrt{3}}{2}$

**35.** (a) $\frac{5}{2}$

(b) 7.6 *cm*

**36.** (a) $\frac{-3GMm}{4R}$

(b) $\frac{3GMm}{28R}$

**37.** $\sqrt{3}v$

**38.** $\sigma_{\circ} = \frac{M}{2\pi R^2}$

**39.** 13.6 *km/s*

**40.** (a) $v_0 = \sqrt{\frac{2GM}{R}}$

(b) $\frac{27}{4}\frac{GMm}{R^2}$

**41.** $4.2 \times 10^4$ *m/s*

**42.** $t = \frac{52}{3}\sqrt{\frac{R}{2g}}$

**43.** (a) $-\frac{GMm}{243R}$

(b) $\frac{14045}{14337}\frac{GMm}{R}$

**44.** (a) $9.3 \times 10^{-6}$ *rad/hr*

(b) 4.6°

**45.** $\frac{r_1}{r_2} = \left(\frac{3}{4}\right)^{2/3}$

**46.** 31.2 *AU*

**47.** 2500 *km*

**48.** $\sqrt{\frac{GM}{a}}$

**49.** $t = \frac{3GMm}{8\eta R}$

**50.** 2*R*

**51.** $u = \sqrt{\frac{2GM}{3R}}$

**52.** $V_0 = \sqrt{\frac{32}{105}\frac{GM}{R}}$

**53.** (1)

**54.** (1/2)

**55.** (a) $\sqrt{\frac{2GM}{r_2}\left(1 + \frac{1}{r_2/r_1}\right)}$

(b) 1.08 *km/s*

**56.** (b) $J = 0.01m\sqrt{\frac{GM}{r}}$

**58.** (a) NO

(b) $r_{\max} = \left(\frac{8+\sqrt{57}}{14}\right)R$

**59.** $\frac{12}{7}\pi R^2$

**60.** (a) $G2\pi\sigma m$

**61.** (b) 0.01 *N*

**63.** (a) $\left(\frac{\sqrt{7}-2}{6}\right)R$

(b) No

**64.** **(a)** $5 \times 10^{-4}\,N$

(b) $\Delta V = 0.25$ *m/s*; $\Delta R = -0.4$ *km*

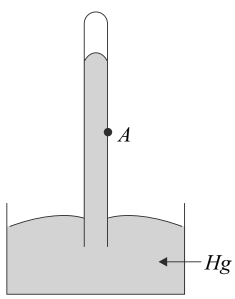
# 08 FLUIDS

## Level 1

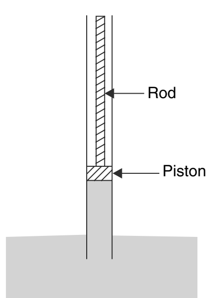
Q. 1. We know that the atmospheric pressure on the surface of the earth is because of weight of the air. The radius of the earth is 6400 *km* and atmospheric pressure on the surface of earth is $1 \times 10^5\ N/m^2$.Estimate the mass of the earth's atmosphere assuming that acceleration due to gravity remains constant at 10 $m/s^2$ over the entire height of the atmosphere.

Q. 2. Why mercury is used in a barometer, though it is costly? Why cannot we use water in place of mercury.

Q. 3 Look at the barometer shown in the figure. If a small hole is developed in the wall of the tube at point *A*, will the mercury leak out of it?




Q. 4.




A tightly fitted piston can slide along the inner wall of a long cylindrical pipe. With the piston at the lower end of the pipe, the lower end of the pipe is dipped into a large tank, filled with water. Now the piston is pulled up with the help of the rod attached to it. Water rises in the pipe along with the piston. Why? To what maximum height water can be raised in the pipe using this method? What will be the answer to your question if water is replaced with mercury? Atmospheric pressure is $P_{atm} = 1.01 \times 10^5\ Pa$.

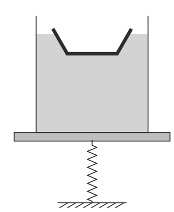
Q. 5. A hypothetical planet has an ocean of water which is 50 *km* deep. The top 5 *km* is frozen as ice (i.e, 45 *km* is water). Radius and average density of the planet are both half the respective values for the earth. There is no atmosphere. Obtain an estimate of the pressure at the bottom of the ocean.

Q. 6. Two identical beakers are filled with water. One of them has an ice block floating in it. The level of water in both the beakers is same. Which beaker will weigh more? Will your answer change if water is replaced with a liquid of higher density in the beakers?

Q. 7. (i) A toy boat made of steel is floating in a beaker having water. The beaker is placed on a spring balance. The boat tilts and sinks into water.

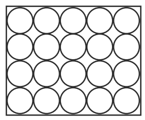
(a) Will the level of water in the beaker go up or fall down?

(b) Will the reading of spring balance decrease or increase?

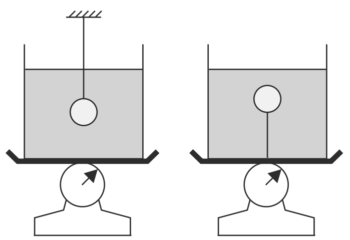


(ii) You are in a boat on a calm lake. There is a floating log near you. You pick the log and put it into the boat. What happens to the level of water in the lake? Does it rise or fall?

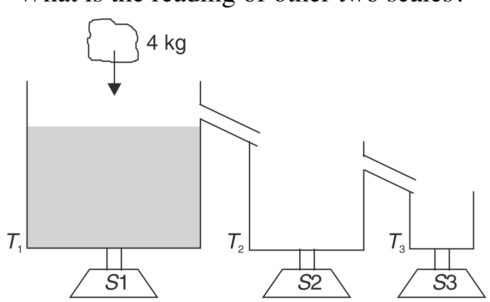
Q. 8. A closed cubical box of negligible mass has large number of spherical balls arranged neatly inside it as shown in the figure. When placed in water, the box floats with 80% of its volume remaining submerged. What is specific gravity of material of the balls? Neglect thickness of the wall of the box.



Q. 9. Two identical containers have the same volume of water in it. Each of them is placed on a balance and readings of the two balances are same. There is a hollow ball and a solid ball that have same volume. The hollow ball floats in water and the solid ball sinks. A string from the ceiling suspends the solid ball so that it remains completely submerged in the water in the first container. The hollow ball is held submerged in the water in the second container and is held by a string fastened to the bottom of the container. Which balance will show higher reading? How will your answer change if the string in the second container is cut?



Q. 10. Three tanks $T_1$, $T_2$ and $T_3$ are sitting on three weighing scales $S_1$, $S_2$ and $S_3$ respectively. Tank $T_1$ has a spout, as shown and water has been filled in it to a level just below the spout. The other two tanks are empty. Reading of the three scales are 20 *kg*, 4 *kg* and 3 *kg* respectively. A 4 *kg* body is put into the tank $T_1$ and it floats in the water. Now the reading of scale $S_3$ was found to be 4.5 *kg*. What is the reading of other two scales?

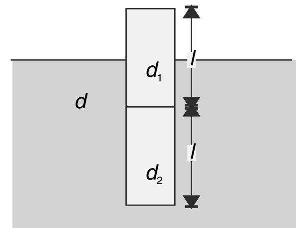



Assume that the water in the system remains inside three tanks.

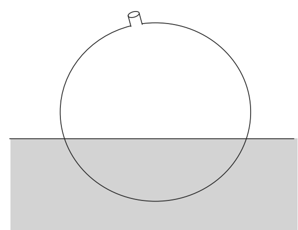
Q. 11. A cylindrical block of length $2l$ is made of two different materials. The upper half has density $d_1$ and lower half, which is heavier, has density $d_2$. The block is floating in a liquid of unknown density $d$ with $\frac{l}{2}$ of its length outside the liquid.

(a) Find $d$

(b) Show that $d > \frac{4d_1}{3}$

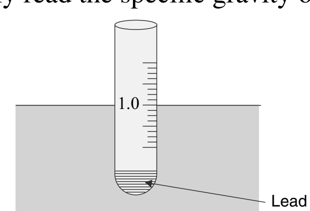



Q. 12. A sealed balloon, filled with air, floats in water with $\frac{1}{3}$ of its volume submerged. It was found that if it is pushed inside water at a depth $h$, it remains in equilibrium, neither sinking nor rising. Find $h$. Given that height of water barometer is 10 *m* and temperature is constant at all depth.



Q. 13. Long back our Earth was made of molten material. Assume it to be a uniform sphere of radius $R$ having density $d$. Take acceleration due to gravity at the surface to be $g$ and calculate the gauge pressure ($P_0$) at the centre of this fluid Earth. Calculate $P_0$ for following data: $R = 6000\ km$; $d = 5500\ kg\ m^{-3}$ and $g = 10\ ms^{-2}$.

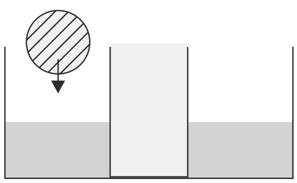
Q. 14. A device used to measure the specific gravity of a liquid is called a hydrometer. In a simple hydrometer there is a cylindrical glass tube with some lead – weight at its bottom. The device floats in liquid while remaining vertical. The top part of the tube extends above the liquid and the divisions marked on the tube allows one to directly read the specific gravity of the liquid.




The scale on the tube is calibrated such that in pure water it reads 1.0 at the water surface and a length $z_0$ of the tube is submerged. Calculate the specific gravity of the liquid if the liquid level is $\Delta z$ above the 1.0 mark. Disregard the curvature of the tube bottom.

Q. 15. A sphere of radius $R$ and having negligible mass is floating in a large lake. An external agent slowly pushes the sphere so as to submerge it completely. How much work was done by the agent? Density of water is $\rho$.

Q. 16. Two identical communicating containers have water filled into them. A spherical ball of ice (relative density = 0.9 ) having volume 100 $cm^3$ is put into the left vessel. Calculate the volume of water flowing into the right container, immediately after placing the ball (i.e., don't consider any melting of the ice ball). Give your answer for following two cases



(i) The ice ball floats in the water in the left container.

(ii) The ice ball gets exactly half immersed in the water.

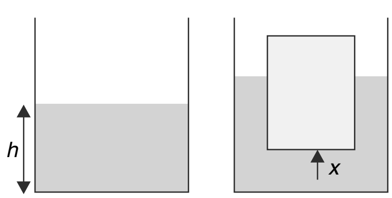
(iii) What will happen to the water level after the ice melts? Answer for both (i) and (ii) above.

Q. 17. An open cylindrical container has a cross sectional area $A_0 = 150\ cm^2$ and water has been filled in it up to a height $h$. A cylinder made of wood (relative density = 0.6) having cross sectional area $A = 125\ cm^2$ and length 10 $cm$ is now placed inside the container with its axis vertical. Find the distance ($x$) of the base of the wooden cylinder from the base of the container in equilibrium for following three cases :

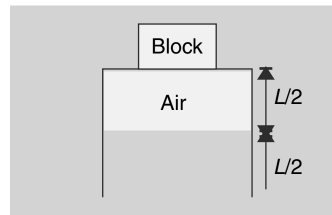
(a) $h = 8\ cm$

(b) $h = 12\ cm$

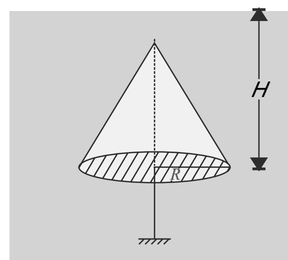
(c) $h = 0.8\ cm$




Q. 18. A light cylindrical tube of length $L = 1.5\ m$ and radius $r = \frac{1}{\sqrt{\pi}}\ m$ is open at one end. The tube containing air is inverted and pushed inside water as shown in figure. A block made of material of relative density 2 has been placed on the flat upper surface of the tube and the whole system is in equilibrium. Neglect the weight of air inside the tube and find the volume of block placed on the tube.



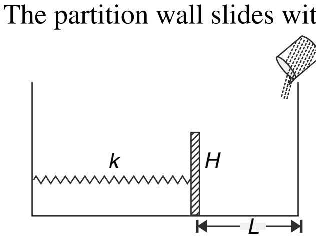

Q. 19. A solid wooden cone has been supported by a string inside water as shown in the figure. The radius of the circular base of the cone is $R$ and the volume of the cone is $v$. In equilibrium the base of the cone is at a depth $H$ below the water surface. Density of wood is $d$ ($< \rho$, density of water).




(a) Find tension in the string.

(b) Find the force applied by the water on the slant surface of the cone. Take atmospheric pressure to be $P_0$

Q. 20. A large container has a sliding vertical wall of height $H$ so as to divide it into two parts. The partition wall is connected to the left container wall by an ideal spring of force constant $k$. When the spring is relaxed the dimensions of the floor of the right part is $L \times b$. Now water (density $\rho$) is slowly poured into the right chamber. What is the maximum volume of water that can be stored in the right chamber without spilling it into the other part. The partition wall slides without friction.



Q. 21. A cylindrical container has cross sectional area of 0.20 $m^2$ and is open at the top. At the bottom, it has a small hole (A) kept closed by a cork. There is an air balloon tied to the bottom surface of the container. Volume of balloon is 2.2 litre. Now water is filled in the container and the balloon gets fully submerged. Volume of the balloon reduces to 2.0 litre. The cork is taken out to open the hole and at the same moment the whole container is dropped from a large height so as to fall under gravity. Assume that the container remains vertical. Find the change in level of water inside the falling container 2 second after it starts falling.

Q. 22. A wooden stick of length $L$, radius $R$ and density $\rho$ has a small metal piece of mass $m$ (of negligible volume) attached to its one end. Find the minimum value for the mass $m$ that would make the stick float vertically in equilibrium in a liquid of density $\sigma$.

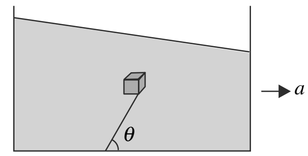
Q. 23. A rod of length 6 m has a mass 12 $kg$. It is hinged at one end at a distance of 3 m below water surface.

(a) What weight must be attached to the other end of the rod so that a length of rod equal to 5 m is submerged in water in equilibrium?

(b) Find the magnitude and direction of the force exerted by the hinge on the rod. (Specific gravity of rod is 0.5).

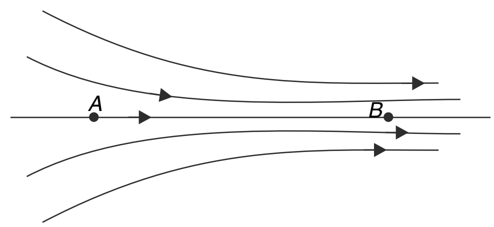
Q. 24. Assume that a car travelling on horizontal straight road with an acceleration of a 5 $ms^{-2}$ has all its windows rolled up and all air vents closed. Length of the car is $L = 3.0\, m$. By considering a horizontal tube of air that extends from the windshield to the rear surface, and applying Newton's Law on it, calculate the difference in pressure of air at the rear and front of the car. How does this pressure compare with the atmospheric pressure ? Density of air $\rho = 1.2\ kg\ m^{-3}$.

Q. 25. A container partially filled with water is moved horizontally with acceleration $a = \frac{g}{3}$. A small wooden block of mass $m$ is tied to the bottom of the container using a string. The block remains inside water with the string inclined at an angle $\theta$ to the horizontal. Assuming that the density of wood is half the density of water, find the angle $\theta$ and the tension in the string.

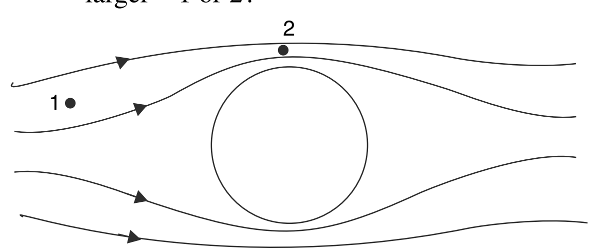



Q. 26. A cylindrical tank having radius $R$ is half filled with water having density $\rho$. There is a hole at the top of the tank. The tank is moved horizontally, perpendicular to its length, with a constant acceleration equal to the acceleration due to gravity ($g$). Find the maximum pressure exerted by water at any point on the tank. Atmospheric pressure is $P_0$. Assume that there is no spillage.

Q. 27. In a steady two dimensional flow of incompressible fluid streamlines are as shown in figure. At which point – $A$ or $B$ – the pressure is higher? Assume the flow to be in a horizontal plane.

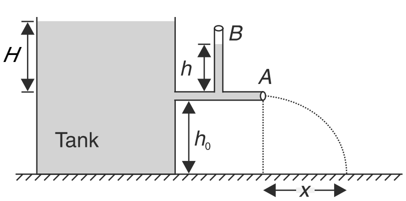



Q. 28. (i) A ball is projected in still air. With respect to the ball the streamlines appear as shown in the figure. At which point is the pressure larger – 1 or 2?




(ii) In the above figure if the ball is also spinning in clockwise sense, in which direction it will get deflected – up or down?

Q. 29. (i) In the arrangement shown in the figure, the tank has a large cross section and the pipes have much smaller cross sections. The opening at $A$ is unplugged and the water jet hits the ground surface at a horizontal distance $x$.



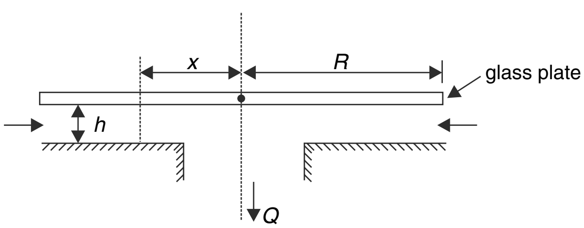
(a) Find the level of water ($h$) in the tube $B$ as water flows out of $A$.

(b) Find $x$.

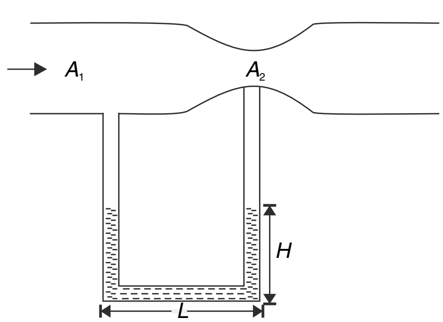
(ii) A flat horizontal surface has a small hole at its centre. A circular glass plate of radius $R$ is placed symmetrically above the hole with a small gap $h$ remaining between the plate and the surface. A liquid enters the gap symmetrically from all sides and after travelling radially through the gap finally exits from the hole. The volume flow rate of the liquid coming out from the hole is $Q$ (in $m^3s^{-1}$).

(a) If the flow speed just inside the circumference of the circular plate is $V_0$ find the speed ($V_x$) of flow inside the gap at a distance $x$ (see figure) from the centre of the hole.

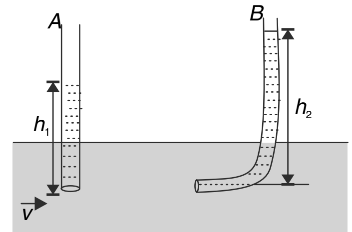
(b) Write $V_x$ in terms of $Q$, $h$ and $x$.




Q. 30. A horizontal tube having cross sectional area $A_1 = 10\ cm^2$ has a venturi connected to it having cross sectional area $A_2 = 4\ cm^2$. A manometer, having mercury as its liquid is connected to the tube as shown in the figure. The manometer tube has uniform cross section and it has a horizontal part of length $L = 10\ cm$. When there is no flow in the tube the height of mercury column in both vertical arms is $H = 12\ cm$. Calculate the minimum flow rate (in $m^3/s$) of air through the tube if it is required that the entire amount of mercury move to one vertical arm of the manometer. Given: density of $Hg = 13.6 \times 10^3\ kg\ m^{-3}$; density of air = $1.2\ kg\ m^{-3}$.



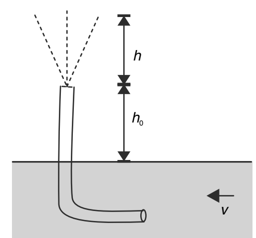

Q. 31. A liquid is flowing in a horizontal pipe of uniform cross section at a speed $v$. Two tubes $A$ an $B$ are inserted into the pipe as shown. Assume the flow to remain streamline inside the pipe.



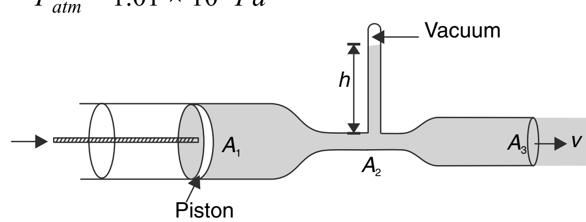

(a) The diagram depicts that height of liquid in tube $B$ ($= h_2$) is more than the height of liquid in tube $A$ ($= h_1$). Is it correct?

(b) Calculate the difference in height of the liquid in two tubes.

Q. 32. Water is flowing in a stream at speed $v$. A $L$ shaped tube is lowered into the stream as shown. The upper end of the tube is held at a height $h_0$ above the surface of the water. To what height '$h$' above the upper end of the tube, will the water jet spurt? Assume that flow remains ideal.




Q. 33. A horizontal glass tube is filled with mercury. The tube has three different cross sections as shown; with $A_1 = 18\ cm^2$, $A_2 = 8\ cm^2$ and $A_3 = 9\ cm^2$. The piston is pushed so as to throw out mercury at a constant speed of $v = 6\ m/s$ at the other end of the tube. Assume that mercury is an ideal fluid with density $\rho_{Hg} = 1.36 \times 10^4\ kg/m^3$. Take $g = 10\ m/s^2$. $P_{atm} = 1.01 \times 10^5\ Pa$



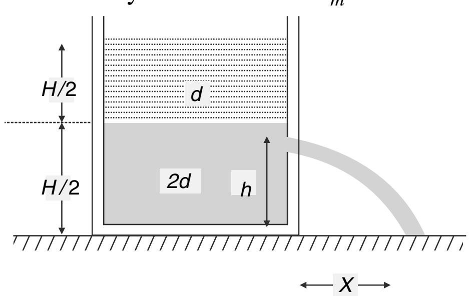

(i) Find the force needed to push the piston assuming that friction force between the piston and the tube wall is $f = 40\ N$

(ii) Find the height ($h$) of mercury column in the attached vertical tube. What happens to this height if the piston is pushed with smaller speed?

Q. 34. A container of large uniform cross-section area $A$ resting on a horizontal surface, holds two immiscible, non-viscous and incompressible liquids of densities $d$ and $2d$ each of height $(H/2)$ as shown in figure. The lower density liquid is open to the atmosphere having pressure $P_0$.

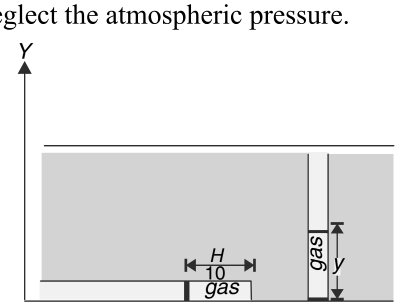
(a) A homogeneous solid cylinder of length $L$ ($L < H/2$), cross-sectional area $(A/5)$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $(L/4)$ in the denser liquid. Determine (i) The density of solid and (ii) The gauge pressure at the bottom of the container.

(b) The cylinder is removed and original arrangement is restored. A tiny hole of area $s(s << A)$ is punched on the vertical side of the container at a height $h(h < H/2)$. Determine (i) the initial speed of efflux of the liquid at the hole (ii) the horizontal distance $x$ travelled by the liquid initially and (iii) the height $h_m$ at which the hole should be punched so that the liquid travels the maximum distance $x_m$ initially. Also calculate $x_m$.




## LEVEL 2

Q. 35. A lake filled with water has depth $H$. A pipe of length slightly less than $H$ lies at the bottom of the lake. It contains an ideal gas filled up to a length of $\frac{H}{10}$. A smooth light movable piston keeps the gas in place. Now the pipe is slowly raised to vertical position (see figure). Assume that temperature of the gas remains Constant and neglect the atmospheric pressure.



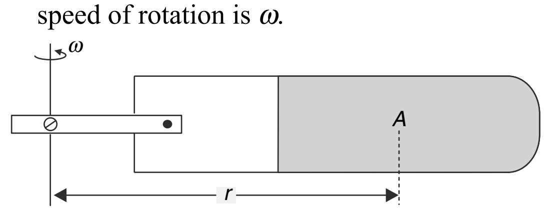

(a) Plot the variation of pressure inside the lake as a function of height $y$ from the base. Let the height of piston from the base, after the pipe is made vertical, be $y$. Plot the variation of gas pressure as a function of $y$ in the first graph itself.

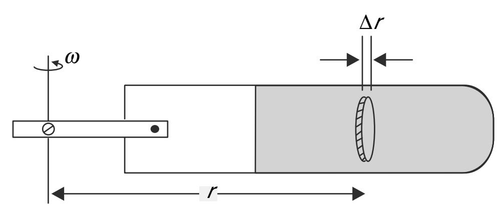
(b) In equilibrium the gas pressure and the pressure due to water on the piston must be equal. Using this solve for equilibrium height $y_0$ of the piston. You get two answers. Which one is correct and why?

Q. 36. A centrifuge has a horizontal cylinder rotating about a vertical axis as shown in the figure. Water inside it has density $\rho$.

(a) Consider a point $A$, inside the liquid, at a distance $r$ from the rotation axis. Liquid pressure at this point is $P$. Write the value of $\frac{dP}{dr}$ when the cylinder, with all its liquid, rotates uniformly at an angular speed $\omega$.

(b) Consider a small disc shaped foreign material inside the centrifuge at point $A$. The area of circular disc is $\Delta S$ and its thickness is $\Delta r$. It is made of material of density $\rho'(> \rho)$. The disc is in position so that its circular face is vertical. Find the radial acceleration of the disc with respect to the cylinder when angular speed of rotation is $\omega$.

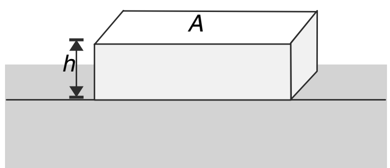


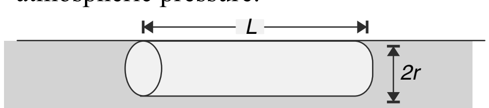

Q. 37. (i) A cubical metal block of side 10 $cm$ is floating in a vessel containing mercury. The vessel has a square cross section of side length 15 $cm$. Water is poured into the vessel so that the metal block just gets submerged. Calculate the mass of water that was poured into the vessel. It is given that relative density of the metal and mercury are 7.3 and 13.6 respectively.

(ii) In the last question, in place of water if we poured another liquid of relative density '$r$' it was found that when the metal block was just completely submerged it was no longer touching mercury. What is value of $r$?

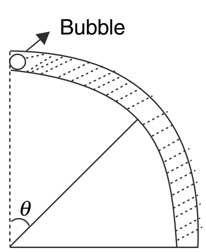
Q. 38. A wooden block of cross sectional area A (in shape of a rectangle) has height $h$. It is held such that its lower surface touches the water surface in a wide and deep tank. The block is released in this position. It oscillates for some time and then settles into its equilibrium position. In equilibrium the block floats with its upper face just on the water surface. Calculate the amount of heat generated in the process assuming that the loss in gravitational potential energy of the system comprising of water and the block gets converted into heat. Density of water is $\rho$.



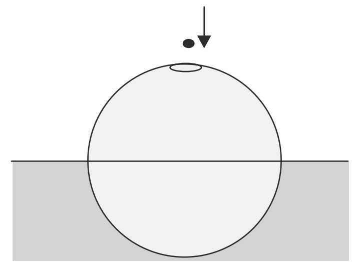

Q. 39. A cylindrical wooden log of length $L$ and radius $r$ is floating in water (density = $\rho$) while remaining completely submerged as shown in figure. Calculate the force of water pressure on the lower half of the cylinder. Exclude contribution due to atmospheric pressure.



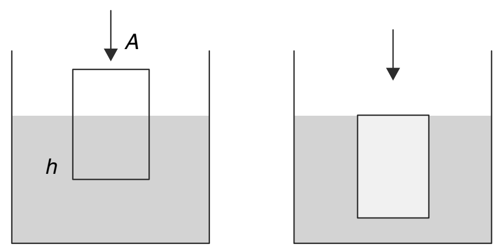

Q. 40. A bent tube contains water. An air bubble is trapped inside the liquid. The tube is held vertical (as shown) and is moved horizontally with an acceleration ($a$) such that the bubble moves to position $\theta$ shown in the diagram. Find the direction and magnitude of $a$.




Q. 41. A spherical pot with a small opening at top has a mass of $M$ = 452.1 gram. It floats in water while remaining exactly half submerged A small coin of mass $m$ = 5 gram is dropped into the pot. Find how much more the pot will sink. Express your answer in terms of change in height of the pot that is under water. Density of water = 1 $g/cc$.



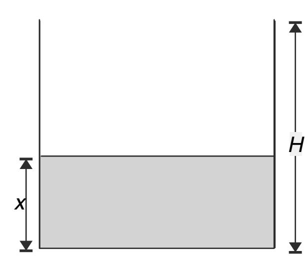
Q. 42. A cylindrical wooden block of density half the density of water is floating in water in a cylindrical container. The cross section of the wooden block and its height are $A$ and $h$ respectively. The cross sectional area of the container is $2A$. The wooden block is pushed vertically so that it gradually gets immersed in water. Calculate the amount of work done in pushing the block . Density of water = $\rho_0$.



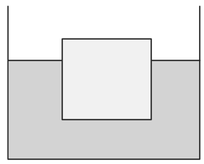

Q. 43. A cylindrical container has mass $M$ and height $H$. The centre of mass of the empty container is at height $\frac{H}{2}$ from the base. A liquid, when completely filled in the container, has mass $\frac{M}{2}$. This liquid is poured in the empty container.

(a) How does the centre of mass of the system (container + liquid) move as the height ($x$) of liquid column changes from zero to $H$? Explain your answer qualitatively. Draw a graph showing the variation of height of centre of mass of the system ($x_{cm}$) with $x$.

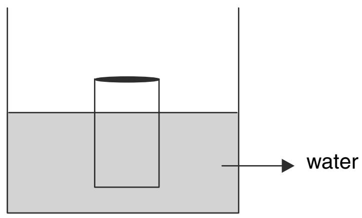
(b) Find the height of liquid column $x$ for which the centre of mass is at its lowest position.




Q. 44. A cubical ice block of side length '$a$' is floating in water in a beaker. Find the change in height of the centre of mass of (water + ice) system when the ice block melts completely. It is given that ratio of mass of water to mass of ice originally in the container is 4 : 1.



Q. 45. A cylindrical ice block is floating in water. 10% of its total volume is outside water. Kerosene oil (relative density = 0.8) is poured slowly on top of water in the container. Assume that the oil does not mix with water. Height of the ice cylinder is *H*.




(a) As kerosene is poured, how does the volume of ice block above the water level change?

(b) What is the thickness of kerosene layer above the water when 20% of the volume of the ice block is above the water surface?

(c) Find the ratio of volume of ice block in kerosene to its volume in water after the kerosene layer rises above the top surface of ice and the block gets completely submerged.

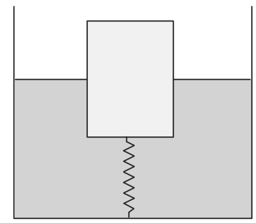
Neglect any melting of ice

Q. 46. A cylindrical container contains water. *A* cubical block is floating in water with its lower surface connected to a spring

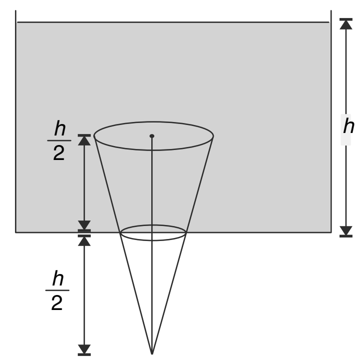
(a) Suppose that the spring is in relaxed state. Now, if the whole container is accelerated vertically upwards, will the spring get compressed?

(b) Suppose that the spring is initially compressed. Now, what will happen to the state of the spring when the container is accelerated upwards?

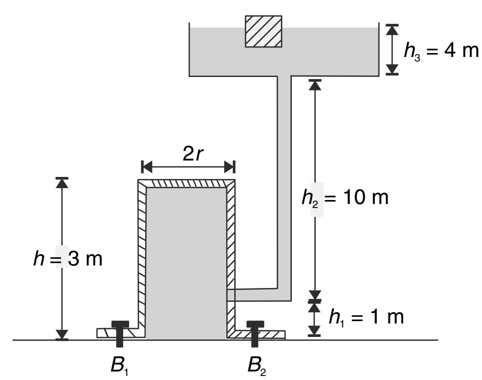
(c) Assume that mass of the block is 1 *kg* and initially the spring (force constant $k$ = 100 *N*/*m*) is compressed by 5 *cm*. When the container is accelerated up by an acceleration of 5 $m/s^2$, the spring has a total compression of 6 *cm*. Calculate the change in volume of block submerged inside water when the container gets accelerated. Density of water is $10^3$ $kg/m^3$.



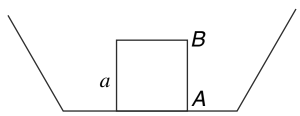
Q. 47. A water tank has a circular hole at its base. A solid cone is used to plug the hole. Exactly half the height of the cone protrudes out of the hole. Water is filled in the tank to a height equal to height of the cone. Calculate the buoyancy force on the cone. Density of water is $\rho$ and volume of cone is $V$.



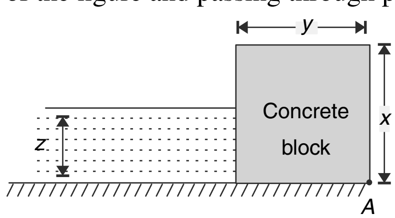

Q. 48. A cylindrical tank has a mass of the 200 *kg* and inner radius of $r$ = 2.0 *m*. The tank has no bottom and is directly bolted to the floor [$B_1$ and $B_2$ are bolts in the figure]. The tank is connected to an elevated open tank and both the tanks are filled with a liquid as shown in the figure. Various heights are as shown. When a small wooden cube of specific gravity 0.6 is placed in the upper tank, it floats while remaining exactly half submerged. Calculate the force applied by the bolts in holding the tank.




Q. 49. (i) A non uniform cube of side length $a$ is kept inside a container as shown in the figure. The average density of the material of the cube is 2 $\rho$ where $\rho$ is the density of water. Water is gradually filled in the container. It is observed that the cube begins to topple, about its edge (into the plane of the figure) passing through point $A$, when the height of the water in the container becomes $\frac{a}{2}$. Find the distance of the centre of mass of the cube from the face $AB$ of the cube. Assume that water seeps under the cube.



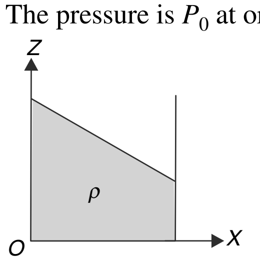

(ii) A rectangular concrete block (specific gravity = 2.5) is used as a retaining wall in a reservoir of water. The height and width of the block are $x$ and $y$ respectively. The height of water in the reservoir is $z = \frac{3}{4}x$. The concrete block cannot slide on the horizontal base but can rotate about an axis perpendicular to the plane of the figure and passing through point $A$



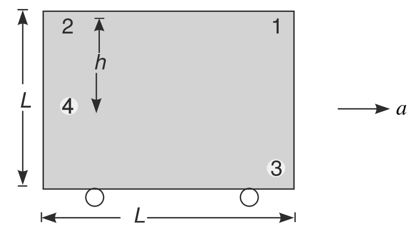

(a) Calculate the minimum value of the ratio $\frac{y}{x}$ for which the block will not begin to overturn about $A$.

(b) Redo the above problem for the case when there is a seepage and a thin film of water is present under the block. Assume that a seal at A prevents the water from flowing out underneath the block.

Q. 50. A container having an ideal liquid of density $\rho$ is moving with a constant acceleration of $\vec{a} = a_x\hat{i} + a_z\hat{k}$ where $x$ direction is horizontal and $z$ is vertically upward. The container is open at the top. In a reference frame attached to the container with origin at bottom corner (see figure), write the pressure at a point inside the liquid at co-ordinates $(x, y, z)$. The pressure is $P_0$ at origin.




Q. 51. A cubical container of side length L is filled completely with water. The container is closed. It is accelerated horizontally with acceleration $a$. Density of water is $\rho$.




(a) Assuming pressure at point 1 [upper right corner] to be zero, find pressure at point 2 [upper left corner]

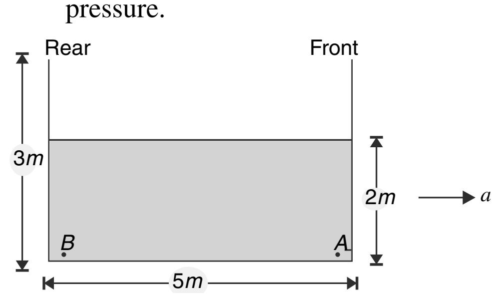
(b) Pressure at point 4, at a distance h vertically below point 2, is same as pressure at lower right corner 3. Find $h$.

Q. 52 An open rectangular tank $5m \times 4m \times 3m$ in dimension is containing water up to a height of $2m$ The tank is accelerated horizontally along the longer side. Assuming water to be an ideal liquid, find -

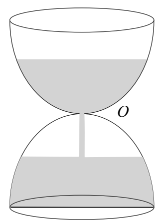
(a) the maximum acceleration with which the tank can be moved so that water does not fall from the rear side.

(b) the gauge pressure at the bottom of the front and back of the tank [points A and B] if the tank is closed at the top and is then accelerated horizontally at 9 $m/s^2$. Assume that the top cover has a small hole at the right side of the tank so that pressure of air inside the tank is maintained at atmospheric pressure.

Gauge pressure at a point is difference in absolute pressure at the point and atmospheric pressure.



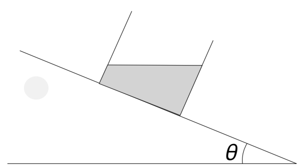

Q. 53. A water clock consist of a vessel which has a small orifice $O$. The upper container is filled with water which trickles down into the lower container. The shape of the (upper or lower) container is such that height of water in the upper container changes at a uniform rate. What should be the shape of the container? Assume that atmospheric air can enter inside the lower container through a hole in it and that the upper container is open at the top. Vessel is axially symmetric.




Q. 54. A rectangular container has been filled with an ideal fluid and placed on an incline plane. The inclination of the incline is $\theta$. Find the angle that the liquid surface will make with the incline surface as the container slides down. Find your answer for following two cases.

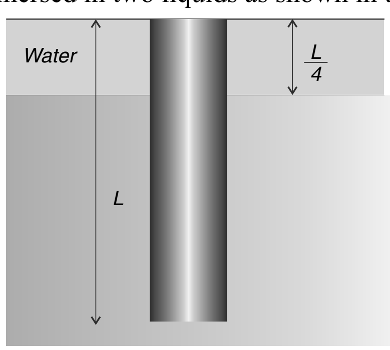
(a) Take the incline to be smooth.

(b) Assume a friction coefficient of μ (< tan $\theta$) between the incline and the container.




Q. 55. A cylindrical vessel of radius $R = 1\ m$ and height $H = 3\ m$ is filled with an ideal liquid up to a height of $h = 2\ m$. The container with liquid is rotated about its central vertical axis such that the liquid just rises to the brim. Calculate the angular speed ($\omega$) of the container.

Q. 56. A cylinder of length $L$ floats with its entire length immersed in two liquids as shown in the fig.



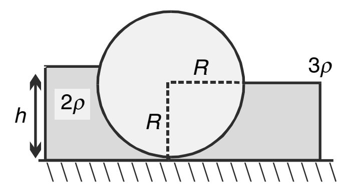

The upper liquid is water and the lower liquid has density twice that of water. The two liquid are immiscible. The cylinder is in equilibrium with its $\frac{3}{4}$ length in the denser liquid and $\frac{1}{4}$ of its length in water. The thickness of water layer is $\frac{L}{4}$ only. Find

(a) the specific gravity of the material of the cylinder.

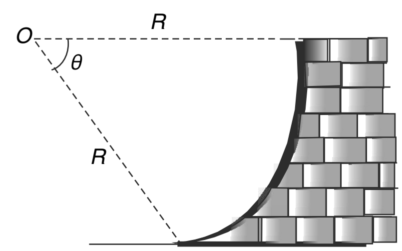
(b) the time period of oscillations if the cylinder is depressed by some small distance $\left(<\frac{L}{4}\right)$ and released.

Neglect viscosity and change in level of liquids when the cylinder moves.

Q. 57. (i) In the figure shown, the heavy cylinder (radius $R$) resting on a smooth surface separates two liquids of densities $2\rho$ and $3\rho$. Find the height '$h$' for the equilibrium of cylinder.




(ii) The cross section of a dam wall is an arc of a circle of radius $R = 20\ m$ subtending on angle of $\theta = 60^\circ$ at the centre of the circle. The centre (O) of the circle lies in the water surface. The width of the dam [i.e., dimension perpendicular to the figure] is $b = 10m$. Neglect atmospheric pressure in following calculations.

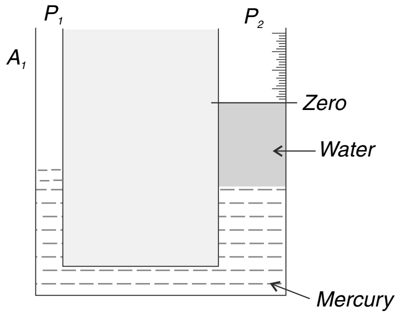



(a) Calculate the vertical component of force ($F_v$) applied by water on the curved dam wall.

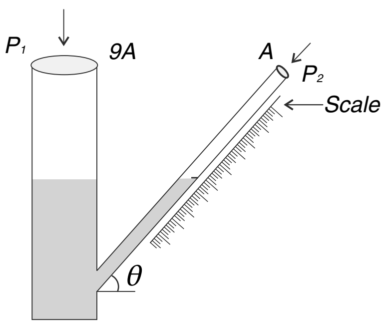
(b) Calculate the horizontal component of force ($F_H$) applied by water on the curved dam wall.

(c) Calculate the resultant force applied by the water on the curved dam wall.

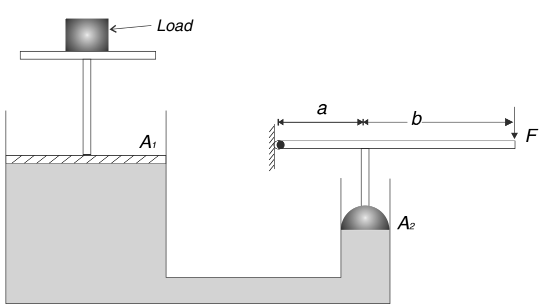
Q. 58. A monometer has mercury and water filled in it as shown in the figure. A scale is marked on the right tube which has a cross sectional area of $A_2$.The other tube has a cross sectional area of $A_1$. When pressures $P_1$ and $P_2$ at both ends of the manometer is same, the level of water in the right side is at the zero of the scale. When the applied pressure $P_1$ is changed by $\Delta P_0$, the level of water surface changes by $\Delta h$. The quantity $\frac{\Delta h}{\Delta P_0}$ can be defined as pressure sensitivity($s$) of the manometer. Calculate the pressure sensitivity of the given manometer. Density of water and mercury is $\rho_\omega$ and $\rho_{Hg}$ respectively.




Q. 59. A manometer has a vertical arm of cross sectional area 9A and an inclined arm having area of cross section A. The density of the manometer liquid has a specific gravity of 0.74. The scale attached to the inclined arm can read up to ± 0.5 $mm$. It is desired that the manometer shall record pressure difference ($P_1 - P_2$) up to an accuracy of ± 0.09 mm of water. To achieve this, what should be the inclination angle $\theta$ of the inclined arm.

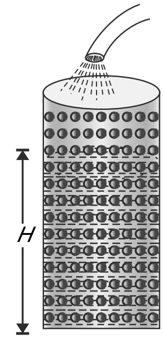



Q. 60. The figure shows a schematic layout of a hydraulic jack. The load to be raised weighs 20,000 $N$. The area of cross section of the two pistons are $A_1 = 50\ cm^2$ and $A_2 = 10\ cm^2$. The force ($F$) is applied at the end of a light lever bar as shown in the figure. Lengths a and b are 4 cm and 36 cm respectively. Find the force ($F$) required to raise the load slowly.




Q. 61. In a two dimensional steady flow the velocity of fluid particle at ($x$, $y$) is given by $\vec{V} = (u_0 + bx)\hat{i} - by\hat{j}$; $u_0$ and $b$ are positive constants. Write the equation of streamlines. Draw few streamlines for $x > 0$.

Q. 62. A cylindrical container having radius $r$ has perforated wall. There are large number of uniformly spread small holes on the vertical wall occupying a fraction $\eta = 0.02$ of the entire area of the wall. To maintain the water level at height $H$ in the container, water is being fed to it at a constant rate $Q$ ($m^3\ s^{-1}$). Find $Q$.




Q. 63. A syringe is filled with water. Its volume is $V_0 = 40\ cm^2$ and cross section of its interior is $A = 8\ cm^2$. The syringe is held vertically such that its nozzle is at the top, and its piston is pushed up with constant speed. Mass of the piston is $M = 50$ $g$ and the water is ejected at a speed of $u_0 = 2\ m/s$. Cross section of the stream of water at the nozzle is $a = 2\ mm^2$. Neglect friction and take the density of water to be $\rho = 10^3\ kg/m^3$

(a) Find the speed of the piston

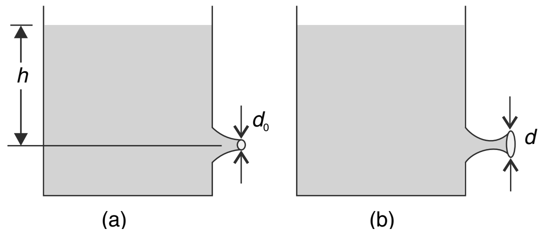
(b) Find the total work done by the external agent in emptying the syringe.

Q. 64. A water tank has a small hole in its wall and a tapering nozzle has been fitted into the hole (figure). The diameter of the nozzle at the exit is $d_0 = 1\ cm$. The height of water in the tank above the central line of the nozzle is $h = 2.0\ m$.

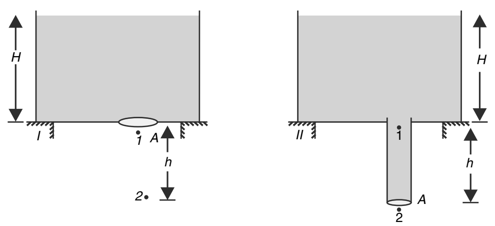
Calculate the discharge rate in $m^3\ s^{-1}$ through the nozzle.

Another nozzle which is diverging outwards is fitted smoothly to the first nozzle. The pressure at the neck of the two nozzles (where diameter is $d_0$) drops to 2.5 $m$ of water. Calculate the exit diameter (d) of the nozzle.

Atmosphere pressure = 10 $m$ of water and $g = 10\ ms^{-2}$




Q. 65. There are two large identical open tanks as shown in figure. In tank I there is a small hole of cross sectional area A at its base. Tank II has a similar hole, to which a pipe of length *h* has been connected as shown. The internal cross sectional area of the pipe can be considered to be equal

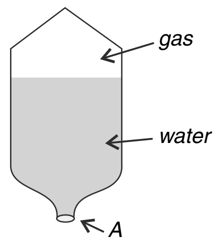



to A. Point 1 marked in both figures, is a point just below the opening in the tank and point 2 marked in both figures, is a point h below point 1 [In fig II, point 2 is just outside the opening in the pipe.].

(a) Find the speed of flow at point 2 in both figures.
(b) Find the ratio of speed of flow at point I is first figure to that in second figure.
(c) Find the difference in pressure at point 1 in both figures.

Q. 66. To illustrate the principal of a rocket, a student designed a water rocket as shown in the figure. It is basically a container having pressurized gas in its upper part and water in its lower part. Pressure of the gas is 4.0 Mpa. Mass of empty container is 1.0 *kg* and mass of its content is also1.0 *kg*. The nozzle at the bottom is opened to impart a vertical acceleration to the container. If it is desired that the initial upward acceleration of the container be 0.5g, what should be the cross sectional area (A) of the exit of the nozzle?

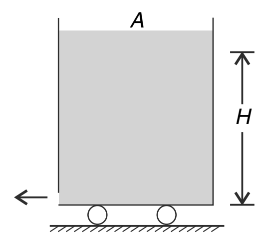
Neglect the pressure due to height of water in the container and take atmospheric pressure to be 1.0 MPa. $g = 10\ ms^{-2}$




Q. 67. An open tank of cross sectional area A contains water up to height *H*. It is kept on a smooth horizontal surface. A small orifice of area $A_0$ is punched at the bottom of the wall of the tank. Water begins to drain out. Mass of the empty tank may be neglected.

(i) Prove that the tank will move with a constant acceleration till it is emptied. Find this acceleration.

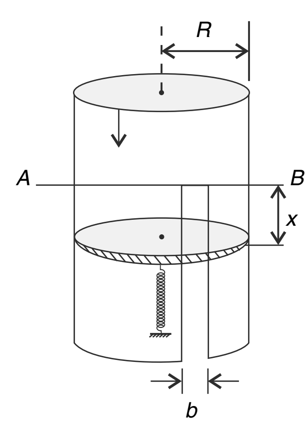
(ii) Find the find speed acquired by the tank when it is completely empty.



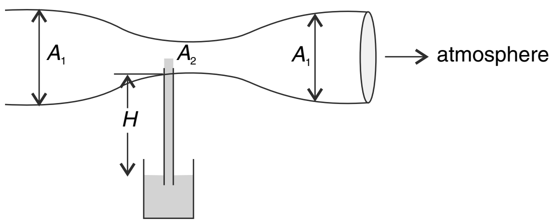

Q. 68. The device shown in figure can be used to measure the pressure and volume flow rate when a person exhales. There is a cylindrical pipe with inside radius *R*. There is a slit of width *b* running down the length of the cylinder. Inside the tube there is a light movable piston attached to an ideal spring of force constant *K*. In equilibrium position the piston is at a position where the slit starts (shown by line *AB* in the figure). A person is made to exhale into the cylinder causing the piston to compress the spring. Assume that slit width b is very small and the outflow area is much smaller than the cross section of the tube; even at the pistons full extension. A person exhales and the spring compresses by *x*. [Density of air = $\rho$]

(a) Calculate the gage pressure in the tube.

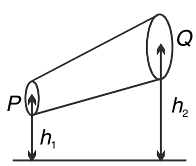
(b) Calculate the volume flow rate (*Q*) of the air.



Q. 69. (i) Air (density = $\rho$) flows through a horizontal venturi tube that discharges to the atmosphere. The area of cross section of the tube is $A_1$ and at the constriction it is $A_2$. The constriction is connected to a water (density = $\rho_0$) tank through a vertical pipe of length $H$. Find the volume flow rate ($Q$) of the air through tube that is needed to just draw the water into the tube.



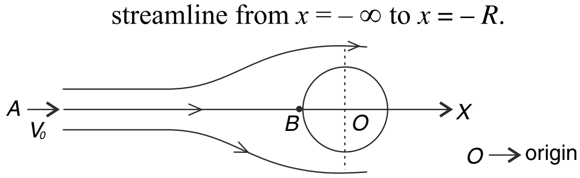

(ii) A non viscous liquid of constant density $\rho$ flows in a streamline motion along a tube of variable cross section. The tube is kept inclined in the vertical plane as shown in the figure. The area of cross section of the tube at two points $P$ and $Q$ at heights of $h_1$ and $h_2$ are respectively $A_1$ and $A_2$. The velocity of the liquid at point $P$ is $v$. Find the work done on a small volume $\Delta V$ of fluid by the neighbouring fluid as the small volume moves from $P$ to $Q$.



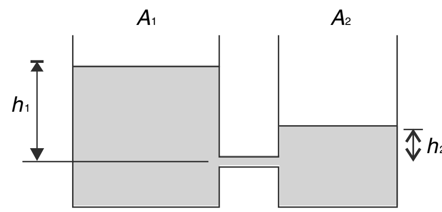

Q. 70. The figure shows a non- viscous, incompressible and steady flow in front of a sphere. $A$-$B$ is a horizontal streamline. It is known that the fluid velocity along this streamline is given by $V = V_0\left(1+\frac{R^3}{x^3}\right)$. $V_0$ is velocity of flow on this streamline when $x \to (-\infty)$. It is given that pressure at $x \to (-\infty)$ is $P_0$ and density of liquid is $\rho$.

(i) Write the variation of pressure along the streamline from point $A$, far away from the sphere, to point $B$ on the sphere.

(ii) Plot the variation of pressure along the streamline from $x = -\infty$ to $x = -R$.




Q. 71. There are two tanks next to each other having cross sectional area $A_1$ and $A_2$. They are interconnected by a narrow pipe having area of cross section equal to $A_0$. Initial height of water in the two tanks is $h_1$ and $h_2$ measured from the level of the pipe. Assume that the flow is ideal and behaves in a way similar to the discharge in air. Calculate the time needed for the water level in two tanks to become same.




Q. 72. A siphon is used to drain water (density = $\rho$) from a wide tank. The inlet and outlet mouth of the siphon are at the same horizontal level and the highest point of the siphon tube is at a height H from the mouth of the tube. Height of water in the tank above the tube mouth is h (see fig). Atmospheric pressure is $P_0$.

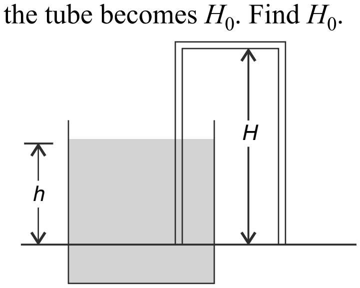
(a) Will the water drain out in this siphon? If yes, at what speed (V)?

(b) Find pressure at the top of the siphon tube (call it $P'$)

(c) Find pressure just inside the left mouth of the tube.

(d) If left part of the tube is slightly cut short, without disturbing anything else, what effect it will have on $V$ and $P'$?

(e) If the right end of the tube is lowered by adding more length of tube, it was observed that flow stops when length of right limb of the tube becomes $H_0$. Find $H_0$.




## LEVEL 3

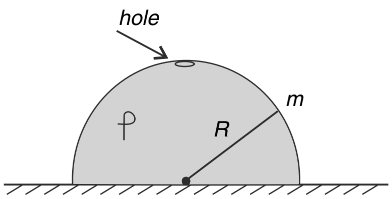
Q. 73. A vessel of volume $V_0$ is completely filled with a salt solution having specific gravity $\sigma_0$. Pure water is slowly added drop by drop to the container and the solution is allowed to overflow.

(a) Find the specific gravity of the diluted solution in the container when a volume $V$ of pure water has been added to it.

(b) If $\sigma_0 = 1.2$ then find the specific gravity of the solution in the container after a volume $V_0$ of pure water has been added to it.

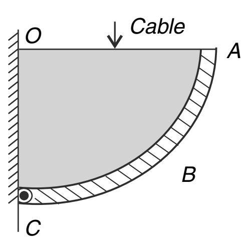
(c) Plot the variation of $\sigma$ with V.

Q. 74. A hemispherical bowl of radius $R$ is placed upside down on a flat horizontal surface. There is a small hole at the top of the inverted bowl. Through the hole, a liquid of density $\rho$ is poured in. Exactly when the container gets full, water starts leaking from between the table and the edge of the container. Find the mass (m) of the container.




Q. 75. A plate is in the shape of a quarter cylinder of radius $R$ and length $L$. This plate is hinged at $C$ to a vertical wall and can rotate freely about $C$. The end $A$ of the plate is tied to the wall using two horizontal cables [the other cables is parallel to $OA$ and the two cables are placed symmetrically]. The space between the wall and the plate is filled completely with water (density = $\rho$). Neglect the weight of the plate and calculate the tension in each cable.

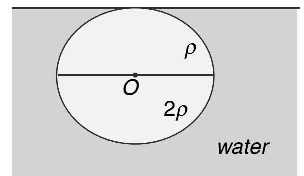
Take $\tan^{-1}\left(\frac{\pi}{2}\right) \simeq 57°$ and $\cos 57° \simeq \frac{1}{2}$



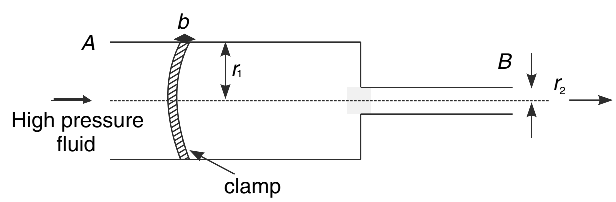

Q. 76. A spherical ball of radius $R$ is made by joining two hemispherical parts. The two parts have density $\rho$ and $2\rho$. When placed in a water tank, the ball floats while remaining completely submerged.

(a) If density of water is $\rho_0$, find $\rho$

(b) Find the time period of small angular oscillations of the ball about its equilibrium position. Neglect viscous forces.




Q. 77. In a machine, a fluid from a compressor, which is at high pressure, is allowed to pass through a nozzle. Cross section of the nozzle is shown in the figure. The nozzle consists of two sections of radii $r_1$ and $r_2$. The nozzle is fixed to a stand with the help of a clamp. The clamp is a circular ring of radius $r_1$ and width $b$ .The fluid from the compressor is at a pressure of $n$ times the atmospheric pressure $P_0$. Assume that the entire system is horizontal, the fluid is ideal and the flow is steady.




(a) What should be the volume flow rate so that pressure of the fluid at end $B$ reduces to half of its value at end $A$?

(b) If the entire system is kept in gravity free space and the net force on the nozzle due to the fluid flow is $F$ then determine the minimum radial pressure that should be applied on the clamp, so the nozzle remains in place. Coefficient of friction between clamp and nozzle is $\mu$.

(c) If a small hole is punched anywhere on the thinner part of the nozzle (close to end $B$) what should be the volume flow rate of the fluid so that it does not gush out?

# ANSWERS

1. $5.14 \times 10^{18}\ kg$
2. Because *Hg* has high density. A water barometer will have a large height.
3. No
4. $10.30\ m$, $0.76\ m$
5. $P = 1.35 \times 10^7\ Pa$
6. Both beakers have same weight.

   Answer does not change.
7. (i) (a) Fall down

   (b) Not change

   (ii) Does not change.
8. 1.54
9. First balance will show higher reading.

   Answer will not change if string in 2nd container is cut.
10. Reading of $S_1 = 20\ kg$; Reading of $S_2 = 6.5\ kg$
11. (a) $d = \frac{2}{3}(d_1 + d_2)$
12. $20\ m$
13. $P_0 = \frac{1}{2} g.d.R = 1.65 \times 10^{11}\ Pa$
14. $\frac{z_0}{z_0 + \Delta z}$
16. (i) 45 cc

    (ii) 25 cc

    (iii) When ice melts there will be no change in water level in case (i) and the water level will rise in case (ii).
17. (a) $7\ cm$

    (b) 0

    (c) 0
18. $0.75\ m^3$
19. (a) $vg\,(d - \rho)$

    (b) $\pi R^2\,[\rho_0 + \rho g H] - v\rho g$
20. $Hb\left(L + \frac{\rho g b H^2}{2k}\right)$
21. Water level will rise by 1 *mm*.
22. $\pi R^2 L\rho\left(\sqrt{\frac{\sigma}{\rho}} - 1\right)$
23. (a) 2.33 kg

    (b) 56.7 N
24. $18\ Nm^{-2}$
25. $\theta = \tan^{-1}(3)$; $T = \frac{\sqrt{10}}{3} mg$
26. $P_0 + \sqrt{2}\,\rho g R$
27. *A*
28. (i) $P_1 > P_2$

    (ii) up
29. (i) (a) $h = 0$

    (b) $x = 2\sqrt{h_0 H}$

    (ii) (a) $V_x = \frac{R}{x} V_0$

    (b) $V_x = \frac{Q}{2\pi h x}$
30. $12.1\ m^3 s^{-1}$
31. (a) Yes

    (b) $h_2 - h_1 = \frac{v^2}{2g}$
32. $\frac{v^2}{2g} - h_0$
33. (i) $371.2\ N$

    (ii) 26 cm. Height h will increase
34. (a) (i) $\frac{5}{4} d$

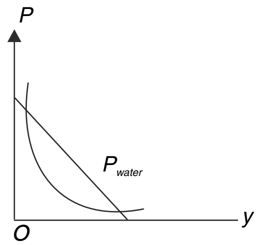
    (ii) $\frac{1}{4}(6H + L)dg$

    (b) (i) $\sqrt{(g/2)(3H - 4h)}$

    (ii) $\sqrt{h(3H - 4h)}$

    (iii) $(3/8)H$, $\frac{3}{4} H$

**35.** (a)




(b) $y = \frac{H}{2} - \frac{\sqrt{15}}{10}H$

**36.** (a) $\frac{dP}{dr} = \rho\omega^2 r$

(b) $\left(1 - \frac{\rho}{\rho'}\right)\omega^2 r$

**37.** (i) 625 $g$
(ii) $r = 7.3$

**38.** $\frac{1}{2}\rho g A h^2$

**39.** $\rho g L r^2 \left(\frac{\pi}{2} + 2\right)$

**40.** $a = g \tan\theta$ towards right

**41.** 0.4 $mm$

**42.** $W = \frac{1}{16}A\rho_0\ gh^2$

**43.** (a) The COM first falls, attains a minimum height and then it rises to original height $\frac{H}{2}$
(b) $x = (\sqrt{6} - 2)H$

**44.** $0.01a$

**45.** (a) First increases then become constant
(b) $\frac{H}{8}$
(c) 1.0

**46.** (a) No
(b) spring will get compressed more
(c) 100 $cm^3$

**47.** $V\rho g/8$

**48.** 179 $KN$

**49.** (i) $x = \frac{9}{8}$
(ii) (a) $\frac{3}{4\sqrt{10}}$
(b) $\frac{3}{4\sqrt{7}}$

**50.** $P = P_o - \rho a_x . x - \rho\ (g + a_z)z$

**51.** (a) $\rho a L$
(b) $h = L\left(1 - \frac{a}{g}\right)$

**52.** (a) $\frac{2g}{5} = 4\ m/s^2$
(b) $P_A = 0, P_B = 0.44\ atm$

**53.** The vessel can be obtained by revolution of a curve $z = kx^4$

**54.** (a) 0
(b) $\tan^{-1}\mu$

**55.** $\omega = 2\sqrt{10}\ rads^{-1}$

**56.** (a) $\frac{7}{4}$
(b) $\pi\sqrt{\frac{7L}{4g}}\left[1 + \frac{1}{\sqrt{2}}\right]$

**57.** (i) $\sqrt{\frac{3}{2}}R$
(ii) (a) $F_v = 1.2 \times 10^7 N$
(b) $F_H = 1.5 \times 10^7 N$
(c) $F = 1.92 \times 10^7 N$

**58.** $s = \frac{1}{\left(1 + \frac{A_2}{A_1}\right)\rho_{Hg}.g}$

**59.** $\theta = \sin^{-1}(0.13)$

**60.** 400 $N$

**61.** $y = \frac{c_0}{u_0 + bx}$ where $c_0$ is a constant. For plot of streamlines see the solution.

**62.** $Q = \frac{4}{3}\pi\eta r\sqrt{2g}\ H^{3/2}$

**63.** (a) $u = \frac{a}{A}u_0 = 5\ mm/s$
(b) 0.115 $J$

**64.** (a) $4.96 \times 10^{-4}\ m^3 s^{-1}$
(b) 1.48 $cm$

**65.** (a) In both figures $v_2 = \sqrt{2g(H + h)}$
(b) $\sqrt{\frac{H}{H + h}}$

(c) $\rho gh$

**66.** $g = 10\ ms^{-2}$

**67.** (i) $\dfrac{2A_0 g}{A}$ (ii) $V = 2\sqrt{2gH}$

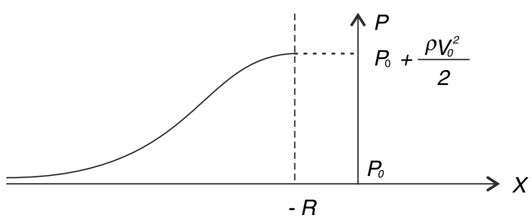
**68.** (a) $\dfrac{Kx}{\pi R^2}$ (b) $bx^{3/2}\sqrt{\dfrac{2K}{\rho \pi R^2}}$

**69.** (i) $Q = \sqrt{\dfrac{2\rho_0 gH}{\rho}\left[\dfrac{1}{A_2^2} - \dfrac{1}{A_1^2}\right]^{-1}}$

(ii) $\dfrac{1}{2}\rho\Delta V\left(\dfrac{A_1^2}{A_2^2} - 1\right)v_1^2 + \rho\Delta Vg(h_2 - h_1)$

**70.** (i) $P = P_0 + \rho V_0^2\left[1 - \left(1 + \dfrac{R^3}{x^3}\right)\right]$

(ii)




**71.** $\sqrt{\dfrac{2}{g}}\dfrac{A_1 A_2\sqrt{h_1 - h_2}}{A_0(A_1 + A_2)}$

**72.** (a) $\sqrt{2gh}$

(b) $P' = P_0 - \rho gH$

(c) $P_0$

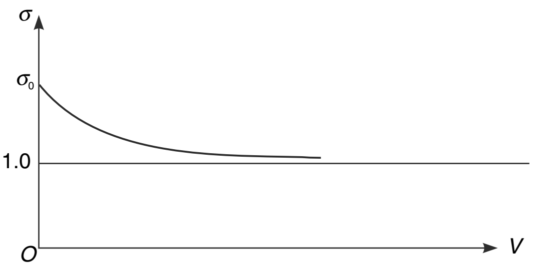
(d) $V$ decreases $P'$ increases

(e) $H_0 = \dfrac{P_0}{\rho g} \simeq 10.3\ m$

**73.** (a) $\sigma = 1 + (\sigma_0 - 1)e^{-v/v_0}$

(b) 1.074

(c)




**74.** $m = \dfrac{\pi R^3 \rho}{3}$

**75.** $\sqrt{\dfrac{4+\pi^2}{8}}\cdot \rho g R^2 L$

**76.** (a) $\rho = \dfrac{2\rho_0}{3}$ (b) $T = 2\pi\sqrt{\dfrac{123R}{40g}}$

**77.** (a) $Q = \pi r_1^2 r_2^2 \sqrt{\dfrac{nP_0}{\rho\{r_1^4 - r_2^4\}}}$

(b) $\dfrac{F}{2\pi\mu r_1 b}$

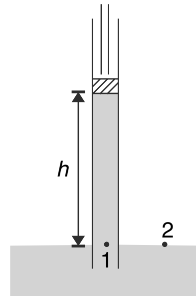
(c) $Q = \pi r_1^2 r_2^2 \sqrt{\dfrac{2(n-1)P_0}{\rho[r_1^4 - r_2^4]}}$

# SOLUTIONS

**1.** $m = \dfrac{pA}{g} = \dfrac{1\times10^5\times4\pi(6.4\times10^6)^2}{10} \simeq 5.14\times10^{18}\,kg$

**3.** The pressure in the tube at the level $A$ will be below the atmospheric pressure. Therefore, the atmospheric pressure will not allow the water to flow out when a hole develops. Air will enter the tube through the hole until the atmospheric pressure is reached inside the tube and the mercury level will sink to the level outside the tube.

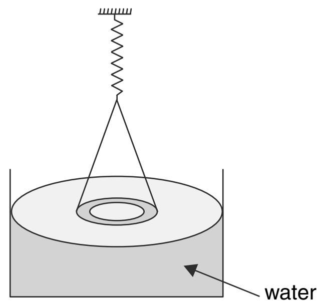
**4.** The water will rise along with the piston till the pressure produced by the water column at point 1 becomes equal to atmospheric pressure.



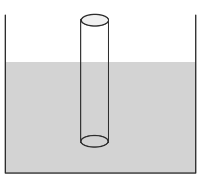
# 09 SURFACE TENSION

## LEVEL 1

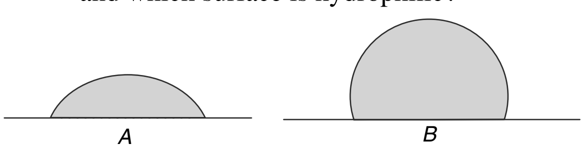
Q.1. A circular ring has inner and outer radii equal to 10 *mm* and 30 *mm* respectively Mass of the ring is $m = 0.7\ g$. It gently pulled out vertically from a water surface by a sensitive spring. When the spring is stretched 3.4 *cm* from its equilibrium position the ring is on verge of being pulled out from the water surface. If spring constant is $k = 0.7\ Nm^{-1}$ find the surface tension of water.



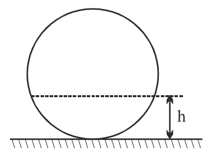

Q.2. A long thin walled capillary tube of mass $M$ and radius $r$ is partially immersed in a liquid of surface tension $T$. The angle of contact for the liquid and the tube wall is 30°. How much force is needed to hold the tube vertically? Neglect buoyancy force on the tube.



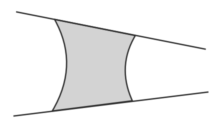
Q.3. (i) Water drops on two surfaces $A$ and $B$ has been shown in figure. Which surface is hydrophobic and which surface is hydrophilic?




(ii) A liquid is filled in a spherical container of radius $R$ till a height $h$. In this position the liquid surface at the edges is also horizontal. What is the contact angle between the liquid and the container wall?




Q.4. A conical pipe shown in figure has a small water drop. In which direction does the drop will tend to move?



Q.5. A narrow tube of length $l$ and radius $r$ is sealed at one end. Its open end is brought in contact with the surface of water while the tube is held vertical. The water rises to a height $h$ in the tube. The contact angle of water with the tube wall is $\theta$, density of water is $\rho$ and the atmospheric pressure is $P_o$. Find the surface tension of the liquid. Assume that the temperature of air inside the tube remains constant and the volume of the meniscus is negligible.

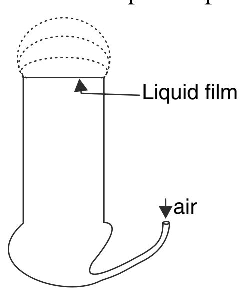
Q.6. The internal radius of one arm of a glass capillary $U$ tube is $r_1$ and for the second arm it is $r_2 (> r_1)$. The tube is filled with some mercury having surface tension $T$ and contact angle with glass equal to $90° + \theta$.

(a) It is proposed to connect one arm of the $U$ tube to a vacuum pump- so that the mercury level in both arms can be equalized. To which arm the pump shall be connected?

(b) When mercury level in both arms is same, how much below the atmospheric pressure is the pressure of air in the arm connected to the pump?

Q.7. In a horizontal capillary tube, the rate of capillary flow depends on the surface tension force as well as the viscous force. Lueas and washburn showed that the length ($x$) of liquid penetration in a horizontal capillary depends on a factor ($k$) apart from time ($t$). The factor is given by $k = \left[\frac{rT\cos\theta}{2\eta}\right]^{\frac{1}{2}}$; where $r$, $T$, $\theta$ and $\eta$ are radius of the capillary tube, surface tension, contact angle and coefficient of viscosity respectively. If the length of liquid in the capillary grows from zero to $x_0$ in time $t_0$, how much time will be needed for the length to increases from $x_0$ to $4x_0$.

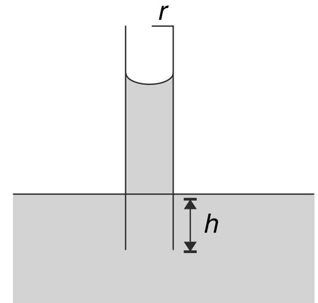
Q.8. A glass tube of radius $R$ is covered with a liquid film at its one end. Air is blown slowly into the tube to gradually increase the pressure inside. What is the maximum pressure that the air inside the tube can have? Assume that the liquid film does not leave the surface (whatever its size) and it does not get punctured. Surface tension of the liquid is $T$ and atmospheric pressure is $P_o$.




Q.9. Why bubbles can be formed using soap water but we do not have bubbles formed out of pure water?

Q.10. A tapering glass capillary tube $A$ of length 0.1 $m$ has diameters $10^{-3}$ $m$ and $5 \times 10^4$ $m$ at the ends. When it is just immersed in a liquid at 0º$C$ with larger radius in contact with liquid surface, the liquid rises $8 \times 10^{-2}$ $m$ in the tube. In another experiment, in a cylindrical glass capillary tube $B$, when immersed in the same liquid at 0º$C$, the liquid rises to $6 \times 10^{-2}$ $m$ height. The rise of liquid in tube $B$ is only $5.5 \times 10^{-2}$ $m$ when the liquid is at 50º$C$. Find the rate at which the surface tension changes with temperature considering the change to be linear. The density of liquid is $(1/14) \times 10^4$ $kg/m^3$ and the angle of contact is zero. Effect of temperature on the density of liquid and glass is negligible.

Q.11. (i) One end of a uniform glass capillary tube of radius $r = 0.025$ $cm$ is immersed vertically in water to a depth $h = 1cm$. Contact angle is 0°, surface tension of water is $7.5 \times 10^{-2}$ $N/m$, density of water is $\rho = 10^3$ $kg/m^3$ and atmospheric pressure is $P_o = 10^5$ $N/m^2$

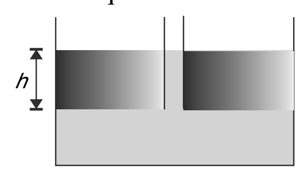



Find the excess pressure to be applied on the water in the capillary tube so that -

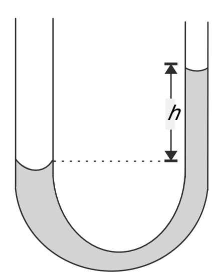
(a) The water level in the tube becomes same as that in the vessel.

(b) Is it possible to blow out an air bubble out of the tube by increasing the pressure?

(ii) A container contains two immiscible liquids of density $\rho_1$ and $\rho_2$ ($\rho_2 > \rho_1$). A capillary of radius $r$ is inserted in the liquid so that its bottom reaches up to denser liquid and lighter liquid does not enter into the capillary. Denser liquid rises in capillary and attain height equal to $h$ which is also equal to column length of lighter liquid. Assuming zero contact angle find surface tension of the heavier liquid.

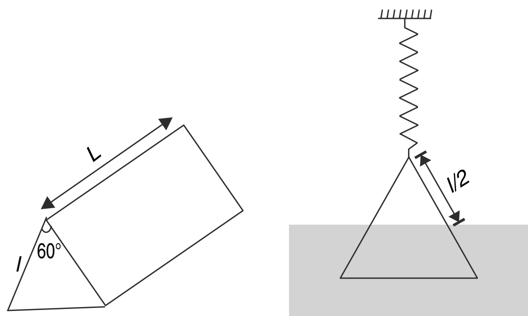



Q.12. The radii of two columns in a $U$ tube are $r_1$ and $r_2$ ($r_1 > r_2$). A liquid of density $\rho$ is filled in it. The contact angle of the liquid with the tube wall is $\theta$. If the surface tension of the liquid is $T$ then plot the graph of the level difference ($h$) of the liquid in the two arms versus contact angle $\theta$. Plot the graph for angle $\theta$ changing from 0° to 90°. Assume the curved surface of meniscus to be part of a sphere.

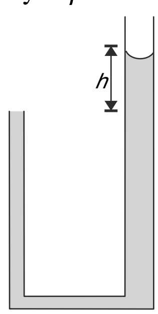


## LEVEL 2

Q.13. A glass prism has its principal section in form of an equilateral triangle of side length $l$. The length of the prism is $L$ (see fig.). The prism, with its base horizontal, is supported by a vertical spring of force constant $k$. Half the slant surface of the prism is submerged in water. Surface tension of water is $T$ and contact angle between water and glass is 0°. Density of glass is $d$ and that of water is $\rho$ ( $< d$). Calculate the extension in the spring in this position of equilibrium.



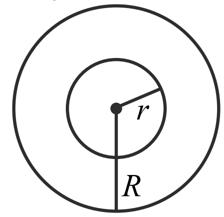

Q.14. Two capillaries of small cross section are connected as shown in the figure. The right tube has cross sectional radius $R$ and left one has a radius of $r$ ($< R$). The tube of radius $R$ is very long where as the tube of radius $r$ is of short length. Water is slowly poured in the right tube. Contact angle for the tube wall and water is $\theta = 0°$. Let $h$ be the height difference between water surface in the right and left tube. Surface tension of water is $T$ and its density is $\rho$.



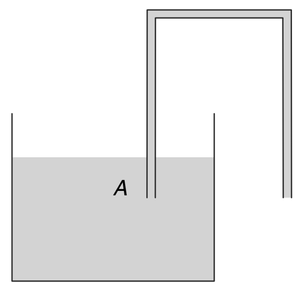

(a) Find the value of $h$ if the water surface in the left tube is found to be flat.

(b) Find the maximum value of $h$ for which water will not flow out of the left tube .

Q.15. A soap bubble of radius $r$ is formed inside another soap bubble of radius $R$ ($> r$). The atmospheric pressure is $P_0$ and surface tension of the soap solution is $T$. Calculate change in radius of the smaller bubble if the outer bubble bursts. Assume that the excess pressure inside a bubble is small compared to $P_0$.




Q.16. In the siphon shown in the figure the ends $A$ and $B$ of the tube are at same horizontal level. Water fills the entire tube but it does not flow out of the end $B$. With the help of a diagram show how the water surface at end $B$ changes if the end $B$ were slightly lower than the position shown.

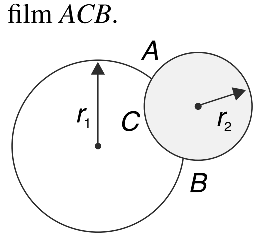



Q.17. A glass capillary tube sealed at the upper end has internal radius $r$. The tube is held vertical with its lower end touching the surface of water. Calculate the length ($L$) of such a tube for water in it to rise to a height $h$ ($< L$). Atmospheric pressure is $P_o$ and surface tension of water is $T$. Assume that water perfectly wets glass (Density of water = $\rho$)

Q.18. In the last question let the length of the tube be $L$ and its outer radius be $R$. Water rises in it to a height $h$. Calculate the vertical force needed to hold the tube in this position. Mass of empty tube is $M$.

Q.19. A glass capillary tube is held vertical and put into contact with the surface of water in a tank. It was observed that the liquid rises to the top of the tube before settling to an equilibrium height $h_0$ in the tube. Assume that water perfectly wets glass and viscosity is small. Is the length of the capillary tube larger then $2h_0$?

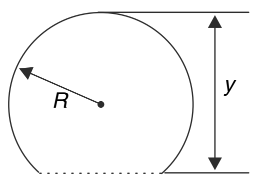
Q.20. Two soap bubbles of radii $r_1$ and $r_2$ are attached as shown. Find the radius of curvature of the common film $ACB$.



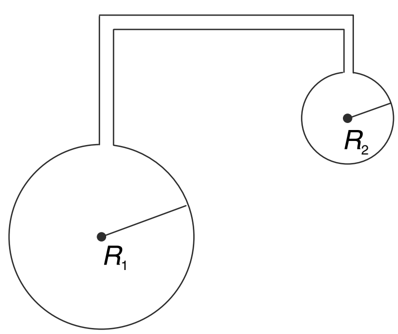
Q.21. (a) In the last question find the angle between the tangents drawn to the bubble surfaces at point *A*.

(b) In the above question assume that $r_1 = r_2 = r$. What is the shape of the common interface *ACB*? Find length *AB* in this case.

(c) With $r_1 = r_2 = r$ the common wall bursts and the two bubbles form a single bubble find the radius of this new bubble. It is given that volume of a truncated sphere of radius *R* and height *y* is $\frac{\pi}{3}y^2(3R - y)$ [see figure]



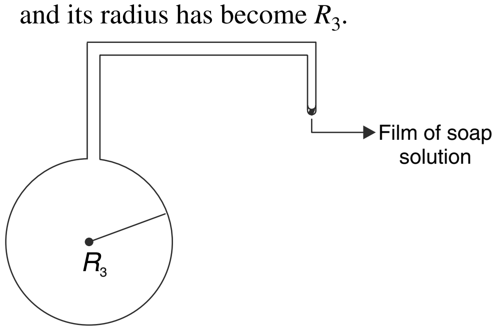

Q.22. Two soap bubbles of radius $R_1$ and $R_2$ ($< R_1$) are joined by a straw. Air flows from one bubble to another and a single bubble of radius $R_3$ remains.




(a) From which bubble does the air flow out ?

(b) Assuming no temperature change and atmospheric pressure to be $P_o$, find the surface tension of the soap solution.

Q.23. In the last problem, one of the bubbles supplies its entire air to the other bubble and a film of soap solution is formed at the end of the straw which keeps it closed. What is the radius of curvature of this film if the bigger bubble has grown in size and its radius has become $R_3$.



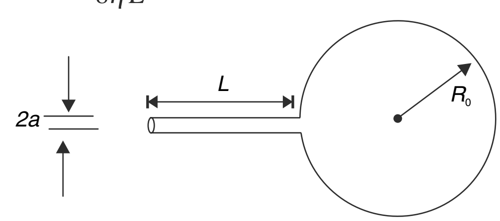

## LEVEL 3

Q.24. Consider a rain drop falling at terminal speed. For what radius (*R*) of the drop can we disregard the influence of gravity on its shape? Surface tension and density of water are *T* and $\rho$ respectively.

Q.25. A soap bubble has radius *R* and thickness of its wall is *a*. Calculate the apparent weight (= true weight - Buoyancy) of the bubble if surface tension of soap solution and its density are *T* and *d* respectively. The atmospheric pressure is $P_0$ and density of atmospheric air is $\rho_0$. By assuming $a = 10^{-6}\ m$, $R = 10\ cm$, $P_0 = 10^5\ Nm^{-2}$, $\rho_0 = 1.2\ kg\ m^{-3}$, $d = 10^3\ kg\ m^{-3}$, $T = 0.04\ Nm^{-1}$; show that the weight of the bubble is mainly because of water in the skin. What is weight of the bubble?

Q.26. A soap bubble is blown at the end of a capillary tube of radius *a* and length *L*. When the other end is left open, the bubble begins to deflate. Write the radius of the bubble as a function of time if the initial radius of the bubble was $R_0$. Surface tension of soap solution is *T*. It is known that volume flow rate through a tube of radius *a* and length *L* is given by Poiseuille's equation-

$$Q = \frac{\pi a^4 \Delta P}{8\eta L}$$



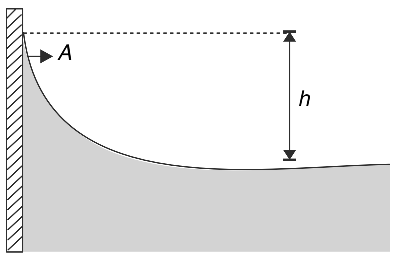

Where $\Delta P$ is pressure difference at the two ends of the tube and $\eta$ is coefficient of viscosity. Assume that the bubble remains spherical.

Q.27. Two blocks are floating in water. When they are brought sufficiently close they are attracted to each other due to surface tension effects. When the experiment is repeated after replacing water with mercury, once again the two blocks are attracted. Explain the phenomena. It is given that water wets the material of the block where as mercury does not.

Q.28. A long thin string has a coat of water on it. The radius of the water cylinder is *r*. After some time it was found that the string had a series of equally spaced identical water drops on it. Find the minimum distance between two successive drops.

Q.29. A liquid having surface tension $T$ and density $\rho$ is in contact with a vertical solid wall. The liquid surface gets curved as shown in the figure. At the bottom the liquid surface is flat.

The atmospheric pressure is $P_o$.

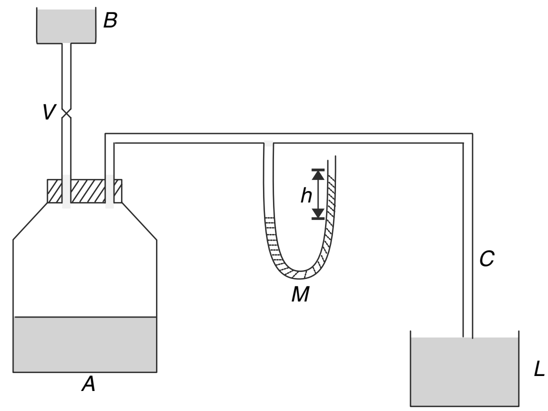


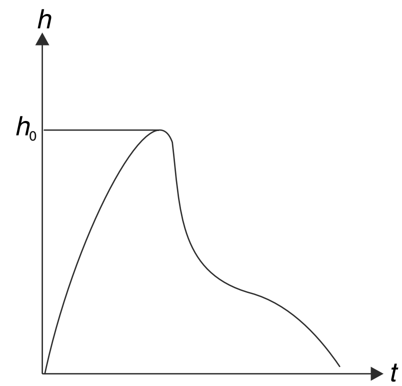

(i) Find the pressure in the liquid at the top of the meniscus (i.e. at $A$)

(ii) Calculate the difference in height ($h$) between the bottom and top of the meniscus.

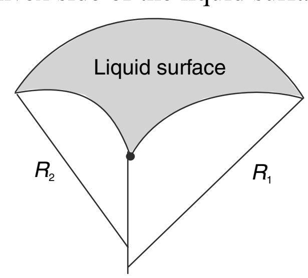
Q.30. Is it possible that water evaporates from a spherical drop of water just by means of surface energy supplying the necessary latent heat of vaporisation? The drop does not use its internal thermal energy and does not receive any heat from outside. It is known that water drops of size less than $10^{-6}$ $m$ do not exist. Latent heat of vaporisation of water is $L = 2.3 \times 10^6$ $Jkg^{-1}$ and surface tension is $T = 0.07$ $Nm^{-1}$.

Q.31. In the arrangement shown in the figure, $A$ is a jar half filled with water and half filled with air. It is fitted with a leak proof cork. $A$ tube connects it to a water vessel $B$. Another narrow tube fitted to A connects it to a narrow tube $C$ via a water monometer $M$. The tip of the tube $C$ is just touching the surface of a liquid $L$. Valve $V$ is opened at time $t = 0$ and water from vessel $B$ pours down slowly and uniformly into the jar $A$. An air bubble develops at the tip of tube $C$. The cross sectional radius of tube $C$ is $r$ and density of water is $\rho$. The difference in height of water ($h$) in the two arms of the manometer varies with time $'t'$ as shown in the graph. Find the surface tension of the liquid $L$.

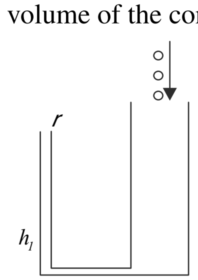



Q.32. A curved liquid surface has radius of curvature $R_1$ and $R_2$ in two perpendicular directions as shown in figure. Surface tension of the liquid is $T$. Find the difference in pressure on the concave side and the convex side of the liquid surface.




Q.33. A capillary tube of radius $r$ and height $h_1$ is connected to a broad tube of large height as shown in the figure. Water is poured into the broad tube – drop by drop. Drops fall at regular intervals. Plot the variation of height of water in both tubes with time. Initially the tube and capillary are empty. Neglect the volume of the connecting pipe.



# ANSWERS

**1.** $0.076\ Nm^{-1}$

**2.** $2\sqrt{3}\pi rT$ + Mg

**3.** (i) $A \rightarrow$ hydrophilic , $B \rightarrow$ hydrophobic

(ii) $\cos^{-1}\left(\frac{R-h}{R}\right)$

**4.** Towards the tapered end.

**5.** $T = \frac{r}{2\cos\theta}\left[\frac{P_\circ h}{\ell - h} + \rho gh\right]$

**6.** (a) To capillary of smaller radius

(b) $\frac{2T\sin\theta(r_2 - r_1)}{r_1 r_2}$

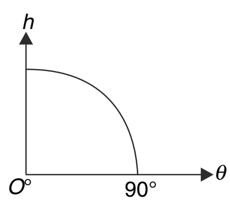
**7.** $15t_0$

**8.** $P_\circ + \frac{4T}{R}$

**10.** $-1.4\times10^{-4}\ \frac{N}{m^\circ C}$

**11.** (i) (a) 600 $P_a$ (b) Yes. (ii) $T = \frac{r}{2}(\rho_2 - \rho_1)gh$

**12.**




**13.** $x = \frac{1}{K}\left[\frac{\sqrt{3}}{4}l^2 L\,d.g - \frac{3\sqrt{3}}{16}l^2\,L\rho g + \sqrt{3}TL + Tl\right]$

**14.** (a) $h = \frac{2T}{R\rho g}$ (b) $\frac{2T}{\rho g}\left(\frac{r+R}{rR}\right)$

**15.** $\Delta r = \frac{4Tr}{3P_0 R}$

**16.** The radius of curvature decreases

**17.** $L = \frac{P_\circ hr}{2T - \rho grh} + h$

**18.** $Mg + \pi P_\circ\left[R^2 - \frac{Lr^2}{L-h}\right] + 2\pi(R+r)T$

**19.** No, $l < 2h_0$

**20.** $\frac{r_1 r_2}{r_1 - r_2}$

**21.** (a) 120° (b) $\sqrt{3}r$ (c) $\frac{3r}{2(2)^{1/3}}$

**22.** (a) From smaller bubble (b) $T = \frac{P_\circ\left(R_3^3 - R_1^3 - R_2^3\right)}{4\left(R_1^2 + R_2^2 - R_3^2\right)}$

**23.** $R_3$

**24.** $R << \sqrt{\frac{T}{\rho g}}$

**25.** $\frac{16\pi}{3}\frac{R^2\rho_\circ}{P_\circ}g + 4\pi R^2 a.d.g$

**26.** $R = R_0\left[1 - \frac{a^4 Tt}{2\eta L R_0^4}\right]^{\frac{1}{4}}$

**28.** $\frac{9}{2}r$

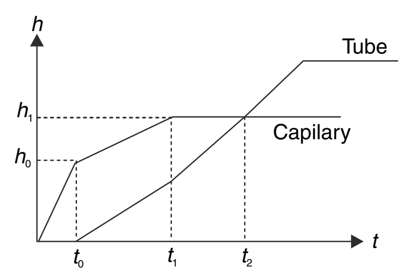
**29.** (i) $P_o - \rho gh$ (ii) $\sqrt{\frac{2T}{\rho g}}$

**30.** No

**31.** $\frac{\rho g h_0 r}{2}$

**32.** $\Delta P = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

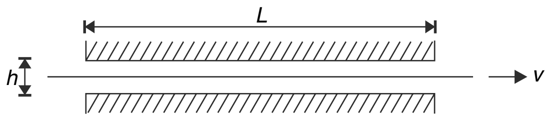
**33.**



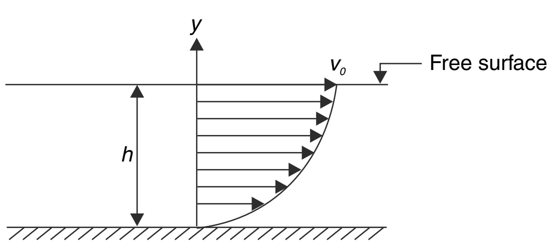
# 10 VISCOSITY

## Level 1

Q.1. During a painting process, a thin, flat tape of width $b$ (dimension perpendicular to the plane of the figure) is pulled through a paint filled channel of length $L$. The density and viscosity of the paint liquid is $\rho$ and $\eta$ respectively. The tape is pulled at a constant speed $v$ and width of the channel is $h$. Find the minimum force needed to pull the tape.




Q.2. A liquid is flowing through a horizontal channel. The speed of flow ($v$) depends on height ($y$) from the floor as $v = v_0\left[2\left(\frac{y}{h}\right) - \left(\frac{y}{h}\right)^2\right]$. Where $h$ is the height of liquid in the channel and $v_0$ is the speed of the top layer. Coefficient of viscosity is $\eta$. Calculate the shear stress that the liquid exerts on the floor.




Q.3. A car having cross sectional area of its front equal to $A$ is travelling on a highway at a speed $v$. The viscous drag force acting on the car is known to be given as $F_v = CA\rho v^2$. Where $\rho$ is density of air and $C$ is a constant which depends on the shape of the car. The petrol used by the car produces $E$ joules of energy per kg of it burnt. Calculate the mileage (in *km/kg*) of the car if the combined efficiency of its engine and transmission is $f$.

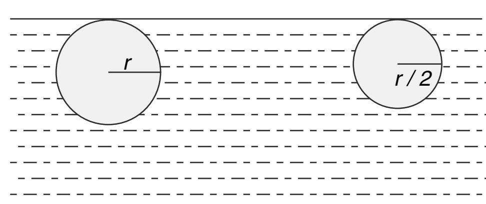
Q.4. An ideal fluid flows through a pipe of circular cross section of radius $r$ at a speed $v_0$. Now a viscous liquid is made to flow through the pipe at the same volume flow rate (measured in $m^3s^{-1}$). Find the maximum speed of a fluid particle in the pipe.

Q.5. A near surface earth satellite is in the shape of a sphere of radius $r$. It encounters cosmic dust in its path. The viscous force experienced by the satellite follows stoke's law. The coefficient of viscosity is $\eta$. Mass and radius of the earth are $M$ and $R$ respectively.

(a) Calculate the power of the rocket engine that must be put on to keep the satellite moving as usual.

(b) Calculate the equilibrium temperature of the surface of the satellite assuming that it radiates like a black body and no outer radiation falls on it. Assume that the heat generated due to viscous force is absorbed completely by the satellite body.

Q.6. Two balls of radii $r$ and $\frac{r}{2}$ are released inside a deep water tank. Their initial accelerations are found to be $\frac{g}{2}$ and $\frac{g}{4}$ respectively. Find the velocity of smaller ball relative to the larger ball, a long time after the two balls are released. Coefficient of viscosity is given to be $\eta$.




Q.7. The coefficient of viscosity $\eta$ of a gas depends on mass of the gas molecule, its effective diameter and its average speed. It is known that diameter of helium atom is $2.1 \times 10^{-10}\ m$ and its coefficient of viscosity, $\eta$ at room temperature is $2.0 \times 10^{-5}$ $kg\ m^{-1}s^{-1}$. Estimate the effective diameter of $CO_2$

molecule if it is known that $\eta$ at room temperature for $CO_2$ is $1.5 \times 10^{-5}\ kg\ m^{-1}s^{-1}$.

Q.8. When hard brakes are applied (so as to lock the wheels) in a car travelling on a wet road it can “hydro-plane”. A film of water is created between the tires and the road and, theoretically, the car can slide a very long distance. [In practice film is destroyed much before such distances can be achieved]. Consider a car of mass $M$ moving on a wet road with speed $v_0$. Hard brakes are applied. Let the area of film under all four tires be $A$ and thickness of the film be $h$. Coefficient of viscosity is $\eta$.

(a) Calculate the distance ($x$) to which the car will slide before coming to rest.

(b) Calculate the value of $x$ for $M = 10^3\ kg$, $A = 0.2\ m^2$, $h = 0.1\ mm$, $v_0 = 20\ ms^{-1}$, and $\eta = 10^{-3}\ kg\ m^{-1}$

## LEVEL 2

Q.9. A spherical ball of radius $r$ and density $d$ is dropped from rest in a viscous fluid having density $\rho$ and coefficient of viscosity $\eta$.

(a) Calculate the power ($P_1$) of gravitational force acting on the ball at a time $t$ after it is dropped.

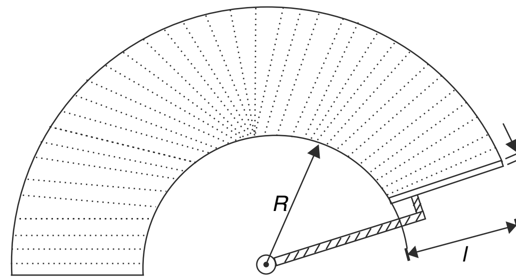
(b) Calculate the rate of heat generation ($P_2$) due to rubbing of fluid molecules with the ball, at time $t$ after it is dropped.

(c) How do $P_1$ and $P_2$ change if the radius of the ball were doubled?

(d) Find $P_1$ and $P_2$ when both become equal.

Q.10. Two balls of same material of density $\rho$ but radius $r_1$ and $r_2$ are joined by a light inextensible vertical thread and released from a large height in a medium of coefficient of viscosity = $\eta$. Find the terminal velocity acquired by the balls. Also find the tension in the string connecting both the balls when both of them are moving with terminal velocity. Neglect buoyancy and change in acceleration due to gravity.

Q.11. A car windshield wiper blade sweeps the wet windshield rotating at a constant angular speed of $\omega$. $R$ is the radius of innermost arc swept by the blade. Length and width of the blade are $l$ and $b$ respectively. Coefficient of viscosity of water is $\eta$. Calculate the torque delivered by the motor to rotate the blade assuming that there is a uniform layer of water of thickness $t$ on the glass surface.




## LEVEL 3

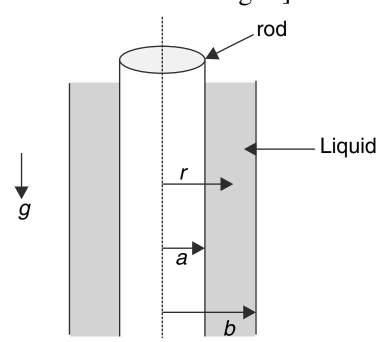
Q.12. A vertical steel rod has radius $a$. The rod has a coat of a liquid film on it. The liquid slides under gravity. It was found that the speed of liquid layer at radius $r$ is given by

$$v = \frac{\rho g b^2}{2\eta}\ln\left(\frac{r}{a}\right) - \frac{\rho g}{4\eta}\left(r^2 - a^2\right)$$

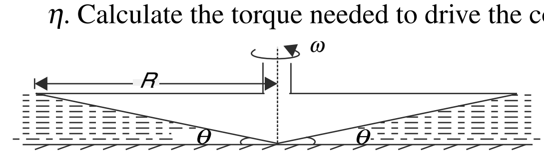
Where $b$ is the outer radius of liquid film, $\eta$ is coefficient of viscosity and $\rho$ is density of the liquid.

(i) Calculate the force on unit length of the rod due to the viscous liquid?

(ii) Set up the integral to calculate the volume flow rate of the liquid down the rod. [you may not evaluate the integral]




Q.13. A viscometer (an instrument used to study characteristics of a non-ideal fluid) consists of a flat plate and a rotating cone. The cone has a large apex angle and the angle $\theta$ shown in figure is very small (typically less than 0.5° ). The apex of the cone just touches the plate and a liquid fills the narrow gap between the plate and the cone. The cone has a base radius $R$ and is rotated with constant angular speed $\omega$. Consider the liquid to be ideal and take its coefficient of viscosity to be $\eta$. Calculate the torque needed to drive the cone.



# ANSWERS

1. $\dfrac{4\eta vLb}{h}$

2. $\dfrac{2\eta v_0}{h}$

3. $\dfrac{fE}{CA\rho v^2}$

4. $\dfrac{3}{2}v_0$

5. (a) $\dfrac{6\pi GM\eta r}{R}$ (b) $\left[\dfrac{3\eta GM}{2\sigma rR}\right]^{1/4}$

6. $\dfrac{11}{54}\dfrac{r^2\rho g}{\eta}$ upwards

7. $4.4\times10^{-10}\,m$

8. (a) $x=\dfrac{Mhv_0}{\eta A}$ (b) 10 $km$

9. (a) $P_1=\dfrac{8\pi}{27}\dfrac{d(d-\rho)g^2r^5}{\eta}\left[1-e^{-\frac{9\eta t}{2dr^2}}\right]$

(b) $P_1=\dfrac{8\pi}{27}\dfrac{(d-\rho)^2g^2r^5}{\eta}\left[1-e^{-\frac{9\eta t}{2dr^2}}\right]$

(c) $P_1$ and $P_2$ become 32 times.

(d) $P_1=P_2=\dfrac{8\pi}{27}\dfrac{(d-\rho)^2g^2r^5}{\eta}$

10. $\dfrac{2}{9}\dfrac{\rho g}{\eta}\left[r_1^2-r_1r_2+r_2^2\right]$ ; $\dfrac{4}{3}\pi\rho g\left|r_1^2r_2-r_2^2r_1\right|$

11. $\dfrac{\eta b\omega R^3}{3t}\left[\left(1+\dfrac{L}{R}\right)^3-1\right]$

12. (i) $\pi\rho ga^2\left[\left(\dfrac{b}{a}\right)^2-1\right]$ (ii) $Q=\displaystyle\int_a^b v\cdot2\pi r\,dr$

13. $\dfrac{2\pi\eta\omega R^3}{3\sin\theta}\simeq\dfrac{2\pi\eta\omega R^3}{3\theta}$

# SOLUTIONS

1. The paint layer in contact with channel wall is at rest and that in contact with the tape is $v$. The viscous force acts on two surfaces of the tape. If gap between the tape and upper surface of the channel is $x$ then velocity gradient at the two surfaces of the tape is

$$\left(\frac{dv}{dh}\right)_{upper}=\frac{v}{x}\text{ and }\left(\frac{dv}{dh}\right)_{lower}=\frac{v}{h-x}$$

Total viscous force on the tape is $F_{vis}=F_{upper}+F_{lower}$

$$=\eta(bL)\cdot\frac{v}{x}+\eta(bL)\cdot\frac{v}{h-x}=\eta bLv\left[\frac{h}{x(h-x)}\right]$$

This force is minimum when $x(h-x)$ is maximum, i.e., when $x=\dfrac{h}{2}$

$$\therefore\ F_{\min}=\frac{4\eta bLv}{h}$$

2. Shear stress is tangential force applied by the liquid on unit area of the floor.

Velocity gradient $=\dfrac{dv}{dy}=\dfrac{2v_0}{h}-\dfrac{2v_0}{h^2}y$

# 11 ELASTICITY

## LEVEL 1

Q.1. Human bones remain elastic if strain is less than 0.5%. However, the young's modulus for compression ($Y_c$) and stretch ($Y_s$) are different. The typical values are $Y_c = 9.4 \times 10^9$ *Pa* and $Y_s = 16 \times 10^9$ *Pa*. The shear modulus of elasticity for the bone is $\eta = 10^{10}$ *Pa*

Answer following questions with regard to a leg bone of length 20 *cm* and cross sectional area 3 $cm^2$

(a) Calculate the maximum stretching force that the bone can sustain and still remain elastic.

(b) A man of mass 60 *kg* jumps from a height of 10 *m* on a concrete floor. Half his momentum is absorbed by the impact of the floor on the particular bone we are talking about. The impact lasts for 0.02 *s*. Will the compressive stress exceed the elastic limit?

(c) How much shearing force will be needed to break the bone if breaking strain is 5°.

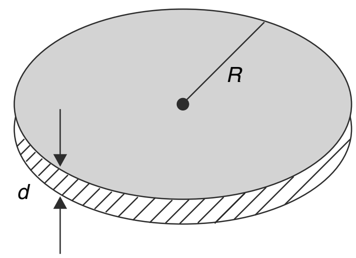
Q.2. The elastic limit and ultimate strength for steel is $2.48 \times 10^8$ *Pa* and $4.89 \times 10^8$ *Pa* respectively. A steel wire of 10 *m* length and 2 *mm* cross sectional diameter is subjected to longitudinal tensile stress. Young's modulus of steel is $Y = 2 \times 10^{11}$ *Pa*

(a) Calculate the maximum elongation that can be produced in the wire without permanently deforming it. How much force is needed to produce this extension?

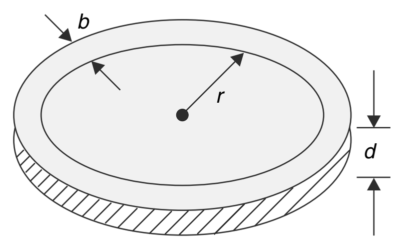
(b) Calculate the maximum stretching force that can be applied without breaking the wire.

Q.3. A steel ring is to be fitted on a wooden disc of radius *R* and thickness *d*. The inner radius of the ring is *r* which is slightly smaller than *R*. The outer radius of the ring is $r + b$ and its thickness is *d* (same as the disc). There is no change in value of *b* and *d* after the ring is fitted over the disc; only the inner radius becomes *R*. If the Young's modulus of steel is *Y*, calculate the longitudinal stress developed in it. Also calculate the tension force developed in the ring.

[Take $b << r$]

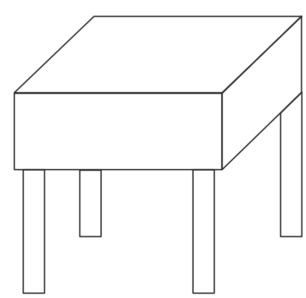



Disc



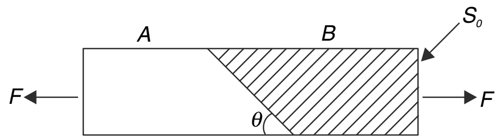

Ring

Q.4. A water tank is supported by four pillars. The pillars are strong enough to sustain ten times the stress developed in them when the tank is completely full. An engineer decides to increase the every dimension of the tank and the pillars by hundred times so as to store more water. Do you think he has taken a right decision? Assume that material used in construction of the tank and pillars remain same.

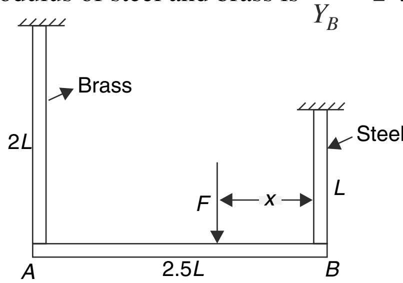


Q.5. Two bars *A* and *B* are stuck using an adhesive. The contact surface of the bars make an angle

$\theta$ with the length. Area of cross section of each bar is $S_0$. It is known that the adhesive yields if normal stress at the contact surface exceeds $\sigma_0$. Find the maximum pulling force $F$ that can be applied without detaching the bars.




Q.6. A very stiff bar *(AB)* of negligible mass is suspended horizontally by two vertical rods as shown in figure. Length of the bar is 2.5 *L*. The steel rod has length *L* and cross sectional radius of *r* and the brass rod has length 2*L* and cross sectional radius of 2*r*. A vertically downward force *F* is applied to the bar at a distance *x* from the steel rod and the bar remains horizontal. Find the value of *x* if it is given that ratio of Young's modulus of steel and brass is $\frac{Y_s}{Y_B} = 2$ .




Q.7. A closed steel cylinder is completely filled with water at 0°*C*. The water is made to freeze at 0°*C*. Calculate the rise in pressure on the cylinder wall. It is known that density of water at 0°*C* is 1000 $kg/m^3$ and the density of ice at 0°*C* is 910 $kg/m^3$. Bulk modulus of ice at 0°*C* is nearly $9 \times 10^9$ *Pa*. [Compare this pressure to the atmospheric pressure. Now you can easily understand why water pipelines burst in cold regions as the winter sets in.]

Q.8. (i) Two identical rods, one of steel, the other of copper, are stretched by an identical amount. On which operation more work is expended?

(ii) Two identical rods, one of steel, the other of copper, are stretched with equal force. On which operation is more work needed?

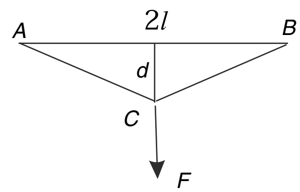
## LEVEL 2

Q.9. A thin ring of radius *R* is made of a wire of density $\rho$ and Young's modulus *Y*. It is spun in its own plane, about an axis through its centre, with angular velocity $\omega$. Determine the amount (assumed small) by which its circumference increases.

Q.10. A steel wire of radius *r* is stretched without tension along a straight line with its ends fixed at *A* and *B* (figure). The wire is pulled into the shape *ACB*. Assume that *d* is very small compared to length of the wire. Young's modulus of steel is *Y*.

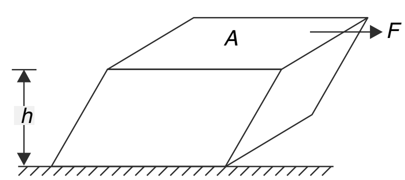
(a) What is the tension (*T*) in the wire?

(b) Determine the pulling force *F*. Is *F* larger than *T*?

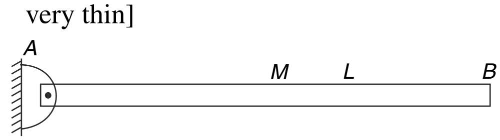



Q.11. A uniform material rod of length *L* is rotated in a horizontal plane about a vertical axis through one of its ends. The angular speed of rotation is $\omega$. Find increase in length of the rod. It is given that density and Young's modulus of the rod are $\rho$ and *Y* respectively.

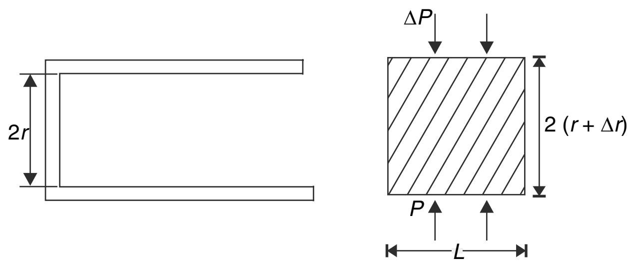
Q.12. A rectangular bar is fixed to a hard floor. Height of the bar is *h* and its area in contact with the floor is *A*. A shearing force distorts the bar as shown. Prove that the work done by the shearing force is $W = \left(\frac{\sigma^2}{2\eta}\right) \times$ volume of the bar. Here $\sigma$ is shear modulus of elasticity. Assume the deformation to be small.




Q.13. A thin uniform rod of mass *M* and length *L* is free to rotate in vertical plane about a horizontal axis passing through one of its ends. The rod is released from horizontal position shown in the figure. Calculate the shear stress developed at the centre of the rod immediately after it is released. Cross sectional area of the rod is *A*. [For calculation of moment of inertia you can treat it to very thin]



Q.14. A rigid cylindrical container has inner radius $r$. A cork having radius $r + \Delta r$ and length $L$ is to be fitted so as to close the container. Uniform pressure ($\Delta P$) is needed on the curved cylindrical surface of the cork. Poisson's ratio of a cork is almost zero, and its bulk modulus is $B$.




(a) Calculate $\Delta P$

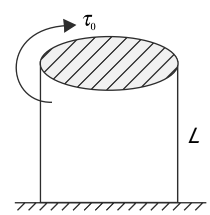
(b) After the cork is fitted how much force will be needed to pull it out of the container? Coefficient of friction between the container and the cork is $\mu$.

Q.15. Assume that the least load which would break a thread when simply suspended from it is $M$ and that this load produces a strain of 1 percent at the moment of breaking. Also assume that Hooke's law applies to the thread right up to breaking-point. A load of mass m is suspended from a thread of length $\lambda$. It is raised to a height and released. Find the least height to which the load must be raised so that it will break the thread when allowed to fall.

Q.16. Atmospheric pressure is $P_0$ and density of water at the sea level is $\rho_0$. If the bulk modulus of water is $B$, calculate the pressure deep inside the sea at a depth $h$ below the surface.

## LEVEL 3

Q.17. A metal cylinder of length $L$ and radius $R$ is fixed rigidly to ground with its axis vertical. A twisting torque $\tau_0$ is applied along the circumference at the top of the cylinder. This causes an angular twist of $\theta_0$ (rad) in the top surface. Calculate the shear modulus of elasticity ($\eta$) of the material of the cylinder.




## ANSWERS

1. (a) $2.4 \times 10^4\ N$ (b) Yes (c) $2.6 \times 10^5\ N$
2. (a) 1.24 $cm$, 779 $N$ (b) 1535 $N$
3. $Y\left(\dfrac{R-r}{r}\right)$; $Ybd\left(\dfrac{R-r}{r}\right)$
4. No
5. $\dfrac{\sigma_0 S_0}{\sin^2\theta}$
6. $x = 1.25\ L$
7. $8.1 \times 10^8\ Pa$
8. (i) Stretching of steel (ii) Stretching of Copper
9. $\dfrac{2\pi\rho R^3\omega^2}{Y}$
10. (a) $T = Y\pi r^2 (d^2/2l^2)$ (b) $2T(d/l)$ where $T$ is given in answer (a). No $F$ is much smaller.
11. $\dfrac{\rho\omega^2 L}{3Y}$
13. $\dfrac{Mg}{16A}$
14. (a) $\Delta P = \dfrac{2B\Delta r}{r}$ (b) $F = 4\pi\mu BL\Delta r$
15. $h = \dfrac{0.01M\ell}{2m}$.
16. $P = P_0 - B\ln\left(1 - \dfrac{\rho_0 gh}{B}\right)$
17. $\dfrac{2l\tau_0}{\pi R^4\theta_0}$

# 12 SIMPLE HARMONIC MOTION

## Level 1

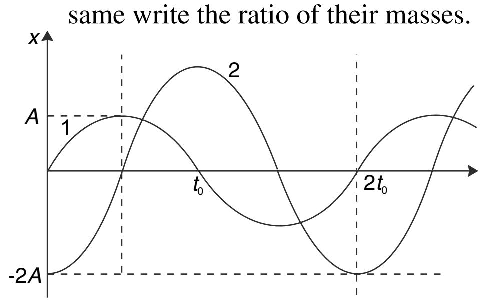
Q. 1. (i) The acceleration ($a$) of a particle moving along a straight line is related to time ($t$) as per the differential equation $\frac{d^2a}{dt^2} = -ba$. $b$ is a positive constant. Is the particle performing SHM? If yes, what is the time period?

(ii) A particle is executing SHM on a straight line. $A$ and $B$ are two points at which its velocity is zero. It passes through a certain point $P$ ($AP < PB$) with a speed of 3 $m/s$ at times recorded as $t$ = 0, 0.5 $s$, 2.0 $s$, 2.5 $s$, 4.0 $s$, 4.5 $s$......Determine the maximum speed of the particle and also the ratio $AP/PB$.

Q. 2. The position – time graph for two particles- 1 and 2- performing SHM along $X$ axis has been shown in the fig.

(a) Write the velocity of the two particles as a function of time.

(b) If the energy of SHM for the two particles is same write the ratio of their masses.




Q. 3. A particle moves along $X$ axis such that its acceleration is given by $a = -\beta(x-2)$, where $\beta$ is a positive constant and $x$ is the position co-ordinate.

(a) Is the motion simple harmonic?

(b) Calculate the time period of oscillations.

(c) How far is the origin of co-ordinate system from the equilibrium position?

Q. 4. A particle is performing simple harmonic motion along the $x$ axis about the origin. The amplitude of oscillation is $a$. A large number of photographs of the particle are shot at regular intervals of time with a high speed camera. It was found that photographs having the particle at $x_1 + \Delta x$ were maximum in number and photographs having the particle at $x_2 + \Delta x$ were least in number. What are values of $x_1$ and $x_2$?

Q. 5. Position vector of a particle as a function of time is given by

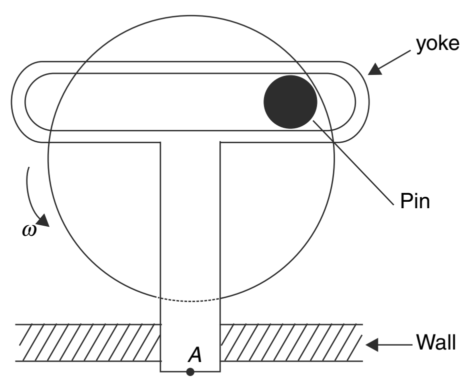
$$\vec{R} = (a \sin \omega t)\,\hat{i} + (a \cos \omega t)\,\hat{j} + (b \sin \omega_0 t)\,\hat{k}$$

The particle appears to be performing simple harmonic motion along $z$ direction, to an observer moving in $xy$ plane.

(a) Describe the path of the observer.

(b) Write the distance travelled by the observer himself in the time interval he sees the particle completing one oscillation.

Q. 6. A wheel is revolving at an angular speed of $\omega$. A pin welded at the circumference of the wheel forces a $T$ shaped body to move up and down. The pin slides freely inside the slot of the yoke as the wheel rotates. The $T$ shaped body is constrained to move vertically by a set of walls.



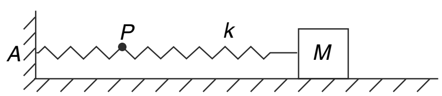

(a) Find the time period of oscillatory motion of point $A$ at the base of the $T$ shaped body

(b) Is the motion of $A$ simple harmonic?

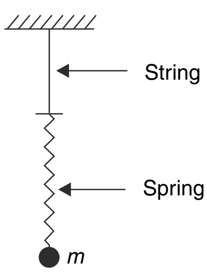
Q. 7. (i) A particle is performing simple harmonic motion with time period $T$. At an instant its speed is 60% of its maximum value and is increasing. After an interval $\Delta t$ its speed becomes 80% of its maximum value and is decreasing. Find the smallest value of $\Delta t$ in terms of $T$.

(ii) A particle is doing SHM of amplitude 0.5 m and period $\pi$ seconds. When in a position of instantaneous rest, it is given an impulse which imparts a velocity of 1 $m/s$ towards the equilibrium position. Find the new amplitude of oscillation and find how much less time will it take to arrive at the next position of instantaneous rest as compared to the case if the impulse had not been applied.

Q. 8. A block of mass $M$ is tied to a spring of force constant $k$ and placed on a smooth horizontal surface. The natural length of the spring is $L$. $P$ is a point on the spring at a distance $\frac{L}{4}$ from its fixed end. The block is set in oscillations with amplitude $A$. Find the maximum speed of point $P$ on the spring.



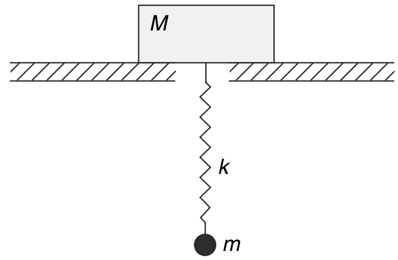

Q. 9. A particle of mass $m$ is suspended with the help of a spring and an inextensible string as shown in the figure. Force constant of the spring is $k$. The particle is pulled down from its equilibrium position by a distance $x$ and released.




(a) Find maximum value of $x$ for which the motion of the particle will remain simple harmonic.

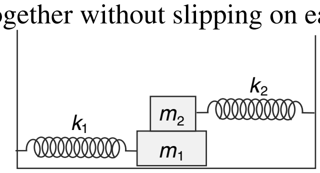
(b) Find maximum tension in the string if $x = \frac{mg}{2k}$.

Q. 10. A block of mass $M$ is placed on top of a hole in a horizontal table. A spring of force constant $k$ is connected to the block through the hole. The other end of the massless spring has a particle of mass $m$ connected to it. With what maximum amplitude can the particle oscillate up and down such that the block does not lose contact with the table?




Q. 11. A block of mass $m$ is moving along positive $x$ direction on a smooth horizontal surface with velocity $u$. It enters a rough horizontal region at $x = 0$. The coefficient of friction in this rough region varies according to $\mu = ax$, where '$a$' is a positive constant and $x$ is displacement of the block in the rough region. Find the time for which the block will slide in this rough region.

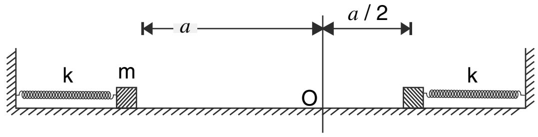
Q. 12. (i) In the shown arrangement, both springs are relaxed. The coefficient of friction between $m_2$ and $m_1$ is $\mu$. There is no friction between $m_1$ and the horizontal surface. The blocks are displaced slightly and released. They move together without slipping on each other.




(a) If the small displacement of blocks is $x$ then find the magnitude of acceleration of $m_2$. What is time period of oscillations?

(b) Find the ratio $\frac{m_1}{m_2}$ so that the frictional force on $m_2$ acts in the direction of its displacement from the mean position.

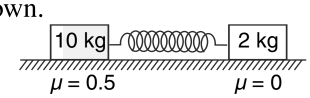
(ii) Two small blocks of same mass $m$ are connected to two identical springs as shown in fig. Both springs have stiffness $K$ and they are in their natural length when the blocks are at point $O$. Both the blocks are pushed so that one of the springs get compressed by a distance $a$ and the other by $a/2$. Both the blocks are released from this position simultaneously. Find the time period of oscillations of the blocks if - (neglect the dimensions of the blocks)




(a) Collisions between them are elastic.

(b) Collisions between them are perfectly inelastic.

Q. 13. Two blocks of mass 10 *kg* and 2 *kg* are connected by an ideal spring of spring constant $K = 800$ *N/m* and the system is placed on a horizontal surface as shown.

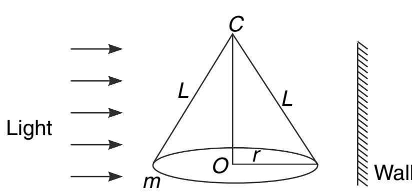



The coefficient of friction between 10 *kg* block and surface is 0.5 but friction is absent between 2 *kg* and the surface. Initially blocks are at rest and spring is relaxed. The 2 *kg* block is displaced to elongate the spring by 1 *cm* and is then released.

(a) Will 10 *kg* block move subsequently?

(b) Draw a graph representing variation of magnitude of frictional force on 10 *kg* block with time. Time $t$ is measured from that instant when 2 *kg* block is released to move.

Q.14. A particle of mass $m$ is tied at the end of a light string of length $L$, whose other end is fixed at point $C$ (fig), and is revolving in a horizontal circle of radius $r$ to form a conical pendulum. A parallel horizontal beam of light forms shadow of the particle on a vertical wall.

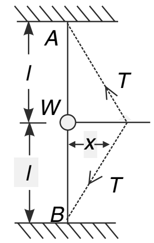



If the tension in the string is $F$ find -

(a) The maximum acceleration of the shadow moving on the wall.

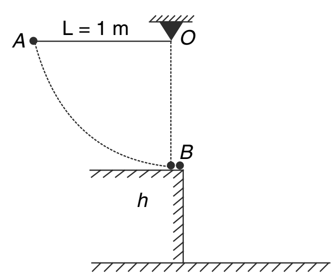
(b) The time period of the shadow moving on the wall.

Q.15. A small ball of mass $m$ is attached to the middle of a tightly stretched perfectly flexible wire $AB$ of length 2 $\ell$ (**figure**). The ball is given a small lateral displacement in horizontal direction and released. The initial tension ($T$) in the wire is high and change in it due to small lateral displacement of the ball can be neglected. Prove that the ball will perform simple harmonic motion, and calculate the period. If there is a device which can change the tension in the wire at will , how will the time period change if tension in the wire is increased?



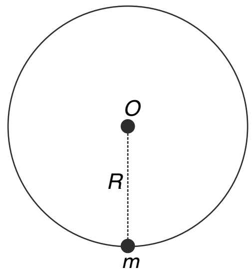

Q. 16. A simple pendulum oscillating with a small amplitude has a time period of $T$ = 1.0$s$. A horizontal thin rod is now placed beneath the point of suspension at a distance equal to half the length of the pendulum. The string collides with the rod once in each oscillation and there is no loss of energy in such collisions. Find the new time period $T'$ of the pendulum.

Q. 17. (i) A small steel ball ($B$) is at rest on the edge of a table of height $h$. Another identical steel ball ($A$) is tied to a light string of length $L$ =1.0 $m$ and is released from the position shown so that it swings like a pendulum. At the lowest position of its path it hits the ball $B$ which is at rest. Ball $B$ flies off the table and hits the ground in time $t$. After collision the ball $A$ keeps moving for a time $t'$ before coming to rest for the first time. Find the value of $h$ if $t = t'$. Collision between the balls is head on and coefficient of restitution is $e = 0.995$.



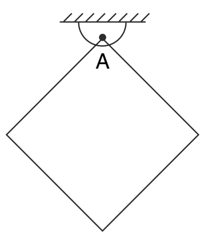

(ii) A pendulum has a particle of mass $m$ attached to a massless rod of length $L$. The rod is released from a position where it makes an angle $\theta_0 \left( < \frac{\pi}{2} \right)$ with the vertical. The time period of oscillation is observed to be $T_0$. Another similar pendulum has a rod of length 2$L$. Time period of this pendulum when released from position $\theta_0$ is $T$. Which is larger $T$ or $T_0$?

Q. 18. A disc of mass $M = 2m$ and radius $R$ is pivoted at its centre. The disc is free to rotate in the vertical plane about its horizontal axis through its centre $O$. A particle of mass $m$ is stuck on the periphery of the disc. Find the frequency of small oscillations of the system about its equilibrium position.




Q. 19. A rigid body is to be suspended like a physical pendulum so as to have a time period of $T = 0.2\pi$ second for small amplitude oscillations. The minimum distance of the point of suspension from the centre of mass of the body is $l_1 = 0.4\ m$ to get this time period. Find the maximum distance $(l_2)$ of a point of suspension from the centre of mass of the body so as to get the same time period. $[g = 10\ m/s^2]$

Q. 20. A square plate of mass $M$ and side length $L$ is hinged at one of its vertex ($A$) and is free to rotate about it. Find the time period of small oscillations if




(a) the plate performs oscillations in the vertical plane of the figure. (Axis is perpendicular to figure.)

(b) the plate performs oscillations about a horizontal axis passing through $A$ lying in the plane of the figure.

## LEVEL 2

**Q. 21**. Two particles $A$ and $B$ are describing SHM of same amplitude ($a$) and same frequency ($f$) along a common straight line. The mean positions of the two SHMs are also same but the particles have a constant phase difference between them. It is observed that during the course of motion the separation between $A$ and $B$ is always less than or equal to $a$.

(a) Find the phase difference between the particles.

(b) If distance between the two particles is plotted with time, with what frequency will the graph oscillate?

Q. 22. (i) A particle of mass $m$ executes SHM in $xy$-plane along a straight line $AB$. The points $A$ $(a, a)$ and $B$ $(-a, -a)$ are the two extreme positions of the particle. The particle takes time $T$ to move from one extreme $A$ to the other extreme $B$. Find the $x$ component of the force acting on the particle as a function of time if at $t = 0$ the particle is at $A$.

(ii) Two particle $A$ and $B$ are performing SHM along $X$-axis and $Y$-axis respectively with equal amplitude and frequency of 2 $cm$ and 1 Hz respectively. Equilibrium positions for the particles $A$ and $B$ are at the coordinate (3, 0) and (0, 4) respectively. At $t = 0$, $B$ is at its equilibrium position and moving toward the origin, while $A$ is nearest to the origin. Find the maximum and minimum distances between $A$ and $B$ during their course of motion.

Q. 23. A particle is performing SHM along $x$ – axis and equation for its motion is $x = a \cos(\pi t)$

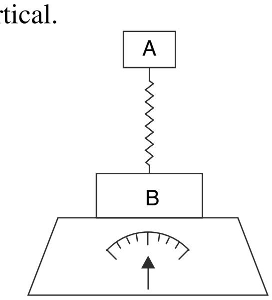
Let the time $t$ be expressed as $\frac{t}{2} = n + m$

Where $n = 0,1,2,3,4....$ and $m$ is a positive fraction.

Calculate the distance travelled by the particle during the interval from t = 0 to $t = t$ if

(a) $m < 0.5$ (b) $m > 0.5$

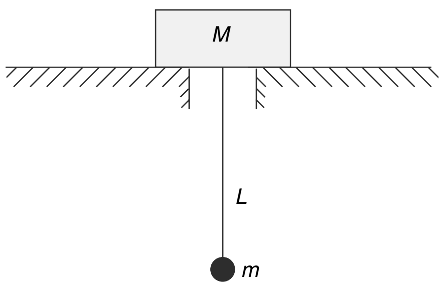
Q. 24. Two blocks $A$ and $B$ having mass $m = 1\ kg$ and $M = 4\ kg$ respectively are attached to a spring and placed vertically on a weighing machine as shown in the figure. Block $A$ is held so that the spring is relaxed. $A$ is released from this position and it performs simple harmonic motion with angular frequency $\omega = 25\ rad\ s^{-1}$. The spring remains vertical.



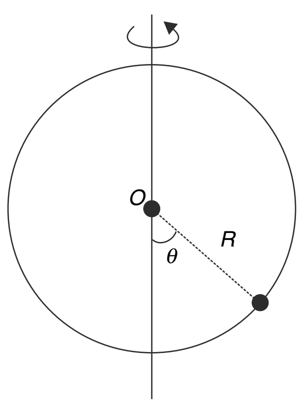
(a) Find the reading of the weighing machine as a function of time. Take $t = 0$ when $A$ is released.

(b) What is the maximum reading of the weighing machine?

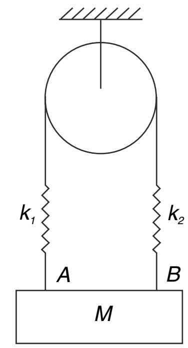
Q. 25. A block of mass $M$ rests on a smooth horizontal table. There is a small gap in the table under the block through which a pendulum has been attached to the block. The bob of the simple pendulum has mass $m$ and length of the pendulum is $L$. The pendulum is set into small oscillations in the vertical plane of the figure. Calculate its time period. The table does not interfere with the motion of the string.



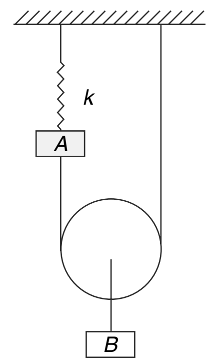

Q. 26. A circular wire frame of radius $R$ is rotating about its fixed vertical diameter. A bead on the wire remains at rest relative to the wire at a position in which the radius makes an angle $\theta$ with the vertical (see figure). There is no friction between the bead and the wire frame. Prove that the bead will perform SHM (in the reference frame of the wire) if it is displaced a little from its equilibrium position. Calculate the time period of oscillation.



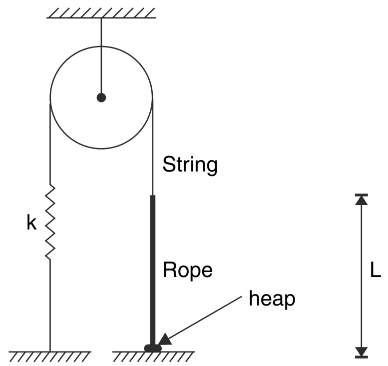

Q. 27. In the system shown in the figure the string, springs and pulley are light. The force constant of the two springs are $k_1 = k$ and $k_2 = 2k$. Block of mass $M$ is pulled vertically down from its equilibrium position and released. Calculate the angular frequency of oscillation. The top surface of the block (represented by line $AB$) always remains horizontal.




Q. 28. (i) In the system shown in figure, find the time period of vertical oscillations of the block $A$. Both the blocks $A$ and $B$ have equal mass of $m$ and the force constant of the ideal spring is $k$. Pulley and threads are massless.

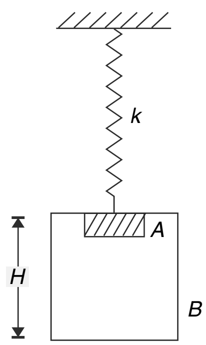



(ii) In the arrangement shown in the figure the spring, string and the pulley are mass less. The force constant of the spring is $k$. A rope of mass per unit length equal to $\lambda$ ($kg\ m^{-1}$) hangs from the string as shown. In equilibrium a length $L$ of the rope is in air and its bottom part lies in a heap on the floor. The rope is very thin and size of the heap is negligible though the heap contains a fairly long length of the rope. The rope is raised by a very small distance and released. Show that motion will be simple harmonic and calculate the time period. Assume that the hanging part of the rope does not experience any force from the heap or the floor (For example there is no impact force while the rope hits the floor while moving downward and there is no impulsive pull when the vertical part jerks a small element of heap into motion).

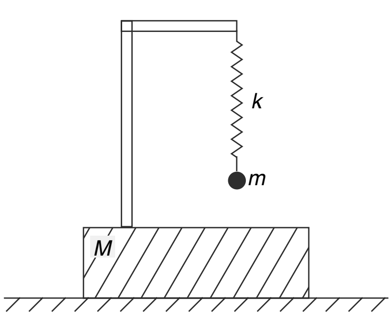



Q. 29. A box $B$ of mass $M$ hangs from an ideal spring of force constant $k$. A small particle, also of mass $M$, is stuck to the ceiling of the box and the system is in equilibrium. The particle gets detached from the ceiling and falls to strike the floor of the box. It takes time '$t$' for the particle to hit the floor after it gets detached from the ceiling. The particle, on hitting the floor, sticks to it and the system thereafter oscillates with a time period $T$. Find the height $H$ of the box if it is given that $t = \frac{T}{6\sqrt{2}}$.

Assume that the floor and ceiling of the box always remain horizontal.



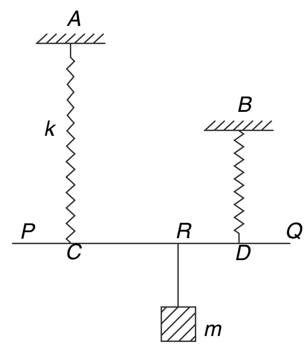

Q. 30. A block has a $L$ shaped stand fixed to it. Mass of the block with the stand is $M$. At the free end of the stand there is a spring which carries a ball of mass $m$. With the spring in its natural length, the ball is released. It begins to oscillate and the stand is tall enough so that the ball does not hit the block.




(a) Find maximum value of mass ($m$) of the ball for which the block will not lose contact with the ground?

(b) If the stand is not tall enough and the ball makes elastic impact with the block, will your answer to part (a) change?

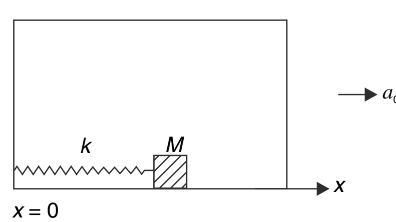
Q. 31. Two ideal springs of same make (the springs differ in their lengths only) have been suspended from points $A$ and $B$ such that their free ends $C$ and $D$ are at same horizontal level. A massless rod $PQ$ is attached to the ends of the springs. A block of mass $m$ is attached to the rod at point $R$. The rod remains horizontal in equilibrium. Now the block is pulled down and released. It performs vertical oscillations with time period $T = 2\pi\sqrt{\frac{m}{3k}}$ where $k$ is the force constant of the longer spring.




(a) Find the ratio of length $RC$ and $RD$.

(b) Find the difference in heights of point $A$ and $B$ if it is given that natural length of spring $BD$ is $L$.

Q. 32. A block of mass $M$ connected to an ideal spring of force constant $k$ lies in equilibrium on the smooth floor of a room. The other end of the spring is fixed to the left wall of the room. The room begins to move to the right with a constant acceleration $a_0$. In the reference frame of the room the block begins to perform simple harmonic motion.



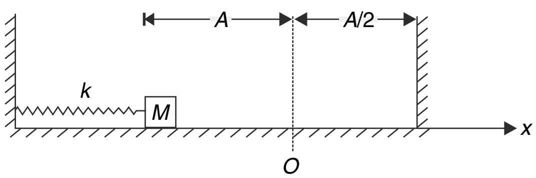

At a certain instant (say $t = t_0$) when the block was at its left extreme, the acceleration of the room vanishes. Plot the $x - t$ graph for the block taking time $t = 0$ when the room started accelerating.

Show the graph till time $t_0$ and beyond that. Take origin to be at the left wall and positive $x$ direction towards right (as shown in figure). Assume no collision of the block with walls.

Q. 33. A block of mass $M$ connected to an ideal spring of force constant $k$, is placed on a smooth surface. The block is pushed to the left so as to compress the spring by a length $A$ and then it is released. The block hits an elastic wall at a distance $\frac{3A}{2}$ from its point of release. Assume the collision to be instantaneous.

(a) Calculate the time required by the block to complete one oscillation

(b) Draw the velocity – time graph for one oscillation of the block.




(c) Find the value of $k$ for which average force experienced by the wall due to repeated hitting of the block is $F_0$.

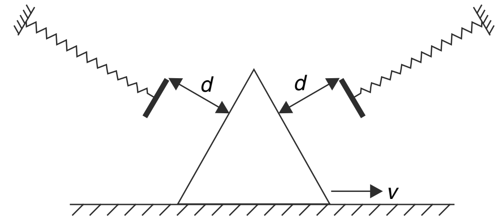
Q. 34. A particle of mass $m$ is constrained to move along a straight line. $A$ and $B$ are two fixed points on the line at a separation of $L$. When the particle is at some point $P$, between $A$ and $B$, it is acted upon by two forces

$$\overrightarrow{F_1} = \left(\frac{6mg}{L}\right)\overrightarrow{PA} \text{ and } \overrightarrow{F_2} = \left(\frac{3mg}{L}\right)\overrightarrow{PB}$$

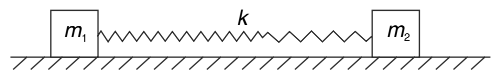
At time $t = 0$, the particle is projected from $A$ towards $B$ with a speed of $\sqrt{gL}$ .

At what time '$t$' will the particle reach at $B$ for the first time?

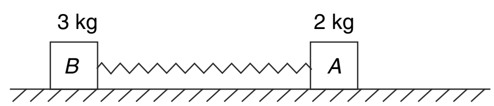

Q. 35. An equilateral prism of mass $m$ is kept on a smooth table between two identical springs each having a force constant of $k$. The two springs have their lengths perpendicular to the inclined faces of the prism and are constrained to remain straight. The ends of the springs have light pads aligned parallel to the faces of the prism, and distance between pads and the incline faces is $d$. The prism is imparted a velocity $v$ to the right. Find time period of its oscillation.




Q. 36. Two blocks rest on a smooth horizontal surface. They are connected by a spring of force constant $k$. If the system is set into oscillation find its time period.




Q. 37. Two blocks $A$ (2 $kg$) and $B$ (3 $kg$) rest on a smooth horizontal surface, connected by a spring of stiffness $k$ = 120 $N/m$. Initially, the spring is relaxed. At $t = 0$, $A$ is imparted a velocity $u = 2$ $m/s$ towards right. Find displacement of block $A$ as a function of time.




Q. 38. A spring has force constant $k$ = 200 $N/m$ and its one end is fixed. There is a block of mass 2 $kg$ attached to its other end and the system lies on a smooth horizontal table. The block is pulled so that the extension in the spring becomes 0.05 $m$. At this position the block is projected with a speed of 1 $m/s$ in the direction of increasing extension of the spring. Consider time $t = 0$ at the moment the block is projected and find

(a) the extension (or compression) in the spring as a function of time.

(b) the maximum extension in the spring and the time at which it occurs for the first time.

(c) the time after which the speed of the block becomes maximum for the first time.

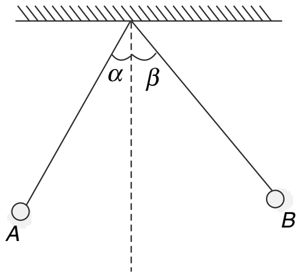
Given: $\sin^{-1}(0.446) = 0.46$ radian

Q. 39. Two identical simple pendulums $A$ and $B$ are fixed at same point. They are displaced by very small angles $\alpha$ and $\beta$ ( = 2 $\alpha$) respectively and are simultaneously released from rest at time $t = 0$. Collisions between the pendulum bobs are elastic and length of each pendulum is $\ell$ .

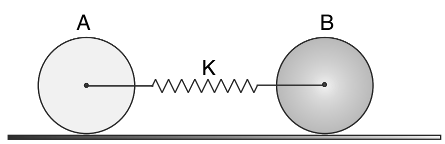
(a) What is the minimum number of collisions between the bobs after which the pendulum $B$ will again reach its original position from where it was released?

(b) Find the time ($t$) at which $B$ reaches its initial position for the first time after the release.

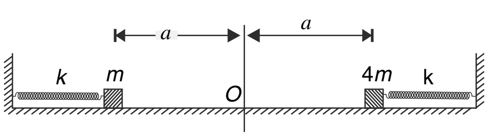
(c) Write the kinetic energy of pendulum $B$ just after $n^{th}$ collision? Take mass of each bob to be $m$.




Q. 40. Two spheres $A$ and $B$ of the same mass $m$ and the same radius are placed on a rough horizontal surface. $A$ is a uniform hollow sphere and $B$ is uniform solid sphere. They are tied centrally to a light spring of spring constant k as shown in figure. $A$ and $B$ are released when the extension in the spring is $x_0$. Friction is sufficient and the spheres do not slip on the surface. Find the frequency and amplitude of SHM of the sphere $A$.




Q. 41. Two small blocks of mass $m$ and $4m$ are connected to two springs as shown in fig. Both springs have stiffness $K$ and they are in their natural length when the blocks are at point $O$. Both the blocks are pushed so that both the springs get compressed by a distance $a$. First the block of mass $m$ is released

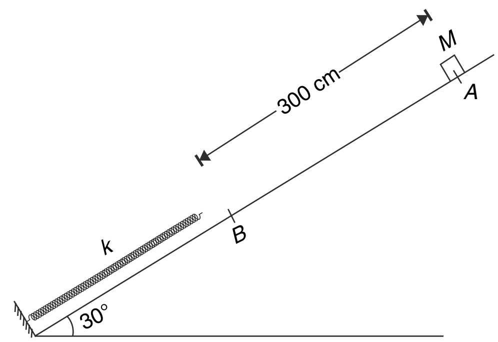



and after it travels through a distance $\left(1-\frac{\sqrt{3}}{2}\right)a$, the second block is also released.

(a) At what distance from point $O$ will the two blocks collide?

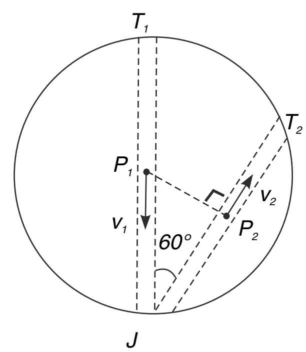
(b) How much time the two blocks need to collide after the block of mass $4m$ is released?

Q. 42. A block of mass $M = 40\ kg$ is released on a smooth incline from point $A$. After travelling through a length of 30 $cm$ it strikes an ideal spring of force constant $K = 1000\ N/m$. It compresses the spring




and then gets pushed back. How much time after its release, the block will be back to point A?

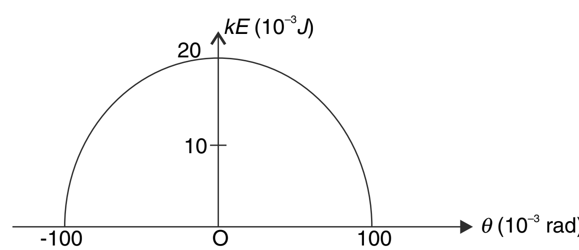
Q. 43. Two tunnels - $T_1$ and $T_2$ are dug across the earth as shown in figure. One end of the two tunnels have a common meeting point on the surface of the earth. Two particles $P_1$ and $P_2$ are oscillating from one end to the other end of the tunnels. At some instant particles are at mid point of their tunnels as shown in figure. Then –



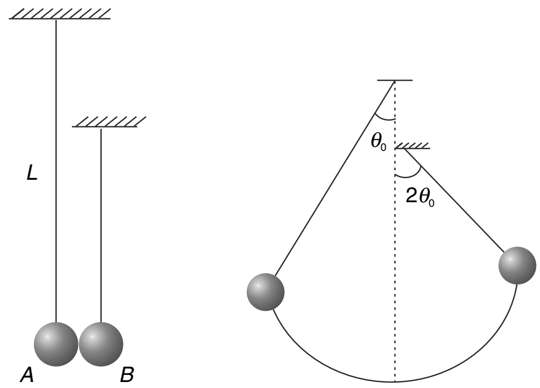

(a) Write phase difference between the particle $P_1$ and $P_2$. Can the two particles ever meet?

(b) Write the ratio of maximum velocity of particle $P_1$ and $P_2$.

Q. 44. The given figure shows the variation of the kinetic energy of a simple pendulum with its angular displacement ($\theta$) from the vertical. Mass of the pendulum bob is $m = 0.2\ kg$. Find the time period of the pendulum. Take $g = 10\ ms^{-2}$.



Q. 45. Two identical small elastic balls have been suspended using two strings of different length (see fig (a)). Pendulum $A$ is pulled to left by a small angle $\theta_0$ and released. It hits ball $B$ head on which swings to angle $2\theta_0$ from the vertical. Calculate the time period of oscillation of $A$ if its length is known to be $L$.

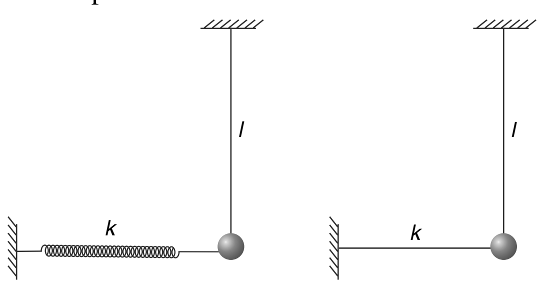


(a) (b)

Q. 46. A simple pendulum of length $L$ has a bob of mass $m$. The bob is connected to light horizontal spring of force constant $k$. The spring is relaxed when the pendulum is vertical (see fig (i)).

(a) The bob is pulled slightly and released. Find the time period of small oscillations. Assume that the spring remains horizontal.

(b) The spring is replaced with an elastic cord of force constant $k$. The cord is relaxed when the pendulum is vertical (see fig (ii)). The bob is pulled slightly and released. Find the time period of oscillations.



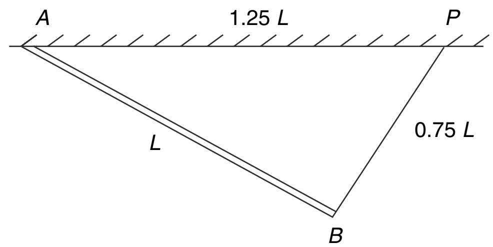
(i) (ii)

Q. 47. A uniform rod $AB$ of mass $m$ and length $L$ is tied, at its end $B$, to a thread which is attached to point $P$ on the ceiling. Length of the thread $PB$ is 0.75 $L$. The other end $A$ of the rod is hinged at a point on the ceiling. Distance $AP = 1.25\ L$. End $B$ of the rod is pushed gently perpendicular to the plane of the figure and it starts oscillating

(a) Find the moment of inertia of the rod about line $AP$.

(b) Assuming that the triangle $APB$ makes a small angle $\theta$ with the vertical plane, write the restoring torque acting on the rod.

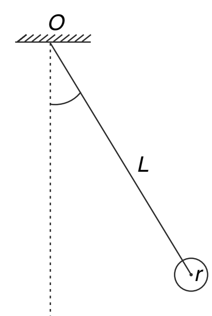
(c) Calculate the time period of small oscillations.




Q. 48. A railway tank wagon with its 2$m$ diameter and 6$m$ long horizontal cylindrical body, half full of petrol is driven around a curve of radius 100$m$, at a speed of 8.33 $m/s$. The curve runs smoothly into a straight track and the train maintains a constant speed. Find the angular amplitude and frequency of subsequent oscillation of the petrol due to this change of motion. Neglect viscosity and consider petrol as a solid semi cylinder sliding inside the tank. Given: $\tan^{-1}(0.07) \approx 4^\circ$

Q. 49. A pendulum consists of an inextensible thread connected to a solid spherical ball of radius $r$. The distance between the point of suspension and the centre of the ball is $L$ ($>> r$). Calculate the percentage difference in the time period of this pendulum to the time period of a simple pendulum of length $L$. How large is this difference for $r = 5\ cm$ and $L = 100\ cm$.

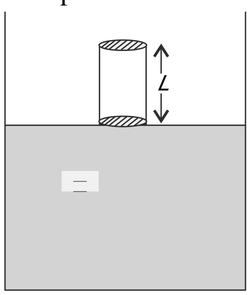
Q. 50. A disc of radius $r$ is connected to a string of length $L$. The string is tied to a point on the circumference of the disc. This system is made to oscillate in vertical plane of the disc with a small angular amplitude $\theta_0$. Find the speed of the lowest point of the disc at the moment the string become vertical.



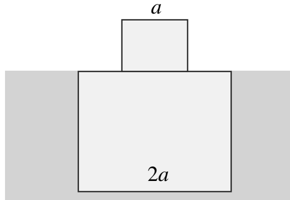

Q. 51. (i) A cylindrical container has area of cross section equal to 4$A$ and it contains a non viscous liquid of density 2$\rho$. A wooden cylinder of cross sectional area $A$ and length $L$ has density $\rho$. It is held vertically with its lower surface touching the liquid. It is released from this position. Assume that the depth of the container is sufficient and the cylinder does not touch the bottom.

(a) Find amplitude of oscillation of the wooden cylinder.

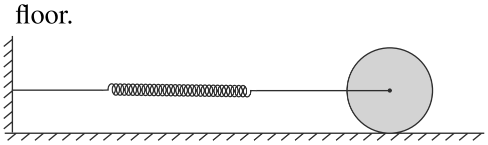
(b) Find time period of its oscillation.




(ii) Two cubical blocks of side length $a$ and $2a$ are stuck symmetrically as shown in the figure. The combined block is floating in water with the bigger block just submerged completely. The block is pushed down a little and released. Find the time period of its oscillations. Neglect viscosity.




Q. 52. A hollow cylindrical shell of radius $R$ has mass $M$. It is completely filled with ice having mass $m$. It is placed on a horizontal floor connected to a spring (force constant $k$) as shown. When it is disturbed it performs oscillations without slipping on the floor.



(a) Find time period of oscillation assuming that the ice is tightly pressed against the inner surface of the cylinder.

(b) If the ice melts into non viscous water, find the time period of oscillations. (Neglect any volume change due to melting of ice)

Q. 53. A particle of mass $m$ is free to move along $x$ axis under the influence of a conservative force. The potential energy of the particle is given by

$U = -ax^n e^{-bx}$ [$a$ and $b$ are positive constants]

Find the frequency of small oscillations of the particle about its equilibrium position

## LEVEL 3

Q. 54. Two particles of mass $m_1$ and $m_2$ are connected by a spring of natural length $L$ and force constant $k$. The masses are brought close enough so as to compress the spring completely and a string is used to tie the system. Assume that length of spring in this position is close to zero. The system is projected with a velocity $V_0$ along the positive $x$ direction. At the instant it reaches origin at time $t = 0$, the string snaps and the spring starts opening.

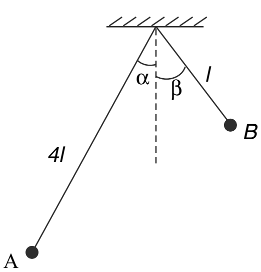

(a) Show that the mass $m_1$ (or $m_2$) will perform SHM in the reference frame attached to the centre of mass of the system. Find the time period of oscillation.

(b) Write the amplitude of $m_1$ and $m_2$ as a function of time.

(c) Write the $X$ co ordinates of $m_1$ and $m_2$ as a function of time

Q. 55. Two simple pendulums $A$ and $B$ have length $4\ell$ and $\ell$ respectively. They are released from rest from the position shown. Both the angles $\alpha$ and $\beta$ are small. Calculate the time after which the two string become parallel for the first time if–

(a) $\alpha = \beta$ (b) $\beta = 1.5\,\alpha$

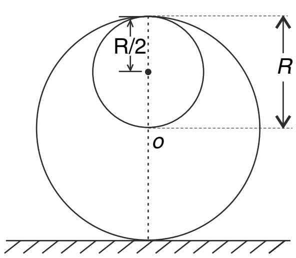



Q. 56. A simple pendulum has a bob of mass $m$ and it is oscillating with a small angular amplitude of $\theta_0$. Calculate the average tension in the string averaged over one time period. [For small $\theta$ take $\cos\theta \simeq 1 - \dfrac{\theta^2}{2}$]

Q. 57. Assume a smooth hole drilled along the diameter of the earth. If a stone is dropped at one end it reaches the other end of the hole after a Time $T_0$. Now instead of dropping the stone, you throw it into the hole with an initial velocity $u$. How big should $u$ be, so that the stone appears at the other end of the hole after a time $\frac{T_0}{2}$. Express your answer in terms of acceleration due to gravity on the surface of the earth ($g$) and the radius of the earth ($R$).

Q. 58. A large horizontal turntable is rotating with constant angular speed $\omega$ in counterclockwise sense. A person standing at the centre, begins to walk eastward with a constant speed $V$ relative to the table. Taking origin at the centre and $X$ direction to be eastward calculate the maximum $X$ co-ordinate of the person.

Q. 59. A spherical cavity of radius $\frac{R}{2}$ is removed from a solid sphere of radius $R$ as shown in fig. The sphere is placed on a rough horizontal surface as shown. The sphere is given a gentle push. Friction is large enough to prevent slippage. Prove that the sphere perform SHM and find the time period.

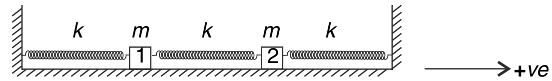



Q. 60. Two blocks 1 and 2, each having mass $m$, are placed on a smooth table connected to three identical springs as shown in the figure. Each spring has a force constant $K$. Initially, all springs are relaxed. The system is disturbed and starts moving. Let $x_1$ and $x_2$ represent the displacements of the two blocks from their respective mean positions.

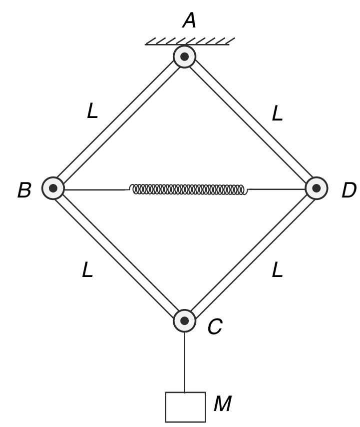
(a) Prove that the quantity $A = x_1 + x_2$ varies sinusoidally and calculate its angular frequency $\omega_a$.

(b) Prove that the quantity $B = x_1 - x_2$ varies sinusoidally and calculate its angular frequency $\omega_b$.

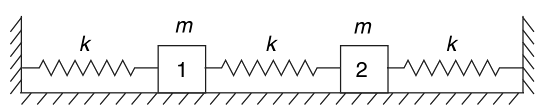
(c) Prove that motion of block 1 is superposition of two SHMs. Write frequency of the component SHMs.




Q. 61. Four identical mass less rods are connected by hinged joints to form a rhombus of side length $L$. Rods can rotate freely about the joints. The joints $B$ and $D$ are connected by a mass less spring of relaxed length 1.5 $L$. The system is suspended vertically with a load of mass $M$ attached at $C$ (see fig). In equilibrium the rods form an angle of $30^\circ$ with the vertical. Find time period of small oscillations of the load.




Q. 62. Two identical blocks 1 and 2, each of mass $m$, are kept on a smooth horizontal surface, connected to three springs as shown in the figure. Each spring has a force constant $k$. Under suitable initial conditions, the two blocks oscillate in phase and their respective displacement from the mean position is given by




$x_1 = A \sin \omega t$ and $x_2 = A \sin \omega t$

(i) Suggest one such initial condition that will result in such oscillation.

(ii) Find $\omega$

# ANSWERS

**1.** (i) $T = \dfrac{2\pi}{\sqrt{b}}$

(ii) $V_{max} = 3\sqrt{2}$ $m/s$, $AP/BP = (\sqrt{2}-1)/(\sqrt{2}+1)$

**2.** (a) $v_1 = A\dfrac{\pi}{t_0}\cos\left(\dfrac{\pi}{t_0}t\right)$ and $v_2 = 2A\dfrac{\pi}{t_0}\sin\left(\dfrac{\pi}{t_0}t\right)$

(b) 2 : 1

**3.** (a) Yes (b) $T = 2\pi\sqrt{\dfrac{1}{\beta}}$ (c) 2 unit

**4.** $x_1 = \pm a$; $x_2 = 0$

**5.** (a) circle of radius $a$. (b) $2\pi a\left(\dfrac{\omega}{\omega_0}\right)$

**6.** (a) $T = \dfrac{2\pi}{\omega}$ (b) yes

**7.** (i) $\Delta T = \dfrac{T}{4}$ (ii) $A = \dfrac{1}{\sqrt{2}}\,m$ ; $t = \dfrac{\pi}{8}\,s$

**8.** $\dfrac{A}{4}\sqrt{\dfrac{k}{M}}$

**9.** (a) $\dfrac{mg}{k}$ (b) $\dfrac{3}{2}mg$

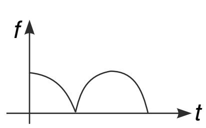
**10.** $\dfrac{Mg}{k} + \dfrac{mg}{k}$

**11.** $\dfrac{\pi}{2}\sqrt{\dfrac{1}{ag}}$

**12.** (a) $a = \left(\dfrac{k_1+k_2}{m_1+m_2}\right)x$ ; $T = 2\pi\sqrt{\dfrac{m_1+m_2}{k_1+k_2}}$

(b) $\dfrac{m_1}{m_2} > \dfrac{k_1}{k_2}$ ; $T = 2\pi\sqrt{\dfrac{m}{k}}$ for both the blocks in both cases.

**13.** (a) No (b)




**14.** (a) $\dfrac{1}{m}\sqrt{F^2-(mg)^2}$ (b) $2\pi\left(\dfrac{m^2r^2}{F^2-(mg)^2}\right)^{1/4}$

**15.** $2\pi\sqrt{\dfrac{m\ell}{2T}}$ , Time period increases.

**16.** $\left(\dfrac{\sqrt{2}+1}{2\sqrt{2}}\right)T$

**17.** (i) $h = 1.25\,m$ (ii) $T > T_0$

**18.** $f = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{2R}}$

**19.** 0.6 $m$

**20.** **(a)** $2\pi\sqrt{\dfrac{2\sqrt{2}}{3}\dfrac{a}{g}}$

(b) $2\pi\sqrt{\dfrac{7}{6\sqrt{2}}\dfrac{a}{g}}$

**21.** (a) $\phi_1 - \phi_2 = \dfrac{2\pi}{3}$ (b) 2f

**22.** (i) $-\dfrac{4\pi^2}{T^2}ma\cos\left(\dfrac{2\pi}{T}t\right)$

(ii) 7 cm and 3 cm.

**23.** (a) $s = 4an + a(1 - \cos \pi t)$

(b) $s = 4an + a(3 + \cos \pi t)$

**24.** (a) $50 - 10\cos(625t)$ (b) 60 $N$

**25.** $T = 2\pi\sqrt{\dfrac{LM}{g(M+m)}}$

**26.** $T = 2\pi\sqrt{\dfrac{R\cos\theta}{g}}$

**27.** $\omega = \sqrt{\dfrac{8k}{3M}}$

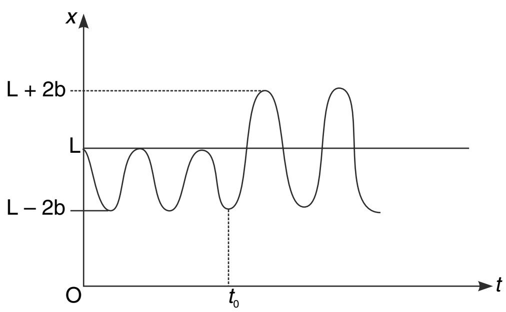
**28.** (i) $T = \pi\sqrt{\dfrac{5m}{k}}$ (ii) $T = 2\pi\sqrt{\dfrac{\lambda L}{k+\lambda g}}$

**29.** $H = \dfrac{Mg}{2k}\left(1+\dfrac{\pi^2}{9}\right)$

**30.** For both (a) and (b) the block will not lose contact with the ground for any value of $m$.

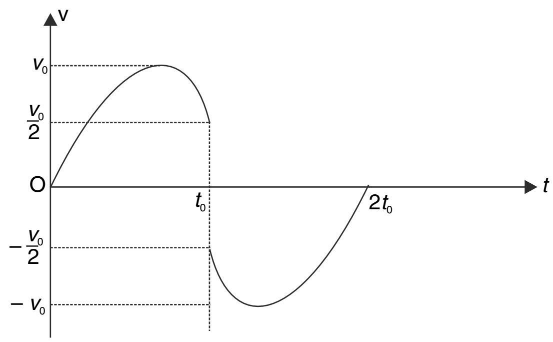
**31.** (a) $\frac{RC}{RD}=\frac{2}{1}$ (b) $L$.

**32.**




**33.** (a) $\frac{4\pi}{3}\sqrt{\frac{M}{k}}$

(b)




(c) $k=\frac{4\pi F_0}{3\sqrt{3}A}$

**34.** $t=\frac{2\pi}{3}\sqrt{\frac{L}{3g}}$

**35.** $\frac{8d}{\sqrt{3}v}+4\pi\sqrt{\frac{m}{3k}}$

**36.** $T=2\pi\sqrt{\frac{\mu}{k}}$ ; where $\mu=\frac{m_1m_2}{m_1+m_2}$

**37.** $x_A = 0.12 \sin(10t) + 0.8t$

**38.** (a) $x = 0.112 \sin(10t + 0.46)\ m$

(b) $0.112\ m$, $0.111\ s$

(c) $t = 0.268\ s$

**39.** (a) 2 (b) $t = 2\pi\sqrt{\frac{\ell}{g}}$

(c) $E=\frac{1}{2}ml\alpha^2 g$ if $n$ is odd and $E=\frac{1}{2}ml\beta^2 g$ if $n$ is even

**40.** $f=\frac{1}{2\pi}\sqrt{\frac{46\ k}{35\ m}}$ ; $A_1=\frac{21}{46}x_0$

**41.** (a) $a\cos\left(\frac{5\pi}{18}\right)$ (b) $\frac{5\pi}{9}\sqrt{\frac{m}{k}}$

**42.** $1.54\ s$

**43.** (a) 180°, No (b) 2 : 1

**44.** $T = 2.80\ s$

**46.** (a) $2\pi\left(\frac{g}{\ell}+\frac{k}{m}\right)^{-\frac{1}{2}}$ (b) $\pi\left[\left(\frac{g}{\ell}+\frac{k}{m}\right)^{-\frac{1}{2}}+\left(\frac{g}{\ell}\right)^{-\frac{1}{2}}\right]$

**47.** (a) $\frac{3}{25}mL^2$ (b) $\frac{3}{10}\ mgL\ \theta$

(c) $2\pi\sqrt{\frac{2L}{5g}}$

**48.** *Amplitude* = $\tan^{-1}(0.07) = 4°$; *frequency* = $0.46\ Hz$.

**49.** $\frac{20r^2}{L^2}$ % ; 0.05%

**50.** $\theta_o(L+2r)\sqrt{\frac{2g(L+r)}{r^2+2(L+r)^2}}$

**51.** (i) (a) $\frac{3L}{8}$ (b) $2\pi\sqrt{\frac{3L}{8g}}$

(ii) $3\pi\sqrt{\frac{2a}{g}}$

**52.** (a) $2\pi\sqrt{\frac{4M+3m}{2K}}$;

(b) $2\pi\sqrt{\frac{2M+m}{K}}$.

**53.** $f=\frac{1}{2\pi}\sqrt{\frac{ae^{-n}n^{n-1}}{mb^{n-2}}}$

**54.** (a) $T=2\pi\sqrt{\frac{m_1m_2}{(m_1+m_2)k}}$

(b) $\frac{m_2L}{m_1+m_2}=A_1$

(c) $X_1 = V_0t + A_1(1 - \cos\omega t)$;

$X_2 = V_0t - A_2(1 - \cos\omega t)$

Where

$A_1 = \frac{m_2 L}{m_1 + m_2}$ ; $A_2 = \frac{m_1 L}{m_1 + m_2}$ ; $\omega = \sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}}$

**55.** (a) $\frac{2\pi}{3}\sqrt{\frac{\ell}{g}}$ (b) $2\sqrt{\frac{\ell}{g}} \cdot \cos^{-1}\left(\frac{\sqrt{19}-1}{6}\right)$

**56.** $T_{av} = mg + \frac{1}{4} mg\, \theta_0^2$

**57.** $u = \sqrt{gR}$

**58.** $\frac{V}{\omega}$

**59.** $T = 2\pi\sqrt{\frac{177R}{10g}}$

**60.** (a) $\omega_a = \sqrt{\frac{k}{m}}$

(b) $\omega_b = \sqrt{\frac{3k}{m}}$

(c) $\omega_a$ and $\omega_b$

**61.** $T = 2\pi\sqrt{\frac{L}{2\sqrt{3}g}}$

**62.** (ii) $\omega = \sqrt{\frac{k}{m}}$

# SOLUTIONS

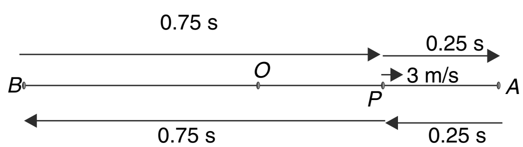
**1.** (i) The given equation has a standard solution given by

$a = a_0 \sin(\omega t + \delta)$. Where $\omega = \sqrt{b}$ .

This is an equation of SHM.

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{b}}$$

(ii) Careful observation of the data tells us that the time period of SHM = 2 $s$




If we consider $t = 0$ when the particle is at origin and travelling in positive direction, we can write the equation of motion as-

$x = A \sin \omega t \Rightarrow v = A\omega \cos \omega t$

Particle will reach at $P$ when time is $t = 1/4$ $s$

$$\therefore 3\ m/s = A\frac{2\pi}{2}\cos\left(\frac{2\pi}{2}.\frac{1}{4}\right) \Rightarrow A = \frac{3\sqrt{2}}{\pi}$$

$v_{max} = A\omega = 3\sqrt{2}$ $m/s$

$$OP = A\sin\omega t = \frac{3\sqrt{2}}{\pi}\sin\left(\frac{\pi}{4}\right) = \frac{3}{\pi}\ m$$

Now, it is easy to work out the ratio $AP/PB$.

**2.** (a) A careful observation of the given graphs reveals that the time period is same for both the particles. Amplitude of 1 and 2 are $A$ and $2A$ respectively and particle 2 lags in phase by $\frac{\pi}{2}$.

Position – time equation for the two particles is –

$x_1 = A \sin \omega t$ and $x_2 = 2A\sin\left(\omega t - \frac{\pi}{2}\right) = -2A \cos \omega t$

$$\Rightarrow \tan\theta = \frac{F_s}{Mg}$$

$\Rightarrow Mg \tan 30° = k\,[1.5L - 2L \sin 30°]$

$$\Rightarrow Mg\frac{1}{\sqrt{3}} = \frac{kL}{2} \Rightarrow \frac{kL}{Mg} = \frac{2}{\sqrt{3}} \qquad \text{-----------(1)}$$

If $y$ changes by $\Delta y$ then we proceed as follows to calculate the restoring force.

Let length of the spring be $\ell$.

$\ell = 2L \sin\theta$

$\Delta\ell = 2L \cos\theta\,\Delta\theta$ ------------(a)

And y = 2L cos $\theta$

$\Delta y = -2L \sin\theta\,\Delta\theta$ ------------(b)

Spring force changes by $k\Delta\ell = 2kL\cos\theta\,\Delta\theta$

$\therefore$ Change in rod tension will be given as

$2\Delta T \sin\theta = 2kL \cos\theta\,\Delta\theta$

$\Delta T = kL \cot\theta.\,\Delta\theta$

$\therefore$ Restoring force on mass $M$ is

$$2\Delta T\cos\theta = 2kL\,\frac{\cos^2\theta}{\sin\theta}\,\Delta\theta$$

$$\therefore M\frac{d^2y}{dt^2} = 2kL\frac{\cos^2\theta}{\sin\theta}\,\Delta\theta$$

$$\frac{d^2y}{dt^2} = -2\frac{kL}{M}\,\frac{\cos^2\theta}{\sin\theta}\cdot\frac{\Delta y}{2L\sin\theta} \qquad \text{[using (b)]}$$

$$= -\frac{k}{M}\frac{\cos^2\theta}{\sin^2\theta}\,\Delta y = -\frac{2g}{\sqrt{3}L}.\left(\sqrt{3}\right)^2\,\Delta y - 2\sqrt{3}\,\frac{g}{L}\,\Delta y$$

$$\therefore \omega^2 = 2\sqrt{3}\,\frac{g}{L}$$

$$T = 2\pi\sqrt{\frac{L}{2\sqrt{3}g}}$$

**62.** (i) If both blocks are simultaneously given equal velocity when they are at their mean positions, they will oscillate as suggested.

(ii) The distance between 1 and 2 does not change. Hence, middle spring does not exert any force on the blocks.

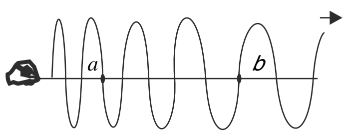
Each block experiences force due to one spring only.

$$\therefore \omega = \sqrt{\frac{k}{m}}$$

# 13 WAVE MOTION

## Level 1

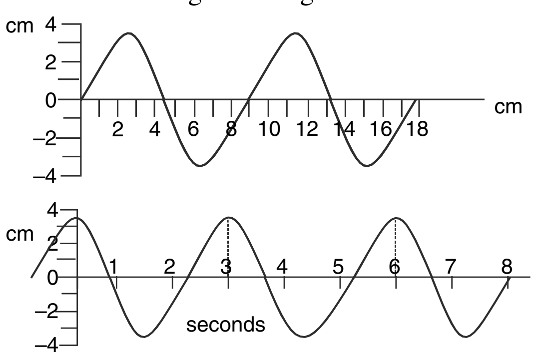
Q. 1. A boy is jerking one end of a taut string. The wave train propagating to the right has been shown in the figure.



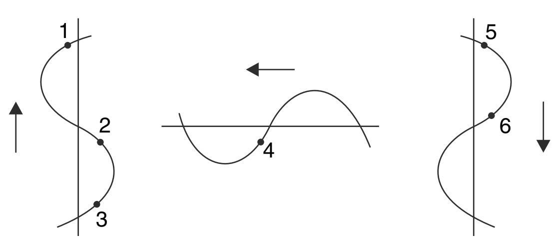

(a) Why the crests are farther apart as we move away from the boy.

(b) Which particle on the string $a$ or $b$ is having higher speed?

Q. 2. A transverse wave is travelling along a horizontal string. The first figure is the shape of the string at an instant of time. The second picture is a graph of the vertical displacement of a point on the string as a function of time. How far does this wave travel along the string in one second?




Q. 3.




The figure shows the shape of three strings on which sinusoidal transverse waves are propagating. The arrows in the diagram indicate the direction of wave propagation. Out of the 6 particles marked (1, 2, 3, 4, 5, 6 ) how many have their instantaneous velocity and acceleration both directed towards their mean position.

Q. 4. Consider a function

$$y = 5.0e^{(-25x^2 - 9t^2 - 30xt)}$$

(a) Does this represent a travelling wave?

(b) What is direction of propagation of the wave?

(c) Find wave speed.

(d) Sketch the wave at $t = 0$

Q. 5. A hypothetical pulse is travelling along positive $x$ direction on a taut string. The speed of the pulse is 10 $cm\ s^{-1}$. The shape of the pulse at $t = 0$ is given as

$y = \frac{x}{6} + 1$ for $-6 < x \leq 0$

$= -x + 1$ for $0 \leq x < 1$

$= 0$ for all other values of $x$

$x$ and $y$ are in $cm$.

(a) Find the vertical displacement of the particle at $x = 1\ cm$ at $t = 0.2\ s$

(b) Find the transverse velocity of the particle at $x = 1\ cm$ at $t = 0.2\ s$.

Q. 6. Which of the following functions does not satisfy the differential wave equation -

(i) $y = 4e^{k(x - vt)}$

(ii) $y = 2 \sin (5t) \cos (6\pi x)$

Q. 7. A transverse harmonic wave of amplitude 4 $mm$ and wavelength 1.5 $m$ is travelling in positive $x$ direction on a stretched string. At an instant, the particle at $x$ = 1.0 $m$ is at $y$ = + 2 $mm$ and is travelling in positive $y$ direction. Find the co-ordinate of the nearest particle ($x > 1.0\ m$) which is at its positive extreme at this instant.

Q. 8. A transverse harmonic wave travels along a taut string having a tension of 57.6 $N$ and linear mass density of 100 $g/m$. Two points $A$ and $B$ on the string are 5 $cm$ apart and oscillate with a phase difference of $\frac{\pi}{6}$. How much does the phase of oscillation of point $A$ change in a time interval of 5.0 $ms$?

Q. 9. A distant source of sound has frequency 800 $Hz$. An observer is facing 90° away from the direction of the source. Estimate the phase difference between the oscillations of her left and right eardrums. Speed of sound in air = 340 $ms^{-1}$.

Q. 10. A sinusoidal wave travels along a taut string of linear mass density 0.1 $g/cm$. The particles oscillate along $y$-direction and the disturbance moves in the positive $x$-direction. The amplitude and frequency of oscillation are 2 $mm$ and 50 $Hz$ respectively. The minimum distance between two particles oscillating in the same phase is 4 $m$.

(a) Find the tension in the string.

(b) Find the amount of energy transferred through any point of the string in one second.

(c) If it is observed that the particle at $x = 2$ $m$ is at $y = 1$ $mm$ at $t = 2$ $s$, and its velocity is in positive y-direction, then write the equation of this travelling wave.

Q. 11. $A$ and $B$ are two point sources of sound (of same frequency) and are kept at a separation. At a point $P$, the intensity of sound is observed to be $I_0$ when only source $A$ is put on. With only $B$ on the intensity is observed to be $2I_0$. The distance $AP$ is higher than distance $BP$ by half the wavelength of the sound. Find the intensity recorded at $P$ with both sources on. Give your answer for following cases:

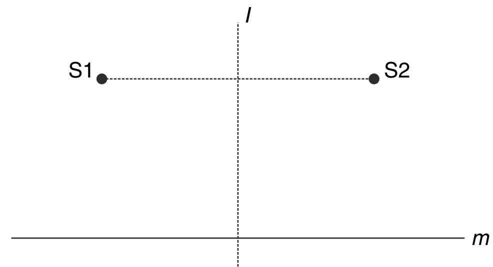
(a) The sources are coherent and in phase.

(b) The sources are coherent and 180° out of phase.

(c) The sources are incoherent.

Q. 12. Two sound sources oscillate in phase with a frequency of 100 $Hz$. At a point 1.74 m from one source and 1.16 $m$ from the other, the amplitudes of sound from the two sources are $A$ and 2 $A$ respectively. Calculate the amplitude of the resultant disturbance at the point. [Speed of sound in air is $v = 348$ $ms^{-1}$]

Q. 13. Two speakers $S1$ and $S2$ are a driven by same source. You walk along a line $l$ that is perpendicular bisector of the line joining the two speakers and record the intensity at different points. Then you walk along a line $m$ that is parallel to the line joining the speakers and record the intensity of sound at various points. On which path you observe the loudness to alternate between faint and loud? Explain.

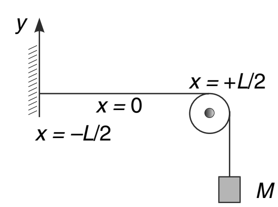



Q. 14. (i) A wire is stretched between two rigid supports. It is observed that the wire resonates at a frequency of 420 $Hz$. If a wooden bridge is placed at the midpoint of the wire (so that the midpoint becomes a node), it was observed that the smallest frequency at which the wire resonates is 420 $Hz$. Find the smallest frequency at which the wire will resonate when there is no wooden bridge.

(ii) A string of length $L$ is fixed at one end and is under tension due to a weight hanging from the other end, as shown in the figure. The point of the string on the pulley behaves as a fixed point. Coordinate axes are chosen so that the horizontal segment of the string runs from $x = -L/2$ to $x = L/2$. The string is vibrating at one of its resonant frequencies with transverse displacement ($y$) given by

$$y(x,t) = 0.05 \cos(12.0x) \sin(360t)$$

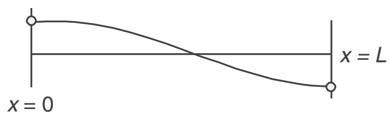
with $x$, $y$ in meter and $t$ in second. Write two smallest possible values of $L$ consistent with the given equation?




Q. 15. Two strings of same material are joined to form a large string and is stretched between rigid supports. The diameter of the second string is twice that of the first. It was observed in an experiment that the whole string was oscillating in 4 loops with a node at the joint. Find the possible lengths of the second string if the length of first string is 90 $cm$.

Q. 16. (i) The equation of wave in a string fixed at both end is $y = 2 \sin \pi t \cos \pi x$. Find the phase difference between oscillations of two points located at $x = 0.4\ m$ and $x = 0.6\ m$.

(ii) A string having length $L$ is under tension with both the ends free to move. Standing wave is set in the string and the shape of the string at time $t = 0$ is as shown in the figure. Both ends are at extreme. The string is back in the same shape after regular intervals of time equal to $T$ and the maximum displacement of the free ends at any instant is $A$. Write the equation of the standing wave.




Q. 17. One type of steel has density 7800 kg/m$^3$ and will break if the tensile stress exceeds $7.0 \times 10^8$ $N/m^2$. You want to make a guitar string using 4.0 g of this type of steel. While in use, the guitar string must be able to withstand a tension of 900 $N$ without breaking.

(a) Determine the maximum length and minimum radius the string can have.

(b) Determine the highest possible fundamental frequency of standing waves on this string, if the entire length of the string is free to vibrate.

Q. 18. A string, of length $L$, clamped at both ends is vibrating in its first overtone mode. Answer the following questions for the moment the string looks flat

(a) Find the distance between two nearest particles each of which have half the speed of the particle having maximum speed.

(b) How many particles in the string have one eighth the speed of the particle travelling at highest speed?

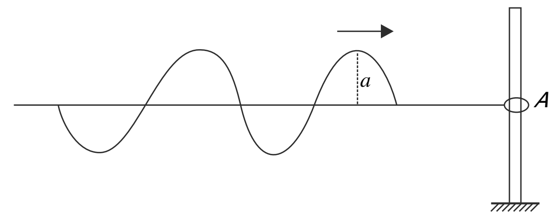
Q. 19. Two transverse waves travel in a medium in same direction.

$$y_1 = a\cos\left(\omega t - \frac{2\pi}{\lambda_1}x\right);\ y_2 = a\cos\left(2\omega t - \frac{2\pi}{\lambda_2}x\right)$$

(a) Write the ratio of wavelengths $\left(\frac{\lambda_1}{\lambda_2}\right)$ for the two waves.

(b) Plot the displacement of the particle at $x = 0$ with time ($t$).

Q. 20. A sine wave is travelling on a stretched string as shown in figure. The end $A$ of the string has a small light ring which can slide on a smooth rod. The wave reaches $A$ at time $t = 0$.

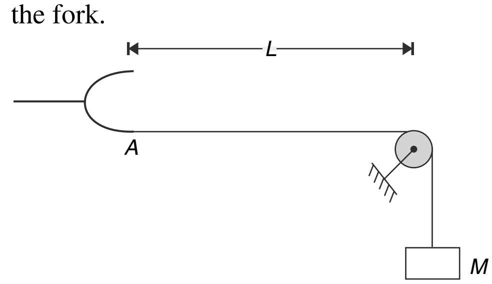



(i) Write the slope of the string at point $A$ as function of time.

(ii) If the incoming wave has amplitude $a$, with what amplitude will the end $A$ oscillate?

Q. 21. Fundamental frequency of a stretched sonometer wire is $f_0$. When its tension is increased by 96% and length decreased by 35%, its fundamental frequency becomes $\eta_1 f_0$. When its tension is decreased by 36% and its length is increased by 30%, its fundamental frequency becomes $\eta_2 f_0$. Find $\frac{\eta_1}{\eta_2}$.

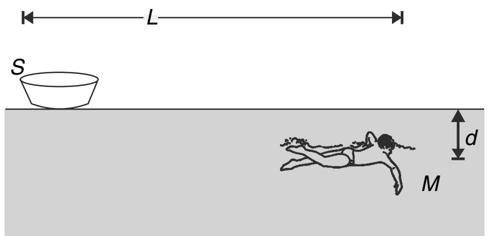
Q. 22. The linear mass density of the string shown in the figure is $\mu = 1$ g/m. One end ($A$) of the string is tied to a prong of a tuning fork and the other end carries a block of mass $M$. The length of the string between the tuning fork and the pulley is $L = 2.0$ m. When the tuning fork vibrates, the string resonates with it when mass $M$ is either 16 kg or 25 kg. However, standing waves are not observed for any other value of $M$ lying between 16 kg and 25 kg. Assume that end $A$ of the string is practically at rest and calculate the frequency of the fork.




Q. 23. Wavelength of two musical notes in air are $\frac{18}{35}\ m$ and $\left(\frac{90}{173}\right) m$. Each note produces four beats per second with a third note of frequency $f_0$. Calculate the frequency $f_0$.

Q. 24. In a science – fiction movie the crew of a ship observes a satellite. Suddenly the satellite blows up. The crew first sees the explosion and after a small time gap hears the sound. Do you think there was a technical lapse?

Q. 25. A man is swimming at a depth $d$ in a sea at a distance $L$ (>> $d$) from a ship ($S$). An explosion occurs in the ship and after hearing the sound the man immediately moves to the surface. It takes $0.8s$ for the man to rise to the surface after he hears the sound of explosion. $0.2s$ after reaching the surface he once again hears a sound of explosion. Calculate $L$.



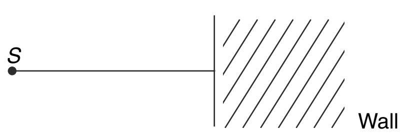

Given: Speed of sound in air = 340 $ms^{-1}$; Bulk modulus of water = $2 \times 10^9$ $Pa$

Q. 26. Speed of sound in air is 331 $ms^{-1}$ at 0°$C$. Prove that it increases at a rate of 0.6 $ms^{-1}{}^\circ C^{-1}$ for small temperature increase.

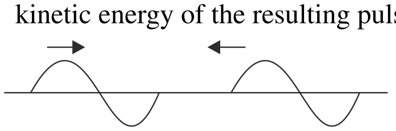
Q. 27. (a) Calculate the speed of sound in hydrogen gas at 300 $K$

(b) At what temperature the speed in oxygen will be same as above. [Assume oxygen molecules to remain diatomic]

Q. 28. A harmonic source (S) is driving a taut string. The other end of the string is tied to a wall that is not so rigid. It is observed that standing waves are formed in the string with ratio of amplitudes at the antinodes to that at the nodes equal to 8. What percentage of wave energy is transmitted to the wall?




Q. 29. (a) Two identical sinusoidal pulses move in opposite directions on a stretched string. Kinetic energy of each pulse is $k$. At the instant they overlap completely, what is kinetic energy of the resulting pulse?



(b) "A string clamped at both ends is vibrating. At the moment the string looks flat, the instantaneous transverse velocity of points along the string, excluding its end-points, must be same everywhere except at nodes." Is this statement correct?

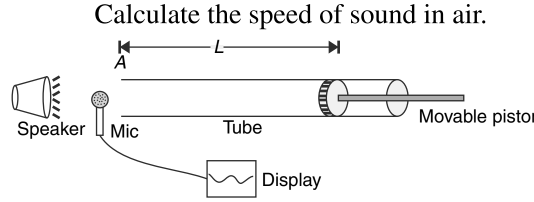
Q. 30 Sound of wavelength 100 cm travels in air. At a given point the difference in maximum and minimum pressure is 0.2 $Nm^{-2}$. If the bulk modulus of air is $1.5 \times 10^5$ $Nm^{-2}$, find the amplitude of vibration of the particles of the medium.

Q. 31. (i) An organ pipe has one end closed and at the other end there is a vibrating diaphragm. The diaphragm is a pressure node. The pipe resonates when the frequency of the diaphragm is 2 $KHz$. Distance between adjacent nodes is 8.0 $cm$. When the frequency is slowly reduced, the pipe again resonates at 1.2 $KHz$.

(a) Find the length of the tube.

(b) Find the next frequency above 2 $KHz$ at which the pipe resonates.

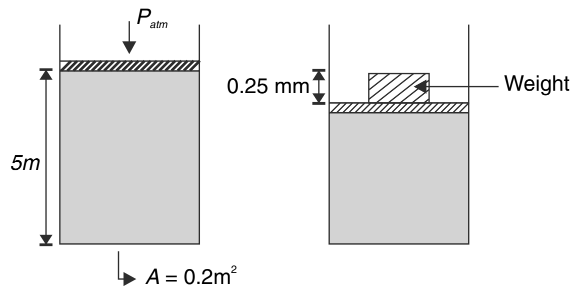
(ii) The figure shows an arrangement for measuring the speed of sound in air. A glass tube is fitted with a movable piston that allows the indicated length $L$ to be adjusted. There is enough gap between the piston and the tube wall to allow the air to pass through it. A speaker is placed near the open end of the tube. A microphone is placed close to the speaker and it is connected to a waveform display. The display is a pure sinusoidal waveform making 750 oscillations in 5 $s$. Initially, the piston is held at end $A$ and is then slowly pulled back. Loud sound is produced by the tube when $L$ = 50 $cm$ and $L$ = 157 $cm$. Calculate the speed of sound in air.




Q. 32. A rigid cylindrical container having a cross sectional area of 0.2 $m^2$ is filled with water up to a height of 5.0 m. There is a piston of negligible mass over the water. Piston can slide inside the container without friction. When a weight of 2000 kg is placed over the piston, it moves down by 0.25 mm compressing the water.

$\rho$ = Density of water = $10^3$ $kg/m^3$; $P_{atm}$

= Atmospheric pressure = $10^5$ $N/m^2$ and $g$ = 10 $m/s^2$. With this information calculate the speed of sound in water.




Q. 33. A point source of sound is located inside sea water. Bulk modulus of sea water is $B_\omega = 2.0 \times 10^9\ N/m^2$. A diver located at a distance of 10 $m$ from the source registers a pressure amplitude of $\Delta P_0 = 3000\ \pi$ N/m$^2$ and gives the equation of sound wave as

$y = A \sin (15\ \pi x - 21000\ \pi t)$, $y$ and $x$ are in meter and $t$ is in second.

(a) Find the displacement amplitude of the sound wave at the location of the diver.

(b) Find the power of the sound source.

Q. 34. A point source of sound is moving uniformly along positive $x$ direction with velocity $V_0$. At time $t$ = 0 the source was at origin and emitted a compression pulse $C_1$. After time $T$ it emitted another compression pulse $C_2$. Write the equation of the wave front representing the compression pulse $C_2$ at time $t$ (>$T$). Speed of sound is $V$.

Q. 35. (i) In a car race sound signals emitted by the two cars are detected by the detector on the straight track at the end point of the race. Frequency observed is 330 $Hz$ and 360 $Hz$. The original frequency of horn is 300 $Hz$ for both cars. Race ends with the separation of 1000 m between the cars. Assume both cars move with constant velocity and velocity of sound is 330 $m/s$. Find the time (in seconds) taken by the winning car to finish the race.

(ii) A source of sound of frequency f is dropped from rest from a height h above the ground. An observer $O_1$ is located on the ground and another observer $O_2$ is inside water at a depth $d$ from the ground. Both $O_1$ and $O_2$ are vertically below the source. The velocity of sound in water is 4$V$ and that in air is $V$. Find

(a) The frequency of the sound detected by $O_1$ and $O_2$ corresponding to the sound emitted by the source initially.

(b) The frequency detected by both $O_1$ and $O_2$ corresponding to the sound emitted by the source at height $h$/2 from the ground.

Q. 36. (i) A source of sound emits waves of frequency $f_0$ = 1200 $Hz$. The source is travelling at a speed of $v_1$ = 30 $m/s$ towards east. There is a large reflecting surface in front of the source which is travelling at a velocity of $v_2$ = 60 $m/s$ towards west. Speed of sound in air is $v$ = 330 $m/s$.

(a) Find the number of waves arriving per second at the reflecting surface.

(b) Find the ratio of wavelength ($\lambda_1$) of sound in front of the source travelling towards the reflecting surface to the wavelength ($\lambda_2$) of sound in front of the source approaching it after getting reflected.

(ii) A sound source ($S$) and an observer ($A$) are moving towards a point $O$ along two straight lines making an angle of 60° with each other. The velocities of $S$ and $A$ are 18 $ms^{-1}$ and 12 $ms^{-1}$ respectively and remain constant with time. Frequency of the source is 1000 $Hz$ and speed of sound is $v$ = 330 $ms^{-1}$.

(a) Find the frequency received by the observer when both the source and observer are at a distance of 180 $m$ from point $O$ (see figure).

(b) Find the frequency received by the observer when she reaches point $O$.

Q. 37. A source of sound, producing a sinusoidal wave, is moving uniformly towards an observer at a velocity of 20 $m/s$. The observer is moving away from the source at a constant velocity of 10 $m/s$. Frequency of the source is 200 $Hz$ and speed of sound in air is of 340 $m/s$.

(a) How many times, in an interval of 10 second, the eardrums of the observer will sense maximum change in pressure?

(b) What will be apparent wavelength of sound for the observer?

Q. 38. Two trains $A$ and $B$ are moving on parallel tracks in opposite direction at same speed of 30 $ms^{-1}$. Just when the engines of the two trains are about to cross, the engine of train $A$ begins to sound a horn. The sound of the horn is composed of components varying in frequency from 900 $Hz$ to

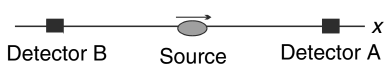
1200 *Hz*. The speed of sound in air is 330 $ms^{-1}$.

(a) Find the frequency spread (range of frequencies) for the sound heard by a passenger in train *A*.

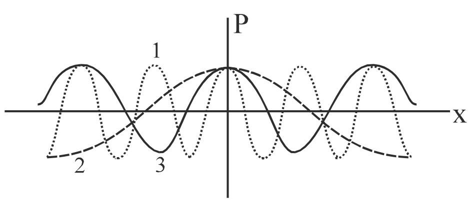
(b) Find the frequency spread heard by a passenger in train *B*.

Q. 39. Two tuning forks produce 4 beats per second when they are sounded together. Now both the forks are moved towards the observer at same speed ($u$). The beat frequency now becomes 5 *Hz*. If the observer also begins to run with speed $u$ towards both the forks, what beat frequency will he hear now?

Q. 40. (i) A sound source emitting sound at a single frequency moves with constant speed along $x$-axis as shown in figure ($a$). $A$ and $B$ are two stationary observers. The three plots shown in figure ($b$) indicate the pressure function $P(x)$ of the sound wave as recorded by the observer $A$, by $B$, and by another observer $C$ who is at rest in the frame of the source. Which plot (marked as 1, 2 and 3) correspond to which observer?

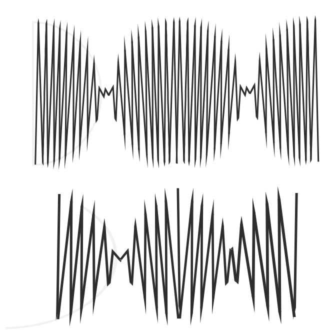



(a)




(b)

(ii) Each of the two figures is rough illustration of the resulting waveform ($y$ versus $t$ ) due to overlapping of two waves. The four component waves have frequencies of 300 *Hz*, 200 *Hz*, 204 *Hz* and an unknown frequency $f = 300 + \Delta f$. Is $\Delta f$ higher than or less than 4 *Hz*?



## LEVEL 2

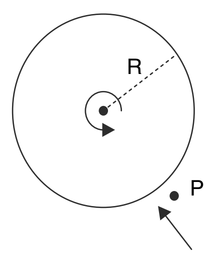
Q. 41. A long taut string is plucked at its centre. The pulse travelling on it can be described as $y(x, t) = e^{-(x+2t)^2} + e^{-(x-2t)^2}$ . Draw the shape of the string at time $t = 0$, a short time after $t = 0$ and a long time after $t = 0$.

Q. 42. A sinusoidal harmonic wave is propagating along a string stretched along $x$ – axis. A particle on the string at $x = 1$ *m* is found to be at its mean position travelling in positive $y$ direction at $t = 1\,s$. The amplitude, wavelength and frequency of the wave are 0.01 *m*, $\frac{\pi}{2}m$ and 20 *Hz* respectively. Write the equation of the wave if-

(a) it is travelling along negative $X$ direction

(b) it is travelling along positive $X$ direction.

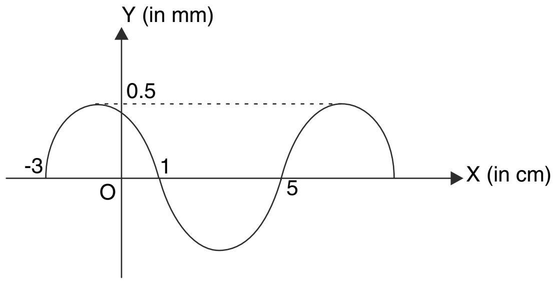
Q. 43. A circular loop of radius $R$ is made of a perfectly elastic wire and is rotating with a constant angular velocity $\omega$ lying on a smooth horizontal table. The rotation axis is vertical passing through the centre. A small radial push given to the loop at a point $P$ on the table causes a transverse pulse to propagate on it. Find the smallest time in which the pulse will be back to its originating point $P$ on the table.




Q. 44. Two waves $y_1 = a\sin\left(\frac{\pi}{2}x - \omega t\right)$ and $y_2 = a$ sin $\left(\frac{\pi}{2}x + \omega t + \frac{\pi}{3}\right)$ get superimposed in the region $x \geq 0$. Find the number of nodes in the region $0 \leq x \leq 6$ *m*.

Q. 45. Two sine waves of same frequency and amplitude, travel on a stretched string in opposite directions. Speed of each wave is 10 *cm/s*. These two waves superimpose to form a standing wave pattern on the string. The maximum amplitude in the standing wave pattern is 0.5 *mm*.

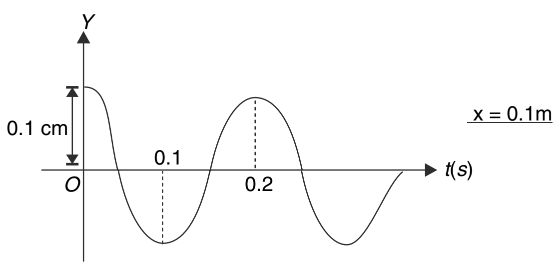
The figure shows the snapshot of the string at $t = 0$.Write the equation of the two travelling waves.

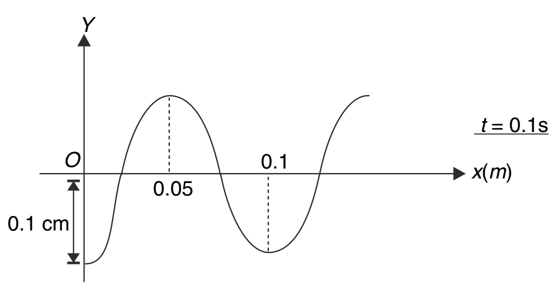



Q. 46. (i) A sinusoidal wave is travelling along positive $x$ direction and the displacements at two positions $x = 0$ and $x = 1\ m$ are given by $y(0, t) = 0.2 \cos(3\pi t)$ and $y(1,t) = 0.2\cos\left(3\pi t + \frac{\pi}{8}\right)$

Find all possible wavelength of the wave if it is known that wavelength is greater than $0.4\,m$.

(ii) A transverse sine wave of amplitude $a = 0.1\ cm$ is travelling along a string laid along the $x$-axis. The displacement ($y$) – time ($t$) graph of the string particle at $x = 0.1\ m$ is shown in first figure. The shape of the string at time $t = 0.1\ s$ is shown in second figure. At this time the particle at $x = 0.11\ m$ is having velocity in positive $y$ direction write the equation of wave.




Q. 47. (i) Two sinusoidal wave are given as $y_1 = a_1 \sin(\omega t + kx + \delta)$ and $y_2 = a_2 \sin(\omega t - kx)$. They superimpose.

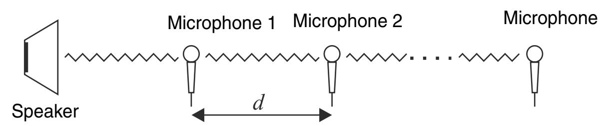
(a) Calculate the resultant amplitude of oscillation at a position $x$. Is amplitude time dependent?

(b) Calculate the ratio of maximum and minimum amplitudes observed.

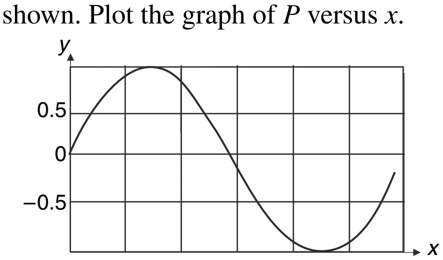
(ii) A speaker (producing a sound of a single wavelength $\lambda$) and a microphone are placed as shown in the figure. The microphone detects the sound and converts it into electrical signal. This way we can obtain the waveform of the sound. Assume that there is no attenuation of the sound. The waveform detected by the microphone is sinusoidal with amplitude $a$. In one experiment 6 microphones are placed in front of the speaker with distance between two neighbouring microphones being $d = \frac{5\lambda}{6}$

(a) The output from all the 6 microphones is superimposed. What is amplitude of the resultant?

(b) If large number of microphone are kept with separation $L$ between two consecutive ones, how will the combined output change with $L$? Given that $L \neq n\lambda$ ($n = 1, 2, 3$...........)




Q. 48. The figure shows $y$ (transverse displacement) vs $x$ (position) graph for a sinusoidal wave travelling along a stretched string. $P$ is power transmitted through a cross section of the string at the instant shown. Plot the graph of $P$ versus $x$.




Q. 49. A string in a guitar is made of steel (density 7962 $kg/m^3$). It is 63.5 $cm$ long, and has diameter of 0.4 $mm$. The fundamental frequency is $f = 247\ Hz$.

(a) Find the string tension ($F$).

(b) If the tension $F$ is changed by a small amount $\Delta F$, the frequency $f$ changes by a small amount $\Delta f$. Show that $\frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta F}{F}$

(c) The string is tuned with tension equal to that calculated in part (a) when its temperature is $18°C$. Continuous playing causes the temperature of the string to rise, changing its vibration frequency. Find $\Delta f$ if the temperature of the string rises to 29°*C*. The steel string has a Young's modulus of $2.00 \times 10^{11}$ *Pa* and a coefficient of linear expansion of $1.20 \times 10^{-5}$ $(°C)^{-1}$. Assume that the temperature of the body of the guitar remains constant. Will the vibration frequency rise or fall?

Q. 50. A long taut string is connected to a harmonic oscillator of frequency *f* at one end. The oscillator oscillates with an amplitude $a_0$ and delivers power $P_0$ to the string. Due to dissipation of energy the amplitude of wave goes on decreasing with distance *x* from the oscillator given as $a = a_0e^{-kx}$. In what length of the string $\left(\frac{3}{4}\right)$th of the energy supplied by the oscillator gets dissipated?

Q. 51. A transverse harmonic wave is propagating along a taut string. Tension in the string is 50 *N* and its linear mass density is 0.02 $kg\ m^{-1}$ The string is driven by a 80 *Hz* oscillator tied to one end oscillating with an amplitude of 1*mm*. The other end of the string is terminated so that all the wave energy is absorbed and there is no reflection

(a) Calculate the power of the oscillator.

(b) The tension in the string is quadrupled. What is new amplitude of the wave if the power of the oscillator remains same?

(c) Calculate the average energy of the wave on a 1.0 *m* long segment of the string.

Q. 52. A small steel ball of mass $m = 5g$ is dropped from a height of 2.0 *m* on a hard floor. 0.001% of its kinetic energy before striking the floor gets converted into a sound pulse having a duration of 0.4 *s*. Estimate how far away the sound can be heard if minimum audible intensity is $2.0 \times 10^{-8}$ $Wm^{-2}$ [Actually it is much less but to account for background sound we are assuming it to be high]. Assume no attenuation due to atmospheric absorption.

Q. 53. Three travelling waves are superimposed. The equations of the wave are

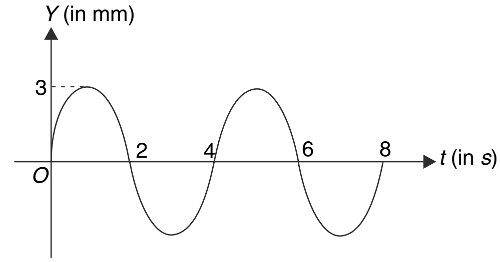
$y_1 = A_0 \sin(kx - \omega t)$, $y_2 = 3\sqrt{2}A_0 \sin(kx - \omega t + \phi)$ and $y_3 = 4A_0 \cos(kx - \omega t)$

Find the value of $\phi$ (given $0 \le \phi \le \pi/2$) if the phase difference between the resultant wave and first wave is $\pi/4$.

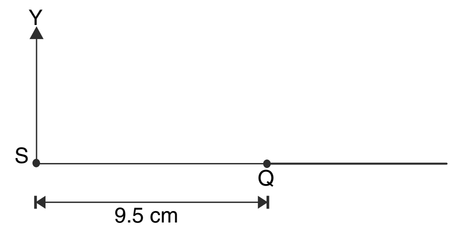
Q. 54. A sinusoidal wave having wavelength of 6 *m* propagates along positive *x* direction on a string. The displacement (*y*) of a particle at $x = 2$ *m* varies with time (*t*) as shown in the graph

(a) Write the equation of the wave

(b) Draw *y* versus *x* graph for the wave at $t = 0$



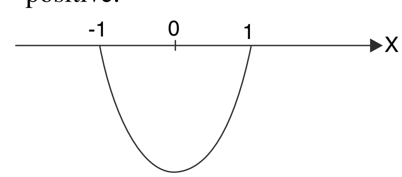

Q. 55. A string *SQ* is connected to a long heavier string at *Q*. Linear mass density of the heavier string is 4 times that of the string *SQ*. Length of *SQ* is 9.5 *cm*. Both the strings are subjected to same tension. *A* 50 *Hz* source connected at *S* produces transverse disturbance in the string. Wavelength of the wave in string *SQ* is observed to be 1 *cm*. If the source is put on at time $t = 0$, calculate the smallest time (*t*) at which we can find a particle in the heavier string that oscillates in phase with the source at *S*.



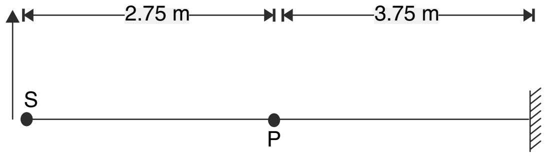

Q. 56. The figure shows the snapshot at time $t = 0$ of a transverse pulse travelling on a string in positive *x* direction.

(a) Sketch the pulse at a slightly later time.

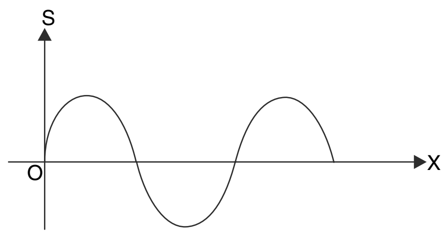
(b) With the help of the given sketch draw a graph of velocity of each string segment versus position. Take upward direction as positive.




Q. 57. A uniform string of length 6.5 *m* is subjected to a tension of 40 *N*. Mass of the string is 162.5 *g*. One end of the string is fixed and the other end is tied to a source (*s*). which produces a transverse oscillation. The displacement of the end of the string tied to the source can be expressed as $y = (3\ mm)\sin(40\pi t)$, where '*t*' is time. Find the displacement of point *P* of the string at a distance of 3.75 *m* from the fixed end, at time $t = 0.3$ *s*.




Q. 58. A longitudinal harmonic wave is travelling along positive $x$ direction. The amplitude, wavelength and frequency of the wave are $8.0 \times 10^{-3}\, m$, 12 $cm$ and 6800 $Hz$ respectively. The displacement ($s$) versus position graph for particles on the $x$ axis at an instant of time has been shown in figure. Find the separation at the instant shown, between the particles which were originally at $x_1 = 1\ cm$ and $x_2 = 3\ cm$




Q. 59. A sinusoidal wave $y = a\sin\left(\frac{2\pi}{\lambda}x - \omega t\right)$ is travelling on a stretched string. An observer is travelling along positive $x$ direction with a velocity equal to that of the wave. Find the angle that the velocity of a particle on the string at $x = \frac{\lambda}{6}$ makes with $-x$ direction as seen by the observer at time $t = 0$.

Q. 60. A standing wave $y = A \sin kx . \cos \omega t$ is established in a string fixed at its ends.

(a) What is value of instantaneous power transfer at a cross section of the string when the string is passing through its mean position?

(b) What is value of instantaneous power transfer at a cross section of the string when the string is at its extreme position?

(c) At what frequency is the power transmitted through a cross section changing with time?

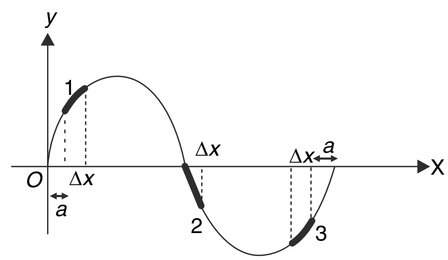
Q. 61. A sinusoidal transverse wave of small amplitude is travelling on a stretched string. The wave equation is $y = a \sin(k x - \omega t)$ and mass per unit length of the string is $\mu$. Consider a small element of length $\Delta x$ on the string at $x = 0$. Calculate the elastic potential energy stored in the element at time $t = 0$. Also write the kinetic energy of the element at $t = 0$.

Q. 62. The figure shows the $y - x$ graph at an instant for a small amplitude transverse wave travelling on a stretched string. Three elements (1, 2 and 3) on the string have equal original lengths ($= \Delta x$). At the given instant-

(i) which element (among 1, 2 and 3) has largest kinetic energy?

(ii) which element has largest energy (i.e., sum of its kinetic and elastic potential energy)

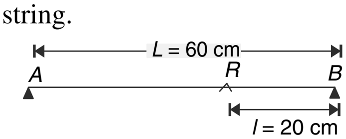
(iii) Prove that energy per unit length $\frac{\Delta E}{\Delta x}$ of the string is constant everywhere equal to $T\left(\frac{\partial y}{\partial x}\right)^2$ where $T$ is tension the string.




Q. 63. A string has linear mass density $\mu = 0.1\ kg / m$. A $L = 60\ cm$ segment of the string is clamped at $A$ and $B$ and is kept under a tension of $T = 160\ N$ [The tension providing arrangement has not been shown in the figure]. A small paper rider is placed on the string at point $R$ such that $BR = 20\ cm$. The string is set into vibrations using a tuning fork of frequency $f$.

(a) Calculate all values of $f$ below 1000 $Hz$ for which the rider will not vibrate at all.

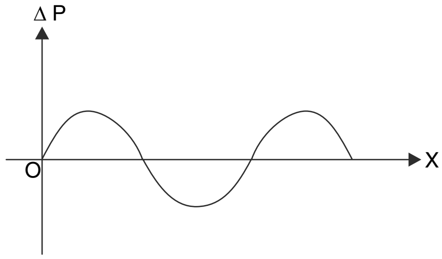
(b) Calculate all values of $f$ below 1000 $Hz$ for which the rider will have maximum oscillation amplitude among all points on the string.



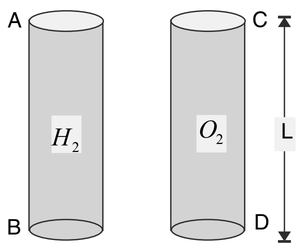

Q. 64. A sinusoidal longitudinal wave is travelling in positive $x$ direction. Wave length of the wave is 0.5 $m$ At time $t = 0$, the change in pressure at various points on the $x$ axis can be represented as shown in figure. Consider five particles of the medium $A$, $B$, $C$, $D$ and $E$ whose $x$ co-ordinates are 0.125 $m$, 0.1875 $m$, 0.250 $m$, 0.375 $m$ and 0.50 $m$ respectively.

(a) Which of the above mentioned five particles of the medium are moving in positive $x$ direction at $t = 0$.

(b) Find the ratio of speed of particles $B$ and $D$ at $t = 0$.




**Q. 65.** (i) Two cylindrical pipes are each of length $L = 30\ cm$. One of them contains hydrogen and the other has oxygen at the same temperature. The ends $A$, $B$, $C$ and $D$ of the pipes are fitted with flexible diaphragms. The diaphragms $A$ and $C$ are set into oscillations simultaneously using the same source having frequency $f = 600\ Hz$. Calculate the difference in phase of oscillations of the diaphragms $D$ and $B$ if it is known that the speed of sound in hydrogen at the temperature concerned is 1200 $m/s$.




(ii) The air column in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 $Hz$. The speed of sound in air is 330 $m/s$. $P_0$ is mean pressure in the pipe and $\Delta P_0$ is maximum amplitude of pressure variation. Neglect end correction.

(a) Find the length $L$ of air column.

(b) What is amplitude of pressure variation at the middle of the column?

(c) What is maximum and minimum pressure at the closed end?

**Q. 66.** Speed of sound in atmosphere at a height $h_0$ is 1080 $km\ hr^{-1}$. The variation of temperature and pressure of the atmosphere with height $h$ from the surface is given by

$$T = T_0 - \beta h \text{ and } P = P_0\left(1 - \frac{\beta h}{T_0}\right)^{\frac{Mg}{R\beta}}$$

Where $T_0$ = temperature at the surface of the earth = 273 $K$,

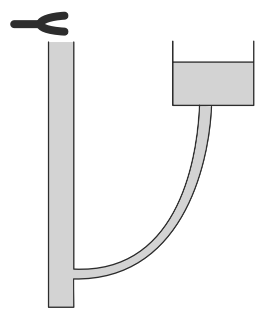
$P_0$ = atmospheric pressure at the surface of the earth,

$\beta$ = 0.006 °C/$m$, $M$ = molar mass of air ≃ 29 $g$ mol$^{-1}$, $g = 9.8\ ms^{-2}$

$R$ = gas constant = 8.31 $J$ mol$^{-1}$ $K^{-1}$

Consider air to be a mixture of diatomic gases and calculate the atmospheric temperature and pressure at height $h_0$. Also find $h_0$. Take $(0.82)^{5.7} = 0.32$.

**Q. 67.** In resonance column experiment a tuning fork of frequency $f = 400\ Hz$ is held above the pipe as shown in figure. The reservoir is raised and lowered to change the level of water and thus the length of the column of air in the tube. The area of cross section of the reservoir is 6 times that of the pipe. Initially, the reservoir is kept so that the pipe is full up to the brim. Tuning fork is sounded and the reservoir is lowered. When the reservoir is lowered by 21 $cm$, first resonance is recorded. When the reservoir is lowered further by 49 $cm$ the second resonance is heard. Find the speed of sound in air.



**Q. 68.** (i) In a travelling sinusoidal longitudinal wave, the displacement of particle of medium is represented by $s = S(x, t)$. The midpoint of a compression zone and an adjacent rarefaction zone are represented by letter '$C$' and '$R$' respectively. The difference in pressure at '$C$' and '$R$' is $\Delta P$ and the bulk modulus of the medium is $B$.

(a) How is $\left|\frac{\partial s}{\partial x}\right|_C$ related to $\left|\frac{\partial}{\partial}\right|$

(b) Write the value of $\left|\frac{\partial s}{\partial x}\right|_C$ in terms of $\Delta P$ and $B$.

(c) What is speed of a medium particle located mid-way between '$C$' and '$R$'.

(ii) A standing wave in a pipe with a length of $L = 3\ m$ is described by $s = A\cos\left(\frac{3\pi x}{L}\right)\sin\left(\frac{3\pi vt}{L}\right)$ where $v$ is wave speed. The atmospheric pressure and density are $P_0$ and $\rho$ respectively.

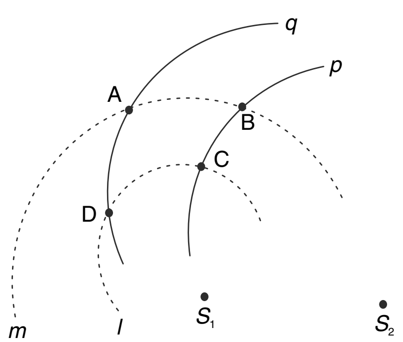
(a) At $t = \frac{L}{18v}$ the acoustic pressure at $x = \frac{L}{2}$ is 0.2 percent of the atmospheric pressure . Find the displacement amplitude $A$.

(b) In which overtone is the pipe oscillating?

Q. 69. Two sources $A$ and $B$ give out sound waves in coherence and in phase. The sources are located at co-ordinates (0, 0) and (0, 9 $m$) in $xy$ plane. There is a detector located at (40 $m$, 0). It was found that the detector records continuous increase in intensity of sound when it is moved in positive $y$-direction for 4.5 $m$ but the intensity was found to fall for some distance when it is moved in negative $y$ direction. What frequency of sound is consistent with these observations? Speed of sound = 340 $ms^{-1}$.

Q. 70. In the figure shown, $S_1$ and $S_2$ are two identical point sources of sound which are coherent 180° out of phase. Taking $S_1$ as centre, two circular arcs $\ell$ and $m$ of radii 1 $m$ and 2 $m$ are drawn. Taking $S_2$ as centres, two circular arcs $p$ and $q$ are drawn having radii 2 $m$ and 4 $m$ respectively. Out of the four intersection points $A$, $B$, $C$ and $D$ which point will record maximum intensity and which will record the least intensity of sound?

It is given that wavelength of wave produced by each source is 4.0 $m$.



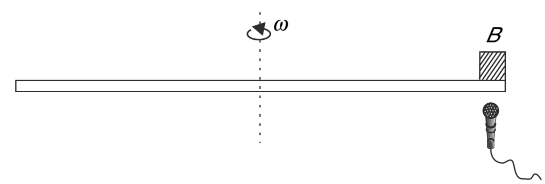

Q. 71. Stationary wave of frequency 5 $K\,Hz$ is produced in a tube open at both ends and filled with air at 300 $K$. The tube is oscillating in its first overtone mode.

(a) Find the length of the tube assuming that air contains only nitrogen and oxygen in molar ratio of 3 : 1.

(b) What shall be the frequency of sound wave used so that the same tube oscillates in its second overtone mode?

Q. 72. The string of a musical instrument was being tuned using a tuning fork of known frequency, $f_0$ = 1024 $Hz$. The tuning fork and the string were set to vibrate together. Both vibrated together for 10 s and no beat was heard. What prediction can be made regarding the frequency of the string?

Q. 73. A wooden platform can be rotated about its vertical axis with constant angular speed $\omega$ with the help of a motor. A buzzer is fixed at the circumference of the platform and it rotates in a circle of radius $R$. The buzzer produces sound of frequency $f_0$. A mic is placed just beneath the platform near its circumference. An electronic frequency analyzer connected to the mic records the frequency ($f$) received by the mic. Take time ($t$) to be zero when the buzzer is just above the mic and express $f$ as a function of time. Plot $f$ versus $t$. Speed of sound = $V_0$.




Q. 74. (i) A harmonic wave in a stationary medium is represented by $y = a \sin (kx - \omega t)$. Write the equation of this wave for an observer who is moving in negative x direction with constant speed $v_0$.

(ii) The Doppler flow meter is a device that measures the speed of blood flow, using transmitting and receiving elements that are placed directly on the skin. The transmitter emits a continuous sound wave whose frequency is 5 $M\,Hz$. When the sound is reflected from the red blood cells, its frequency is changed in a kind of Doppler effect. The cells are moving with the same velocity as the blood. The receiving element detects the reflected sound, and an electronic counter measures its frequency, which is Doppler-shifted relative to the transmitter frequency. From the change in frequency the speed of the blood flow can be determined. Typically, the change in frequency is around 600 $Hz$ for flow speeds of about 0.1 $m/s$. Assume that the red blood cell is directly moving away from the source and the receiver.

(a) Estimate the speed of the sound wave in the blood?

(b) A segment of artery is narrowed down by plaque to half the normal cross-sectional area. What will be the Doppler change in frequency due to reflection from the red blood cell in that region?

Q. 75. A sound source emits waves of frequency $f_0$ and wavelength $\lambda_0$ in still air. When there is a wind blowing with speed $u$ from left to right what will be wavelength of sound to the right of the source and to the left of the source.

Q. 76. There are two horns $H1$ and $H2$ in a car. When sounded together, the driver records 35 beats in 10 second. With horn $H2$ blowing and car moving towards a wall at a speed of 5 $ms^{-1}$, the driver noticed a beat frequency of 5 $Hz$ with the echo. When frequency of $H1$ is decreased the beat frequency with two horns sounded together increases. Calculate the frequency of two horns. Speed of sound $= 332\ ms^{-1}$

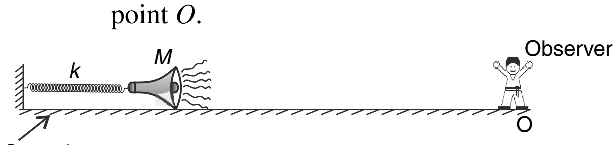
Q. 77. A toy train in a children amusement park runs on an elliptical orbit having major and minor axis in the ratio of 4 : 3. The length of the train is exactly equal to half the perimeter of the elliptical track. The train is travelling at a constant speed of 20 $ms^{-1}$. The engine sounds a whistle when its acceleration is minimum. The whistle has a frequency of $f_0 = 3460\ Hz$ and speed of sound in air is $V = 330\ ms^{-1}$

(a) What frequency of whistle is received by a passenger in the last compartment of the train?

(b) What frequency of whistle is received by a passenger sitting in the central compartment of the train?

Q. 78. A small source of sound has mass $M$ and is attached to a spring of force constant $K$. It is oscillating with amplitude $A = \dfrac{V}{20}\sqrt{\dfrac{M}{K}}$ where $V$ is speed of sound in air. The source of sound produces a sound of frequency $f_0 = 399\ Hz$.

(a) Find the frequency of sound registered by a stationary observer standing at a distant point $O$.




(b) Let $\Delta t_1$ be the time interval during which the registered frequency changes from 420 $Hz$ to $\left(399 \times \dfrac{40}{39}\right) Hz$ and $\Delta t_2$ be the time interval during which the observed frequency changes from 399 $Hz$ to $\left(399 \times \dfrac{40}{41}\right) Hz$. Which is larger $\Delta t_1$ or $\Delta t_2$?

Q. 79. (i) A straight railway track is at a distance '$d$' from you. A distant train approaches you travelling at a speed $u$ (< speed of sound) and crosses you. How does the apparent frequency ($f$) of the whistle change with time ($f_0$ is the original frequency of the whistle). Draw a rough $f$ vs $t$ graph.

(ii) A bat is tracking a bug. It emits a sound, which reflects off the bug. The bat hears the echo of the sound 0.1 seconds after it originally emitted it. The bat can tell if the insect is to the right or left by comparing when the sound reaches its right ear to when the sound reaches its left ear. Bat's ears are only 2 $cm$ apart. Bats also use the frequency change of the sound echo to determine the flight direction of the bug. While hovering in the air (not moving), the bat emits a sound of 40.0 $kHz$. The frequency of the echo is 40.4 $kHz$. Assume that the speed of sound is 340 $m/s$.

(a) How far away is the bug?

(b) How much time delay is there between the echo reaching the two ears if the bug is directly to the right of the bat?

(c) What is the speed of the bug?

Q. 80. A source of sound is located in a medium in which speed of sound is $V$ and an observer is located in a medium in which speed of sound is $2V$. Both the source and observer are moving directly towards each other at velocity $\dfrac{V}{5}$. The source has a frequency of $f_0$.

(a) Find the wavelength of wave in the medium in which the observer is located.

(b) Find the frequency received by the observer.

## LEVEL 3

Q. 81. A transverse wave $y = A \sin \omega \left(\dfrac{x}{V_1} - t\right)$ is travelling in a medium with speed $V_1$. Plane $x = 0$ is the boundary of the medium. For $x > 0$ there is a different medium in which the wave travels at a different speed $V_2$. Part of the wave is reflected and part is transmitted. For $x < 0$ the wave function is described as $y_- = A_1 \sin\omega\left(\frac{x}{V_1} - t\right) + A_2 \sin\omega\left(\frac{x}{V_2} + t\right)$, while

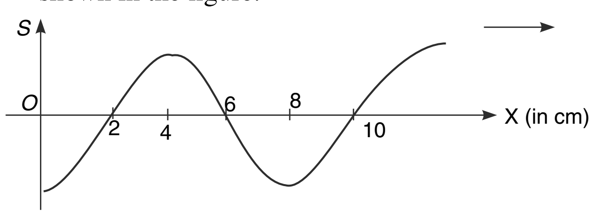
for $x > 0$ ; $y_+ = A_3 \sin\omega\left(\frac{x}{V_2} - t\right)$

(a) Using the fact that the wave function must be continuous at $x = 0$, show that $A_1 - A_2 = A_3$

(b) Using the fact that $\frac{\partial y}{\partial x}$ must be continuous at $x = 0$, prove that $\frac{V_2}{V_1} A_1 = A_3 - A_2$

(c) Show that $A_3 = \frac{2V_2 A_1}{V_1 + V_2}$ and $A_2 = \left(\frac{V_1 - V_2}{V_1 + V_2}\right) A_1$

Q. 82. A longitudinal wave is travelling at speed $u$ in positive $x$ direction in a medium having average density $\rho_0$. The displacement ($s$) for particles of the medium versus their position ($x$) has been shown in the figure.




Answer following questions for $0 < x \le 10$ cm

(a) Write $x$ co-ordinates of all positions where the particles of the medium have maximum negative acceleration. What is density at these locations – higher than $\rho_0$, less than $\rho_0$ or equal to $\rho_0$?

(b) Write $x$ co-ordinates of all locations where the particles of the medium have negative maximum velocity. What do you think about density at these positions?

(c) Knowing that the change in density ($\Delta\rho$) is proportional to negative of the slope of $s$ versus $x$ graph, prove that $\frac{d\rho}{dx} \propto -a$, where $a$ is acceleration of the particles at position $x$. At which point $(0 < x \le 10)$ is $\frac{d\rho}{dx}$ positive maximum.

Q. 83. Two sound waves, travelling in same direction can be represented as

$$y_1 = (0.02\,mm)\sin\left[\left(400\pi\,rad\,s^{-1}\right)\left(\frac{x}{330ms^{-1}} - t\right)\right]$$

And

$$y_2 = (0.02\,mm)\sin\left[\left(404\pi\,rad\,s^{-1}\right)\left(\frac{x}{330ms^{-1}} - t\right)\right]$$

The waves superimpose.

(i) Find distance between two nearest points where an intensity maximum is recorded simultaneously.

(ii) Find the time gap between two successive intensity maxima at a given point.

Q. 84. There are three sinusoidal waves $A$, $B$ and $C$ represented by equations-

$$A \to y = A\sin kx;\ B \to y = \frac{A}{2}\sin 2kx;$$

$$C \to y = \frac{A}{2}\sin 3kx$$

(a) To get a waveform of nearly the shape given in fig (a) which of the two waves $B$ or $C$ shall be superimposed with wave $A$?

(b) To get a waveform close to that in fig (b) which of the two waves $B$ or $C$ shall be superimposed with $A$?

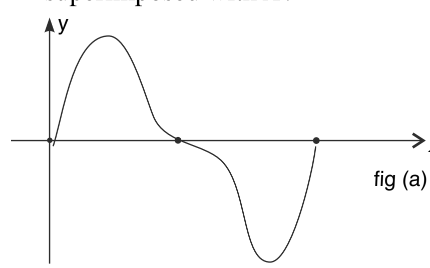



fig (a)

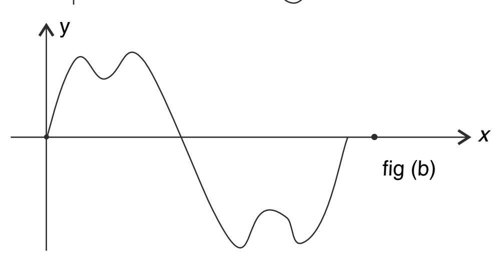
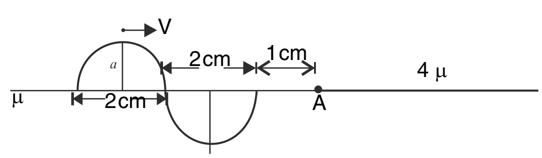



fig (b)

Q. 85. A taut string is made of two segments. To the left of A it has a linear mass density of $\mu$ $kg/m$ and to




the right of A its linear mass density is 4 $\mu$ $kg/m$.

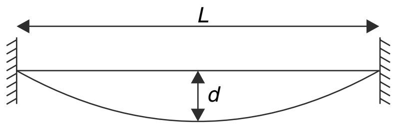
A sinusoidal pulse of amplitude $a$ is travelling towards right on the lighter string with a speed $V = 2\ cm/s$.

Draw the shape of the string after

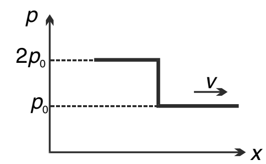
(a) 1 $s$

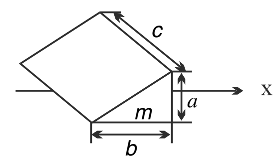
(b) 2.5 $s$

Q. 86. A wire having mass per unit length $\mu$ and length $L$ is fixed between two fixed vertical walls at a separation $L$. Due to its own weight the wire sags. The sag in the middle is $d$ ($<< L$). Assume that tension is practically constant along the wire, owing to its small mass. Calculate the speed of the transverse wave on the wire.




Q. 87. A shock wave is a region of high acoustic pressure propagating at speed of sound ($v$). Assume that the pressure in one such shock wave is $2P_0$ where $P_0$ is the atmospheric pressure. This shock wave is travelling horizontally along $x$ direction and hits a small wedge whose dimensions are as shown in the figure. The wedge has a mass m and is lying on a smooth horizontal surface. Determine the velocity $u$ acquired by the wedge immediately after the shock wave passes through it. The velocity acquired by the wedge should be assumed to be much lower than the velocity of the wave ($u << v$).




## ANSWERS

**1.** (a) The boy is jerking the string with gradually increasing frequency

(b) Particle '$a$'

**2.** 3.09 $cm$

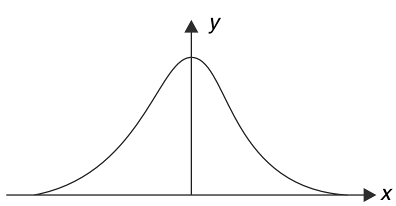
**3.** Three

**4.** (a) Yes

(b) Negative $x$ direction

(c) 0.6 unit

(d)




**5.** (a) $\frac{5}{6}\ cm$

(b) $-\frac{5}{3}\ cm\ s^{-1}$

**6.** Both satisfy the wave equation

**7.** 2.25 $m$

**8.** $\frac{2\pi}{5}$

**9.** $\frac{16}{17}\pi$

**10.** (a) 400 $N$

(b) $\frac{\pi^2}{25}\ J$

(c) $y = 2\sin\left(\frac{\pi x}{2} - 100\pi t - 30°\right)$

**11.** (a) $0.2\ I_0$

(b) $(3 + 2\sqrt{2})I_0$

(c) $3\ I_0$

**12.** $\sqrt{7}\ A$

**13.** Along path $m$ the loudness alternates between faint and loud due to phenomena of interference.

**14.** (i) 210 $Hz$

(ii) $\frac{\pi}{12}, \frac{\pi}{4}$

**15.** 15 $cm$, 4 $cm$, 135 $cm$

**16.** (i) $\pi$

(ii) $y = A\cos\frac{\pi x}{L}\cos\frac{2\pi}{T}t$

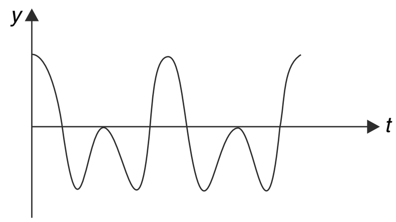
**17.** (a) $0.40\ m$, $6.4 \times 10^{-4}\ m$

(b) 376 $Hz$

**18.** (a) $\frac{L}{6}$

(b) 4

**19.** (a) $\frac{\lambda_1}{\lambda_2} = \frac{2}{1}$

(b) 


**20.** (i) zero

(ii) $2a$

**21.** 3.5

**23.** 696 $Hz$

**24.** Yes, sound cannot travel in free space.

**25.** 447.6 $m$

**27.** (a) 1320 $ms^{-1}$

(b) 4800 $K$

**28.** 39.5%

**29.** (a) $4k$

(b) No

**30.** 0.1 $\mu m$

**31.** (i) (a) 20 $cm$

(b) 2800 $Hz$

(ii) 321 $m/s$

**32.** 1414 $m/s$

**33.** (a) 0.1 $\mu m$

(b) $3.9 \times 10^4$ watt

**34.** $(x - V_0T)^2 + y^2 + z^2 = V^2 (t - T)^2$

**35.** (i) 40 $s$

(ii) (a) $f_2 = f_1 = \dfrac{2vf^2}{2vf - g}$

(b) $f_1 = f_2 = \dfrac{2vf^2}{2(v - \sqrt{gh})f - g}$

**36.** (i) (a) 1560

(b) $\dfrac{13}{9}$

(ii) (a) 1047 $Hz$

(b) 991 $Hz$

**37.** (a) 4125

(b) 1.6 $m$

**38.** (a) 900 $Hz$ to 1200 $Hz$

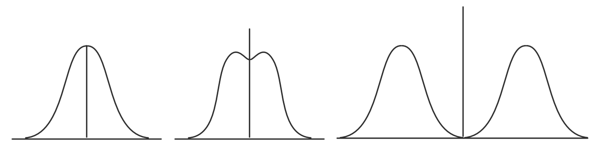
(b) 1080 $Hz$ to 1440 $Hz$

**39.** 6 $Hz$

**40.** (i) $A$ - 1 $B$ - 2 $C$ - 3

(ii) Less than 4 $Hz$

**41.**



**42.** (a) $y = 0.01 \sin [40\pi (t - 1) + 4(x - 1)]$

(b) $y = 0.01 \sin [40\pi (t - 1) - 4(x - 1)]$

**43.** $\dfrac{\pi}{\omega}$

**44.** 3

**45.** $y_1 = 0.25 \sin\left(\dfrac{\pi}{4}x + \dfrac{5\pi}{2}t + \dfrac{3\pi}{4}\right)$;

$y_2 = 0.25 \sin\left(\dfrac{\pi}{4}x - \dfrac{5\pi}{2}t + \dfrac{3\pi}{4}\right)$

**46.** (i) $\dfrac{16}{15}m$ and $\dfrac{16}{31}m$

(ii) $y =$ (0.1 $cm$) cos $[(20\pi m^{-1}x] + (10\pi s^{-1})t]$

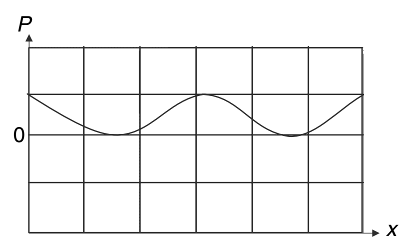
**47.** (i) (a) $\sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos(2kx + \delta)}$

(b) $\dfrac{A_{\max}}{A_{\min}} = \dfrac{a_1 + a_2}{|a_1 - a_2|}$

(ii) (a) zero

(b) zero

**48.**




**49.** (a) 98.4 $N$

(c) 4.2 $Hz$ decrease.

**50.** $\dfrac{\ln 2}{k}$

**51.** (a) 0.126 $W$

(b) $\dfrac{1}{\sqrt{2}}mm$

(c) 2.5 $mJ$

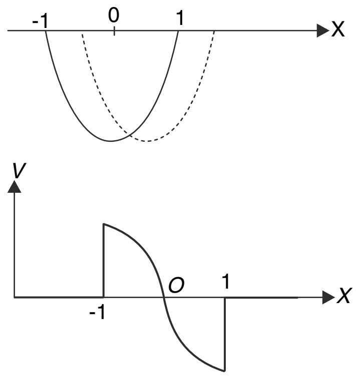
**52.** 3.15 $m$

**53.** $\phi = \dfrac{\pi}{12}$

**54.** (a) $y = (3mm)\sin\left(\frac{\pi}{3}x - \frac{\pi}{2}t + \frac{\pi}{3}\right)$

**55.** 0.2 *s*

**56.**




**57.** Zero

**58.** 2.4 *cm*

**59.** $\theta = \tan^{-1}\left(\frac{a\pi}{\lambda}\right)$

**60.** (a) Zero

(b) Zero

(c) $\frac{\omega}{\pi}$

**61.** $\Delta U = \frac{1}{2}\mu a^2\omega^2\Delta x$ ; $\Delta k = \frac{1}{2}\mu a^2\omega^2\Delta x$

**62.** (i) 2

(ii) 2

**63.** (a) 100, 200, 300.........900 *Hz*

(b) It is not possible to have antinode at *R*

**64.** (a) *A* and *B*

(b) $\frac{1}{\sqrt{2}}$

**65.** (i) 0.9 $\pi$ *rad*

(ii) (a) 93.75 *cm*

(b) $\frac{\Delta P_0}{\sqrt{2}}$

(c) $P_{max} = P_0 + \Delta P_0$ ; $P_{min} = P_0 - \Delta P_0$

**66.** $T = 224\ K$, $P = 0.32\ P_0$, $h_0 = 8167\ m$

**67.** 336 $ms^{-1}$

**68.** (i) (a) $\left|\frac{\partial s}{\partial x}\right|_C = \left|\frac{\partial s}{\partial x}\right|_R$

(b) $\left|\frac{\partial s}{\partial x}\right|_C = \frac{\Delta P}{2B}$

(c) Zero

(ii) (a) $\frac{0.0004 L P_0}{3\pi v^2 \rho}$,

(b) Second

**69.** $f < 170\ Hz$

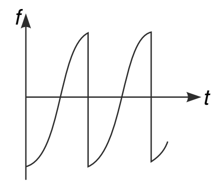
**70.** Maximum at *C* and least at *B*

**71.** (a) 6.9 *cm*

(b) 7543 *Hz*.

**72.** $1023.9\ Hz < f_{string} < 1024.1\ Hz$

**73.**




**74.** (i) $y_0 = a \sin[(\omega + kv_0)t - kx]$

(ii) (a) 1700 *m/s*

(b) 1200 *Hz*

**75.** $\lambda_0 + \frac{u}{f_0}$ ; $\lambda_0 - \frac{u}{f_0}$

**76.** $f_1 = 160\ Hz, f_2 = 163.5\ Hz$

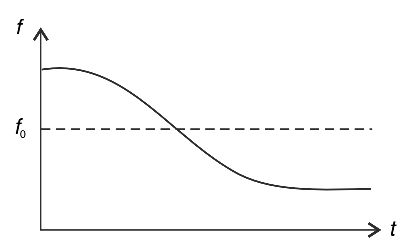
**77.** (a) 3460 *Hz*

(b) 3420 *Hz*

**78.** (a) $380\ Hz \le f \le 420\ Hz$

(b) $\Delta t_1 > \Delta t_2$

**79.** (i)




(ii) (a) 17 *m*

(b) $5.9 \times 10^{-5}\ s$

(c) 1.7 *m/s*

**80.** (a) $\frac{8}{5}\frac{V}{f_0}$

(b) $\frac{11}{8}f_0$

**82.** (a) $x = 4$; equal to $\rho_0$

(b) $x = 2, x = 10$ ; $\Delta\rho$ is maximum negative

(c) $x = 4$.

**83.** (i) 150 $m$

(ii) 0.5 $s$

**84.** (a) $B$

(b) $C$

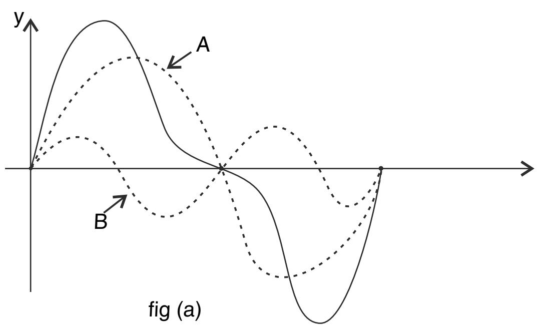


fig (a)

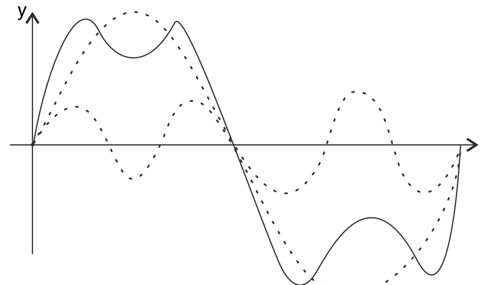
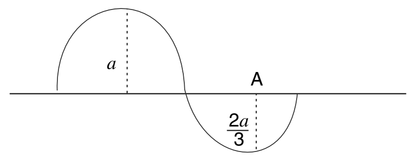
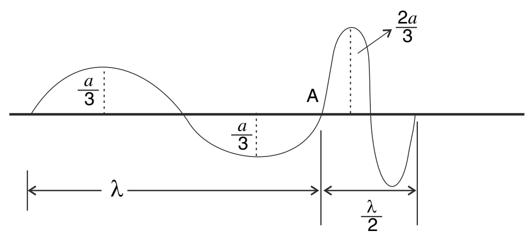


fig (b)

**85.** (a)




(b)




**86.** $v = L\sqrt{\frac{g}{8d}}$

**87.** $\frac{P_0 abc}{2mv}$

## SOLUTION

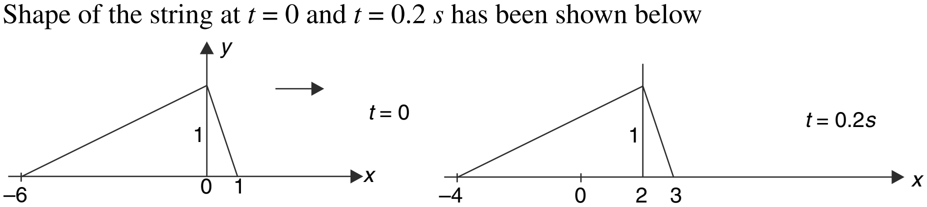
**2.** The wavelength is = 9 $cm$

The particle makes 2.75 oscillations in 8 $s$, hence time period is $T = \frac{8}{2.75} = 2.91\ s$

Wave speed, $v = \frac{\lambda}{T} = \frac{9}{2.91} = 3.09\,cms^{-1}$

**3.** All particles perform $SHM$. Their acceleration is always towards their mean position. Velocity of 3, 4 and 5 are towards mean position. This can be observed by looking at the direction of wave and predicting the position of a particle a moment later.

**5.** Shape of the string at $t = 0$ and $t = 0.2\ s$ has been shown below




(a) From the diagram

$y = \frac{5}{6}\,cm$ at $x = 1\ cm$

(b) Velocity of the particle